ANOMALY CANCELLATION ON D-BRANES AND O-PLANES

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ANOMALY INFLOW MECHANISM

Anomalies

Anomalies depend on characteristic classes of the tangent and gauge bundles. In units of 2π:

$$\mathrm{ch}(F) = \mathrm{tr}\,\exp iF$$

$$\widehat{A}(R) = \prod_{a=1}^{D/2} \frac{\lambda_a/2}{\sinh \lambda_a/2}\,, \quad \widehat{L}(R) = \prod_{a=1}^{D/2} \frac{\lambda_a}{\tanh \lambda_a}\,, \quad e(R) = \prod_{a=1}^{D/2} \lambda_a$$

The anomaly $\mathcal{A}$ has to satisfy the WZ consistency condition. This implies that it is the WZ descent of some closed form $I(F,R)$. Defining $I = dI^{(0)}$ and $\delta I^{(0)} = dI^{(1)}$, one has:

$$\boxed{\mathcal{A} = 2\pi i \int I^{(1)}}$$

Inflow

It can happen that a consistent theory admits as vacuum a top. defect carrying chiral zero modes. The anomaly arising on the world-volume must be canceled by an inflow from the bulk.

Callan, Harvey

This is the case of consistent superstring vacua with D-branes and O-planes, where no anomaly can arise but zero modes occur.

In general, there can be a net world-volume quantum anomaly. By consistency, this must be canceled by an equal and opposite classical inflow of anomaly.

Classical anomalies arise in magnetic interactions. Consider some defects M_i in spacetime X, with the RR couplings:

$$S = -\sum_i \mu_i \int_{M_i} C \wedge Y_i$$

with $C = \sum_p C_{(p)}$ and $Y = Y(F, R)$.

This is written as an integral over X by using the currents τ_{M_i}. Locally, $\tau_{M_i} \sim \delta(x^{d_i})\, dx^{d_i} \wedge \ldots \wedge \delta(x^D)\, dx^D$, but globally τ_{M_i} is determined by $N(M_i)$. The RR action is then

$$S = -\frac{1}{2} \int_X H \wedge {}^*H - \sum_i \mu_i \int_X \tau_{M_i} \wedge \left(C - H \wedge Y_i^{(0)}\right)$$

For consistency, the total top-form charge must vanish, and the equation of motion and Bianchi identity are

$$d^*H = \sum_i \mu_i\, \tau_{M_i} \wedge Y_i$$

$$dH = -\sum_i \mu_i\, \tau_{M_i} \wedge \bar{Y}_i$$

Due to the modified Bianchi identity:

$$H = dC - \sum_i \mu_i\, \tau_{M_i} \wedge \bar{Y}_i^{(0)}$$

Since this must be gauge invariant, C must transform as

$$\delta C = \sum_i \mu_i\, \tau_{M_i} \wedge \bar{Y}_i^{(1)}$$

Consequently, the RR couplings are anomalous:

$$\mathcal{A} = -\,i \sum_{i,j} \mu_i \, \mu_j \int_X \tau_{M_i} \wedge \tau_{M_j} \wedge \left(Y_i \wedge \bar{Y}_j \right)^{(1)}$$

The magnetic interaction of M_i and M_j has therefore an anomaly localized on the intersections M_{ij}. This follows from the property

$$\tau_{M_i} \wedge \tau_{M_j} = \tau_{M_{ij}} \wedge e[N(M_{ij})]$$

Finally, the classical anomaly inflow on each intersection M_{ij} can be written as $\mathcal{A}_{ij} = 2\pi i \int_{M_{ij}} I_{ij}^{(1)}$ in terms of

$$\boxed{I_{ij} = -\frac{\mu_i \, \mu_j}{2\pi} Y_i \wedge \bar{Y}_j \wedge e[N(M_{ij})]}$$

and must cancel the corresponding quantum anomaly.

Green, Harvey, Moore; Cheung, Yin

ANOMALY CANCELLATION ON D-BRANES AND O-PLANES

Consider two parallel Dp-branes and/or Op-planes on M. The anomalous fields living on their world-volumes can be read from the potentially divergent one-loop amplitudes:

BB : Annulus $\Rightarrow$ Chiral R spinors in the adjoint

BO : Möbius strip $\Rightarrow$ Chiral R spinors in the fundamental

OO : Klein bottle $\Rightarrow$ Self-dual RR forms

These fields are dimensionally reduced from $D = 10$ to $D = p + 1$. Chirality and anomalies only when $N(M)$ is non-trivial.

The anomalies for these fields can be computed à la Fujikawa. They are topological indices, which can be computed using index theorems or via a path-integral representation in SQM.

Alvarez-Gaumé, Witten

Anomaly for a reduced chiral spinor

The anomaly of a chiral spinor reduced from X to M is

$$\mathcal{A} = \lim_{t\to 0} \mathrm{Tr}\left[\Gamma^{D+1}\,\delta\, e^{-t(i\not{D})^2}\right]$$

By exponentiating δ, this can be written as $\mathcal{A} = 2\pi i Z^{(1)}$, where

$$Z = \lim_{t\to 0} \mathrm{Tr}\left[\Gamma^{D+1}\, e^{-t(i\not{D})^2}\right]$$

Mathematically, Z is the index of a twisted spin complex:

$$Z = \mathrm{index}(i\not{D})$$

A (anti-)chiral spinor on X is a section of $S^{\pm}_{T(X)}$. On $M \subset X$, these decompose into

$$E^{\pm} = \left(S^{\pm}_{T(M)} \otimes S^{+}_{N(M)}\right) \oplus \left(S^{\mp}_{T(M)} \otimes S^{-}_{N(M)}\right)$$

Considering also a gauge bundle, we have the two-term complex

$$i\not{D}: \ \Gamma\Big[M, E^{+} \otimes V\Big] \to \Gamma\Big[M, E^{-} \otimes V\Big]$$

The index theorem reads

$$\mathrm{index}(i\not{D}) = \int_M \mathrm{ch}(V)\, \mathrm{ch}(E^{+} \ominus E^{-})\, \frac{\mathrm{Td}[T(M^C)]}{e[T(M)]}$$

Explicit evaluation yields

$$Z = \int_M \mathrm{ch}(F) \wedge \frac{\widehat{A}(R)}{\widehat{A}(R')} \wedge e(R')$$

Physically, Z is a partition function. If we find some SQM with $Q = i\not{D}$ and $(-1)^F = \Gamma^{D+1}$, then Z becomes a Witten index:

$$Z = \mathrm{Tr}\left[(-1)^F\, e^{-tH}\right]$$

The appropriate SQM model is obtained by dimensionally reducing the SNSM with ($M, N, \ldots$: X, $\mu, \nu, \ldots$: M, $i, j, \ldots$: N):

$$x^i = 0$$

$$\psi_1^{\mu} = \psi_2^{\mu} = \psi^{\mu}\ , \quad \psi_1^{i} = -\psi_2^{i} = \psi^{i}$$

The Lagrangian is:

$$L = \frac{1}{2} g_{\mu\nu}\, \dot{x}^\mu \dot{x}^\nu + \frac{i}{2}\psi_{\underline{\mu}}\left(\dot{\psi}^{\underline{\mu}} + \omega_\rho{}^{\underline{\mu}}{}_{\underline{\nu}}\, \dot{x}^\rho\, \psi^{\underline{\nu}}\right)$$
$$+ \frac{i}{2}\psi_{\underline{i}}\left(\dot{\psi}^{\underline{i}} + \omega_\rho{}^{\underline{i}}{}_{\underline{j}}\, \dot{x}^\rho\, \psi^{\underline{j}}\right) + \frac{1}{4} R_{\underline{\mu}\underline{\nu}\underline{i}\underline{j}}\, \psi^{\underline{\mu}}\psi^{\underline{\nu}}\psi^{\underline{i}}\psi^{\underline{j}}$$

Gauge backgrounds can be taken into account as in the standard case, through additional terms.

Due to $(-1)^F$, all the fields are periodic and

$$Z = \int_P \mathcal{D}x^\mu \int_P \mathcal{D}\psi^{\underline{\mu}} \int_P \mathcal{D}\psi^{\underline{i}}\; e^{-S}$$

For $t \to 0$, Z is dominated by constant paths:

$$x^\mu = x_0^\mu + \xi^\mu$$
$$\psi^{\underline{\mu}} = \psi_0^{\underline{\mu}} + \lambda^{\underline{\mu}}\ , \quad \psi^{\underline{i}} = \psi_0^{\underline{i}} + \lambda^{\underline{i}}$$

It is enough to keep quadratic interactions and only terms with the maximum number of ψ_0's. Introducing also a gauge background, one finds:

$$L^{eff} = \frac{1}{2}\left(\dot{\xi}_{\underline{\mu}}\dot{\xi}^{\underline{\mu}} + i\lambda_{\underline{\mu}}\dot{\lambda}^{\underline{\mu}} + i\lambda_{\underline{i}}\dot{\lambda}^{\underline{i}} + iR_{\underline{\mu}\underline{\nu}}\,\dot{\xi}^{\underline{\mu}}\xi^{\underline{\nu}} + R'_{\underline{i}\underline{j}}\,\lambda^{\underline{i}}\lambda^{\underline{j}}\right)$$
$$+ \frac{1}{2} R'_{\underline{i}\underline{j}}\,\psi_0^{\underline{i}}\psi_0^{\underline{j}} + iF$$

where

$$R_{\underline{\mu}\underline{\nu}} = \frac{1}{2} R_{\underline{\mu}\underline{\nu}\underline{\rho}\underline{\sigma}}(x_0)\,\psi_0^{\underline{\rho}}\psi_0^{\underline{\sigma}}\ , \quad R'_{\underline{i}\underline{j}} = \frac{1}{2} R_{\underline{i}\underline{j}\underline{\rho}\underline{\sigma}}(x_0)\,\psi_0^{\underline{\rho}}\psi_0^{\underline{\sigma}}$$
$$F = \frac{1}{2} F_{\underline{\mu}\underline{\nu}}(x_0)\,\psi_0^{\underline{\mu}}\psi_0^{\underline{\nu}}$$

Evaluating the path-integral one finds:

$$Z = \int dx_0^\mu \int d\psi_0^\mu \,\mathrm{tr}\exp\{iFt\}\, \overbrace{\frac{\det_P(i\eta_{\mu\nu}\partial_\tau)}{\det_P(\eta_{\mu\nu}\partial_\tau^2 + iR_{\mu\nu}\partial_\tau)}}^{(2\pi t)^{-\frac{d}{2}} \prod_{a=1}^{d/2} \frac{\lambda_a t/2}{\sinh \lambda_a t/2}}$$

$$\underbrace{\det_P(i\eta_{ij}\partial_\tau + R'_{ij})}_{\prod_{a=1}^{d/2} \frac{\sinh \lambda'_a t/2}{\lambda'_a t/2}} \; \underbrace{\int d\psi_0^i \, \exp\left\{\frac{t}{2} R'_{ij}\psi_0^i\psi_0^j\right\}}_{\prod_{a=d/2}^{D/2} \lambda'_a t}$$

Finally, one obtains:

$$\boxed{Z = \int_M \mathrm{ch}(F) \wedge \frac{\hat{A}(R)}{\hat{A}(R')} \wedge e(R')}$$

Cheung, Yin; Scrucca, Serone

Anomaly for a reduced self-dual tensor

The anomaly of a self-dual tensor reduced from X to M can be written as

$$A = \frac{1}{4} \lim_{t \to 0} \mathrm{Tr}\left[I *_D \delta\, e^{-t\mathcal{D}^2}\right]$$

where $*_D$ is the Hodge operator and $\mathcal{D} = d + d^\dagger$ on all of X. The dynamics is constrained to $M \subset X$ thanks to the transverse reflection I.

By exponentiating δ, this can be written as $\mathcal{A} = 2\pi i Z^{(1)}$, with

$$Z = -\frac{1}{8} \lim_{t\to 0} \mathrm{Tr}\left[I *_D e^{-t\mathcal{D}^2} \right]$$

Mathematically, Z is a G-index of the signature complex:

$$Z = -\frac{1}{8}\,\mathrm{index}(\mathcal{D}_+^G)$$

More precisely, we have:

$$\mathcal{D}_+ : \ \Gamma\Big[X, {}^{+}\!\wedge T^*X\Big] \longrightarrow \Gamma\Big[X, {}^{-}\!\wedge T^*X\Big]$$

$$G : X \longrightarrow X \ \Big(I : (x^\mu, x^i) \longrightarrow (x^\mu, -x^i) \Big)$$

$G = \mathbf{Z}_2$ is orientation-preserving since D and d must be even. It leaves $M \subset X$ fixed and acts as $+\mathbf{1}$ in $T(M)$ and $-\mathbf{1}$ in $N(M)$.

The G-signature theorem gives:

$$\mathrm{index}(\mathcal{D}_+^G) = \int_M \frac{\mathrm{ch}(E^+ \ominus E^-)\,\mathrm{ch}(F^+ \ominus F^-)}{\mathrm{ch}(F)} \frac{\mathrm{Td}[T(M^C)]}{e[T(M)]}$$

where

$$E^\pm = {}^{\pm}\!\wedge T^*M \ , \quad F^\pm = {}^{\pm}\!\wedge N^*M$$

$$F = \oplus_i (-1)^i \wedge^i N^*M$$

By explicit evaluation one finds:

$$Z = -\frac{1}{8} \int_M \frac{\widehat{L}(R)}{\widehat{L}(R')} \wedge e(R')$$

Physically, Z is again a partition function. We need a SQM with $H = \mathcal{D}^2$ and a symmetry $\Omega = *_D$, so that Z becomes a SUSY index:

$$Z = -\frac{1}{8}\mathrm{Tr}\left[I\,\Omega\,e^{-tH}\right]$$

The appropriate SQM is the trivial dimensional reduction of the SNSM:

$$L = \frac{1}{2}g_{MN}(x)\dot{x}^M\dot{x}^N + \frac{i}{2}\sum_{\alpha=1,2}\psi_{\alpha\underline{M}}\Big(\dot{\psi}_\alpha^{\underline{M}} + \omega_M{}^{\underline{M}}{}_{\underline{N}}(x)\psi_\alpha^{\underline{N}}\,\dot{x}^M\Big)$$
$$+\frac{1}{4}R_{\underline{MNPQ}}(x)\psi_1^{\underline{M}}\psi_1^{\underline{N}}\psi_2^{\underline{P}}\psi_2^{\underline{Q}}$$

where:

$$\Omega : (\psi_1, \psi_2) \longrightarrow (-\psi_1, \psi_2)$$
$$I : (x^\mu, x^i; \psi_\alpha^{\underline{\mu}}, \psi_\alpha^{\underline{i}}) \longrightarrow (x^\mu, -x^i; \psi_\alpha^{\underline{\mu}}, -\psi_\alpha^{\underline{i}})$$

Due to $\Omega\,I$, the fields acquire non-standard periodicities and

$$Z = -\frac{1}{8}\int_P \mathcal{D}x^\mu \int_A \mathcal{D}x^i \int_P \mathcal{D}\psi_1^{\underline{\mu}} \int_A \mathcal{D}\psi_1^{\underline{i}} \int_A \mathcal{D}\psi_2^{\underline{\mu}} \int_P \mathcal{D}\psi_2^{\underline{i}}\; e^{-S}$$

For $t \to 0$, Z is dominated by constant paths:

$$x^\mu = x_0^\mu + \xi^\mu\ , \quad x^i = \xi^i$$
$$\psi_1^{\underline{\mu}} = \psi_0^{\underline{\mu}} + \lambda_1^{\underline{\mu}}\ , \quad \psi_2^{\underline{\mu}} = \lambda_2^{\underline{\mu}}$$
$$\psi_1^{\underline{i}} = \lambda_1^{\underline{i}}\ , \quad \psi_2^{\underline{i}} = \psi_0^{\underline{i}} + \lambda_2^{\underline{i}}$$

Again, it is enough to keep terms quadratic in the fluctuations and with a maximum number of fermionic zero modes.

One finds:

$$L^{eff} = \frac{1}{2}\Big[\dot\xi_\mu\dot\xi^\mu + \dot\xi_i\dot\xi^i + i\lambda_{1\mu}\dot\lambda_1^\mu + i\lambda_{1i}\dot\lambda_1^i + i\lambda_{2\mu}\dot\lambda_2^\mu + i\lambda_{2i}\dot\lambda_2^i$$
$$+R_{\mu\nu}\left(i\,\dot\xi^\mu\xi^\nu + \lambda_2^\mu\lambda_2^\nu\right) + R'_{ij}\left(i\,\dot\xi^i\xi^j + \lambda_2^i\lambda_2^j\right)\Big]$$
$$+\frac{1}{2}R'_{ij}\,\psi_0^i\psi_0^j$$

where

$$R_{\mu\nu} = \frac{1}{2}R_{\mu\nu\rho\sigma}(x_0)\psi_0^\rho\psi_0^\sigma\ , \quad R'_{ij} = \frac{1}{2}R_{ij\rho\sigma}(x_0)\psi_0^\rho\psi_0^\sigma$$

Evaluating the path-integral one finds:

$$Z = -\frac{1}{8}\int dx_0^\mu \int d\psi_0^\mu\, \overbrace{\frac{\det_P(i\eta_{\mu\nu}\partial_\tau)\det_A(i\eta_{\mu\nu}\partial_\tau + R_{\mu\nu})}{\det_P(\eta_{\mu\nu}\partial_\tau^2 + iR_{\mu\nu}\partial_\tau)}}^{(\pi t)^{-\frac{d}{2}}\prod_{a=1}^{d/2}\frac{\lambda_a t/2}{\tanh\lambda_a t/2}}$$
$$\underbrace{\frac{\det_A(i\eta_{ij}\partial_\tau)\det_P(i\eta_{ij}\partial_\tau + R'_{ij})}{\det_A(\eta_{ij}\partial_\tau^2 + iR'_{ij}\partial_\tau)}}_{2^{\cancel{-\frac{D-d}{2}}}\prod_{a=1}^{d/2}\frac{\tanh\lambda'_a t/2}{\lambda'_a t/2}} \underbrace{\int d\psi_0^i \exp\left\{\frac{t}{2}R'_{ij}\psi_0^i\psi_0^j\right\}}_{2^{\cancel{\frac{D-d}{2}}}\prod_{a=d/2}^{D/2}\lambda'_a t/2}$$

Finally, this can be rewritten as:

$$\boxed{Z = -\frac{1}{8}\int_M \frac{\widehat{L}(R)}{\widehat{L}(R')}\wedge e(R')}$$

Scrucca, Serone

Anomalous couplings

The anomalies on parallel Dp-branes and/or Op-planes on M are

$$I_{BB} = \mathrm{ch}_{\mathbf{n}\otimes\bar{\mathbf{n}}}(F) \wedge \frac{\widehat{A}(R)}{\widehat{A}(R')} \wedge e(R')$$

$$I_{BO} = \mathrm{ch}_{\mathbf{n}\oplus\bar{\mathbf{n}}}(2F) \wedge \frac{\widehat{A}(R)}{\widehat{A}(R')} \wedge e(R')$$

$$I_{OO} = -\frac{1}{8}\frac{\widehat{L}(R)}{\widehat{L}(R')} \wedge e(R')$$

Assigning the anomalous couplings:

$$S_{B,O} = \sqrt{2\pi}\int C \wedge Y_{B,O}$$

one gets the inflows

$$I_{BB} = -Y_B \wedge \bar{Y}_B \wedge e(R')$$

$$I_{BO} = -\left(Y_B \wedge \bar{Y}_O + Y_O \wedge \bar{Y}_B\right) \wedge e(R')$$

$$I_{OO} = -Y_O \wedge \bar{Y}_O \wedge e(R')$$

Anomaly cancellation requires

$$Y_B = \mathrm{ch}_{\mathbf{n}}(F) \wedge \sqrt{\frac{\widehat{A}(R)}{\widehat{A}(R')}}$$

$$Y_O = -2^{p-4}\sqrt{\frac{\widehat{L}(R/4)}{\widehat{L}(R'/4)}}$$

STRING THEORY COMPUTATION OF THE ANOMALOUS COUPLINGS

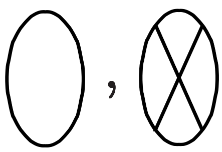
Duality arguments

The presence of most of the anomalous couplings was predicted by various string dualities.

Bershadski, Sadov, Vafa; Dasgupta, Jatkar, Mukhi

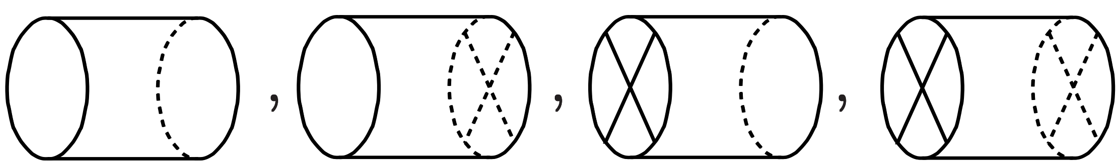
Direct computation of the couplings

The actual appearance of anomalous couplings for D-branes and O-planes can be checked on the disk and the crosscap.



Li; Craps, Roose; Stefanski

One can also compute topological RR magnetic interactions on the annulus, Möbius strip and Klein bottle in the odd spin structure, and extract the couplings by factorization. Technically, this is very similar to the anomaly computation.



Morales, Scrucca, Serone

Direct computation of anomalies and inflows

One can compute anomalies by evaluating amplitudes with external photons and/or gravitons, one of them being pure gauge. This measures the clash of gauge invariance and gives directly the anomaly. Only potentially divergent diagrams can contribute.

In string theory, these amplitudes are the annulus, Möbius strip and Klein bottle, in the RR odd spin-structure. Tadpole cancellation guarantees finiteness and implies anomaly cancellation.

The amplitudes we want to compute have the form:

$$\mathcal{A} = \int_0^\infty dt \left\langle V_1^{phy.} V_2^{phy.} \dots V_n^{phy.} V^{unphy.} (T_F + \tilde{T}_F) \right\rangle$$

The insertion of $T_F + \tilde{T}_F$ is due to the gravitino zero mode, and the vertices must have total superghost charge -1. The ghosts determine the measure in moduli space, but drop out from the correlation.

We take all the $V^{phy.}$'s in the 0-picture, with transverse polarizations ξ_M or ξ_{MN}:

$$V_\gamma^{phy.} = \xi_M \oint d\tau \left(\dot{X}^M + ip\cdot\psi\,\psi^M\right) e^{ip\cdot X}$$

$$V_g^{phy.} = \xi_{MN} \int d^2z \left(\partial X^M + ip\cdot\psi\,\psi^M\right)\left(\bar{\partial} X^N + ip\cdot\tilde{\psi}\,\tilde{\psi}^N\right) e^{ip\cdot X}$$

The $V^{unphy.}$ must then be in the -1-picture, with longitudinal polarization $\xi_M = p_M \eta$ or $\xi_{MN} = p_M \eta_N + p_N \eta_M$.

Interesting, it can then be written as

$$V^{unphy.} = \left[Q + \tilde{Q}, \hat{V}^{unphy.}\right]$$

Omitting the ghosts:

$$\hat{V}_{\gamma}^{unphy.} = i\eta \oint d\tau \, e^{ip\cdot X}$$

$$\hat{V}_{g}^{unphy.} = 2i\eta_M \int d^2 z \left[(\partial + \bar{\partial})X^M + ip\cdot(\psi - \tilde{\psi})(\psi - \tilde{\psi})^M\right] e^{ip\cdot X}$$

Using standard arguments, one can move $Q + \tilde{Q}$ onto the other operators in the correlation. The $V^{phy.}$'s are supersymmetric, but

$$\left[Q + \tilde{Q}, T_F + \tilde{T}_F\right] = T_B + \tilde{T}_B$$

The net effect of $T_B + \tilde{T}_B$ is to take the derivative of the remaining correlation with respect to t.

We are then left with a total derivative in moduli space:

$$\mathcal{A} = \int_0^\infty dt \, \frac{d}{dt} \left\langle V_1^{phy.} \, V_2^{phy.} \ldots V_n^{phy.} \, \hat{V}^{unphy.} \right\rangle$$

In consistent models, this vanishes, reflecting a cancellation between one-loop anomalies and tree-level inflows associated to the same surface.

At finite p's, only the ultraviolet boundary $t \to 0$ can contribute. This should vanish, but the computation is too difficult. To get a field theory interpretation, we can restrict to the leading order in $p \to 0$.

The correlation becomes then t-independent and there are two equal contributions from $t \to 0$ and $t \to \infty$ cancel, reflecting anomaly cancellation through the inflow mechanism.

In this limit, since the correlation vanishes unless all the fermionic zero modes are inserted, one can use

$$V_\gamma^{eff.} = \oint d\tau \, F$$

$$V_g^{eff.} = \oint d^2 z \, R_{MN} \left[X^M (\partial + \bar{\partial}) X^N + (\psi - \tilde{\psi})^M (\psi - \tilde{\psi})^N \right]$$

These hold both for physical and unphysical vertices:

$$\text{Phy.} \quad : \; F = \frac{1}{2} F_{\mu\nu} \, \psi_0^\mu \psi_0^\nu \; , \quad R_{MN} = \frac{1}{2} R_{MN\mu\nu} \, \psi_0^\mu \psi_0^\nu$$

$$\text{Unphy.} : \; F = \eta \; , \quad R_{MN} = p_M \eta_N + p_N \eta_M$$

The generating functional is a twisted partition function in the backgrounds $F + \eta$ and $R_{MN} + p_M \eta_N + p_N \eta_M$. The correct number of physical vertices is automatically selected, the unphysical one being obtained by restricting to the term linear in η.

The role of the unphysical vertex is to take the descent of the remaining partition function, and the anomaly polynomial is finally

$$\boxed{I = Z'}$$

This is the analog of Fujikawa's method.

Scrucca, Serone

The anomaly polynomials on D-branes and O-planes are then given by:

$$I_{BB} = Z'_A = \frac{1}{4}\,\mathrm{Tr}'_R\left[(-1)^F\, e^{-tH}\right]$$

$$I_{BO} = Z'_M = \frac{1}{4}\,\mathrm{Tr}'_R\left[\Omega\, I\,(-1)^F\, e^{-tH}\right]$$

$$I_{OO} = Z'_K = \frac{1}{8}\,\mathrm{Tr}'_{RR}\left[\Omega\, I\,(-1)^{F+\tilde{F}}\, e^{-tH}\right]$$

These are supersymmetric indices, and only massless modes constant in σ contribute. Effectively, one recovers precisely the SQM models seen before.

One therefore reproduces the results for the anomalies and the anomalous couplings.

ORIENTIFOLD MODELS

In $\mathbf{Z}_N$ orientifolds, there are in general D-branes and F-planes.

The element $g^k \in \mathbf{Z}_N$ acts as a rotation of $2\pi k v_i$ in the i-th compact two-torus, and as conjugation by the matrix γ_k on the Chan-Paton factors associated to D-branes. Similarly, Ω is accompanied by some matrix γ_Ω.

The twist has the form $v_i = (n_1, n_2, \dots)/N$, $\sum_i n_i = 0 \mod 2$. The tadpole cancellation conditions fixes γ_k, γ_Ω and the spectrum. One needs always 32 D9-branes to cancel the 09-plane tadpole, and for N even also 32 D5-branes to cancel the 05-plane tadpoles.

The anomalies on the D-branes and F-planes are given by:

$$I_{BB} = Z_A' = \frac{1}{4N} \sum_{k=0}^{N-1} \sum_{p,q=9,5} \mathrm{Tr}_R'^{pq} \left[g^k \, (-1)^F \, e^{-tH} \right]$$

$$I_{BF} = Z_M' = \frac{1}{4N} \sum_{k=0}^{N-1} \sum_{p=9,5} \mathrm{Tr}_R'^{p} \left[\Omega \, g^k \, (-1)^F \, e^{-tH} \right]$$

$$I_{FF} = Z_K' = \frac{1}{8N} \sum_{k=0}^{N-1} \sum_{m=0}^{N-1} \mathrm{Tr}_{RR}'^{(m)} \left[\Omega \, g^k \, (-1)^{F+\tilde{F}} \, e^{-tH} \right]$$

For open strings, a trace over Chan-Paton factors is understood. For closed strings, only the $m = 0, N/2$ twists really contribute. In each sector, there is a degeneracy $N_{k,m}$, the # of F-points fixed under k and m twists.

Anomalies

The evaluation of the partition functions is straightforward, and one finds:

$$I_{BB}^{pq} = \frac{1}{4N}\sum_{k=0}^{N-1} C_k^{pq}(v_i)\,\mathrm{ch}_{\gamma_k}(F_p)\wedge \mathrm{ch}_{\gamma_k}(F_q)\wedge \widehat{A}(R)$$

$$I_{BF}^{p} = \frac{1}{4N}\sum_{k=0}^{N-1} C_k^{p}(v_i)\,\mathrm{ch}_{\gamma_{2k}}(2F_p)\wedge \widehat{A}(R)$$

$$I_{FF}^{(m)} = \frac{1}{8N}\sum_{k=0}^{N-1} C_k^{(m)}(v_i)\,\widehat{L}(R)$$

where $\mathrm{ch}_{\gamma_k}(F) = \mathrm{tr}\,[\gamma_k \exp iF]$ is the $\mathbf{Z}_N$-twisted Chern class in the Chan-Paton representation, and the $C_k(v_i)$'s are given by:

$$C_k^{pq}(v_i) = \begin{cases} \prod_i (2\sin\pi k v_i)\ , & pq = 99, 55 \\ \prod_{i//5} (2\sin\pi k v_i)\ , & pq = 95, 59 \end{cases}$$

$$C_k^{p}(v_i) = \begin{cases} \prod_i (2\sin\pi k v_i)\ , & p = 9 \\ \prod_{i//5} (2\sin\pi k v_i)\prod_{i\perp 5}(2\cos\pi k v_i)\ , & p = 5 \end{cases}$$

$$C_k^{(m)}(v_i) = \begin{cases} \prod_i (2\sin 2\pi k v_i)\ , & m = 0 \\ \delta_{k;0,N/2}\prod_i 4 + \min_{k,k+N/2}\prod_i (2\sin\pi k v_i)^2\ , & m = N/2 \end{cases}$$

One can check that the anomalies from charged open string states and neutral closed string states are indeed given by

$$I_{charged} = \sum_{p,q=5,9} I^{pq}_{BB} + \sum_{p=5,9} I^{p}_{BF}$$
$$I_{neutral} = \sum_{m=0,N/2} I^{(m)}_{FF}$$

For $I_{charged}$, this is easy to understand qualitatively and in general, since the only charged anomalous particles are chiral spinors.

For the simplest models, the gauge group has the form

$$G = [U(r_1) \times U(r_2) \times ... \times SO(s_1) \times SO(s_2) \times ...]^2$$

The Chan-Paton representation is a sum of fundamental representations: $\rho_{CP} = (\mathbf{r_1} \oplus \mathbf{\bar{r}_1}) \oplus (\mathbf{r_2} \oplus \mathbf{\bar{r}_2}) \oplus ... \oplus \mathbf{s_1} \oplus \mathbf{s_2} \oplus ...$.

All the representations appearing in the spectrum can be decomposed into fundamentals, and the Chern classes decompose accordingly. In particular:

$$\mathrm{ch}_{\mathbf{\frac{n(n\pm1)}{2}}}(F) = \frac{1}{2}\left[\mathrm{ch}^2_{\mathbf{n}}(F) \pm \mathrm{ch}_{\mathbf{n}}(2F)\right]$$

For $I_{neutral}$, the situation is more complicated, since all the types of anomalous particle can arise. Interestingly, the total is always proportional to the anomaly of a self-dual tensor.

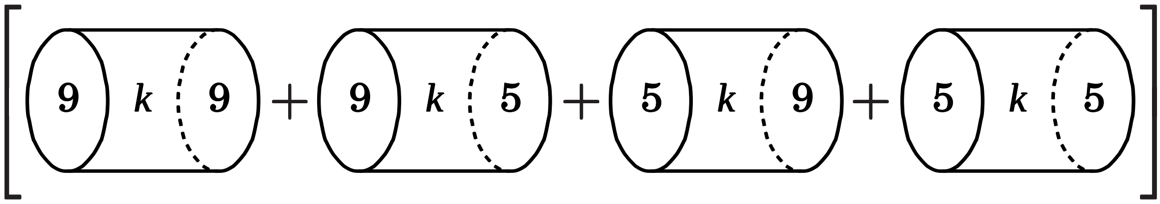
This is due to the fact that a subset of gravitational anomalies associated to the torus amplitude vanishes.

Anomalous couplings

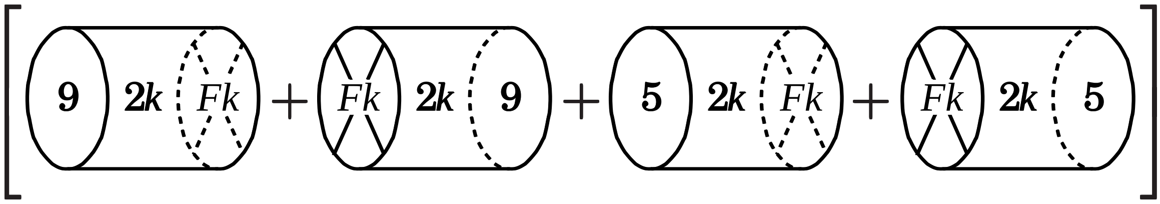
In the transverse channel, the string amplitudes are interpreted as inflows mediated by RR fields in all the twisted sectors.

The factorization is as follows:

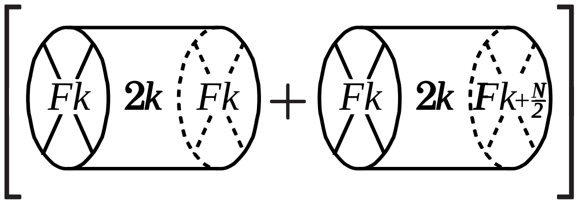
$$I_{BB} = \sum_{k=0}^{N-1}$$



$$I_{BF} = \sum_{k=0}^{N-1}$$



$$I_{FF} = \sum_{k=0}^{N-1}$$



This implies anomalous couplings for D-branes and F-planes:

$$S_{Dp}^{(k)} = -\sqrt{\frac{\pi}{N}} \sum_{i_k=1}^{N_k^p} \int C^{(k)i_k} \wedge Y_{Dp}^{(k)}$$

$$S_{Fk}^{(2k)} = -\sqrt{\frac{\pi}{N}} \sum_{i_k=1}^{N_k} \int C^{(2k)i_k} \wedge Y_{Fk}^{(2k)}$$

where $C^{(k)i_k} = \sum_p C_{(p)}^{(k)i_k}$ and

$$N_k^p = \prod_{i//p} (2 \sin \pi k v_i)^2$$

$$N_k = \prod_i (2 \sin \pi k v_i)^2$$

The corresponding inflow is

$$I_{BB}^{pq} = \sum_{k=0}^{N-1} N_k^{p\cap q}\, Y_{Dp}^{(k)} \wedge Y_{Dq}^{(k)}$$

$$I_{BF}^{p} = 2\sum_{k=0}^{N-1} N_k^{p}\, Y_{Dp}^{(2k)} \wedge Y_{Fk}^{(2k)}$$

$$I_{FF}^{(m)} = \sum_{k=0}^{N-1} N_{k,m}\, Y_{Fk}^{(2k)} \wedge Y_{F(k+m)}^{(2k+2m)}$$

It is then possible to extract the Y's in the anomalous couplings. Defining $\epsilon_k^p = \mathrm{sign}\sqrt{N_k^p}$, $\epsilon_k = \mathrm{sign}\sqrt{N_k}$, one finds:

$$Y_{Dp}^{(k)} = \epsilon_k^p \frac{\prod_{i\perp p}\sqrt{|2\sin\pi k v_i|}}{\prod_{i//p}\sqrt{|2\sin\pi k v_i|}}\, \mathrm{ch}_{\gamma_k}(\epsilon_k\, F_p) \wedge \sqrt{\widehat{A}(R)}$$

$$Y_{Fk}^{(2k)} = -\,2^{\frac{D}{2}}\,\epsilon_k \prod_i \sqrt{|\cot\pi k v_i|}\,\sqrt{\widehat{L}(R/4)}$$

Scrucca, Serone

GS mechanism of anomaly cancellation

All one-loop anomalies are automatically canceled by a tree-level inflow, through a GS mechanism involving all the $C^{(k)i_k}$.

The GS term is given by the sum of all the D-branes and F-planes anomalous couplings.

$N=1$ $D=6$ models

These models have been constructed as Type IIB on $T^4/\{\Omega, \mathbf{Z}_N\}$, for $N=2,3,4,6$. The spectrum is:

Open: - Vector: $I^{\rho}_{1/2}$ $\quad (G=\prod_a U(n_a)\times\prod_b SO(n_b))$

- Hyper: $-I^{\rho}_{1/2}$ $\quad (\rho = \mathbf{n_a}(\mathbf{n_a}-\mathbf{1})/\mathbf{2}, (\mathbf{n_a},\mathbf{n_b}))$

Closed: - Gravitational: $I_{3/2}+I_A$

- Hyper (n_H): $-I_{1/2}$

- Tensor (n_T): $-I_{1/2}-I_A$ $\quad (n_H+n_T=21)$

Bianchi,Sagnotti; Gimon,Polchinski; Gimon,Johnson

Using the $D=6$ relation $I_{3/2}-21I_{1/2}-8I_A=0$, one gets:

$$I_{charged} = \sum\nolimits_\rho \mathrm{ch}_\rho(F)\,\widehat{A}(R)$$

$$I_{neutral} = -\frac{1}{8}(9-n_T)\,\widehat{L}(R)$$

The inflow has a factorized structure:

$$I^{(8)}_{GS} = \underbrace{\sum\nolimits_t I^{(4)}_t\wedge I^{(4)}_t}_{\text{RR tensors}} + \underbrace{\sum\nolimits_h I^{(2)}_h\wedge I^{(6)}_h}_{\text{RR scalars}}$$

The corresponding GS couplings are:

$$S_{GS} = \sum\nolimits_t \int b_t\wedge I^{(4)}_t + \sum\nolimits_h \int \left(\phi_h\, I^{(6)}_h + \tilde{\phi}_h \wedge I^{(2)}_h\right)$$

The $I_h^{(2)}$ coupling modifies the kinetic terms of the RR scalars ϕ_h: the $U(1)$ fields A_h associated to $I_h^{(2)}$ enter as shifts in the field strengths $H_h = d\phi_h - A_h$, and the RR scalars are gauge-variant. The SUSY partners of the $I_h^{(2)}$ couplings are D-terms involving the other three NSNS scalars φ_h of each neutral hyper multiplet.

The corresponding $U(1)$ vector multiplets become massive through super-Higgsing of the hyper multiplets.

Harvey,Moore;

Berkooz, Leigh, Polchinski, Schwarz, Seiberg, Witten

The $I_t^{(4)}$ couplings are related by SUSY to the gauge kinetic terms $\sum_a \frac{1}{4} f_a(\varphi_t)\, \mathrm{tr}\, F_a^2$, which depend on the NSNS scalar partners φ_t of the RR tensors b_t in the tensor multiplets.

Sagnotti

One can deduce the gauge kinetic functions $f_a(\varphi_t)$ from $I_t^{(4)}(F_a)$.

The results for $f_a(\varphi_t)$ are in agreement with a direct computation.

Antoniadis,Bachas,Dudas

At the points $f_a(\langle\varphi_t\rangle) = 0$ in moduli space, the effective gauge coupling diverges, and tensionless strings appear in the spectrum.

Duff, Minasian, Witten

$N=1$ $D=4$ models

These models have been constructed as Type IIB on $T^6/\{\Omega, \mathbf{Z}_N\}$, for $N = 3, 6, 7, 12$. The spectrum is:

Open: - Vector: $I^{\rho}_{1/2}$ $\quad (G = \prod_a U(n_a) \times \prod_b SO(n_b))$

- Chiral: $I^{\rho}_{1/2}$ $\quad (\rho = \mathbf{n_a}(\mathbf{n_a}-\mathbf{1})/\mathbf{2}, (\mathbf{n_a}, \mathbf{n_b}))$

Closed: - Gravitational: $I_{3/2}$

- Chiral: $I_{1/2}$

Angelantonj,Bianchi,Pradisi,Sagnotti,Stanev; Kakushadze,Shiu; Aldazabal,Font,Ibáñez,Violero

On gets:

$$I_{charged} = \sum \mathrm{ch}_{\rho}(F)\,\hat{A}(R)$$

$$I_{neutral} = 0$$

The inflow contains only one type of term:

$$I^{(6)}_{GS} = \underbrace{\textstyle\sum_c I^{(2)}_c \wedge I^{(4)}_c}_{\text{RR scalars}}$$

The corresponding GS terms are:

$$S_{GS} = \textstyle\sum_c \int \left(\phi_c\, I^{(4)}_c + \tilde{\phi}_c \wedge I^{(2)}_c\right)$$

The $I_c^{(2)}$ couplings do again shift the field-strengths of the RR scalars ϕ_c, and spontaneously break the corresponding $U(1)$'s. Their SUSY partners are FI D-terms involving the NSNS scalar φ_c of each neutral chiral multiplet.

Ibáñez, Rabadán,Uranga

The $I_c^{(4)}$ couplings are as before related by SUSY to gauge kinetic terms, $\sum_a \frac{1}{4} f_a(\varphi_c) \,\mathrm{tr}\, F_a^2$, which now depend on the same NSNS scalars φ_c.

Again, the results for $f_a(\varphi_c)$ are in agreement with a direct computation.

Antoniadis,Bachas,Dudas

In these models, the same scalars φ_c occur both in the FI terms and in the gauge kinetic functions, and their vev's $\langle\varphi_c\rangle$ are fixed.

CONCLUSIONS

- The anomalous couplings are determined by anomaly cancellation.
  World-volume anomalies are canceled through the inflow mechanism.
- Anomalies and inflows can be computed directly in string theory.
  The anomalous couplings follow then by factorization.
- Anomalies, inflows and anomalous couplings can be studied in generic orientifold models.
  The GS mechanism is realized as inflow mechanism.
  Important informations on the effective action of orientifold models can be obtained.