SOFT SCALAR MASSES AND SEQUESTERING IN CALABI-YAU STRING MODELS

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- Soft scalar masses in supergravity
- Sequestering and global symmetries
- Calabi-Yau heterotic string models
- Kähler potential and contact terms
- Structure of scalar masses

Based on works with C. Andrey

SOFT SCALAR MASSES IN SUPERGRAVITY

General structure of soft scalar masses

Kaplunovsky, Louis, 1993
Brignole, Ibanez, Munoz 1993

In a supergravity theory with Kähler potential K and superpotential W, the soft scalar masses induced for the visible superfields Q^α when the hidden superfields X^i break supersymmetry are given by:

$$m^2_{\alpha\bar\beta} = -\Big(R_{\alpha\bar\beta i\bar\jmath} - \frac{1}{3} g_{\alpha\bar\beta} g_{i\bar\jmath}\Big) F^i \bar F^{\bar\jmath}$$

This can be written in terms of the Kähler function $\Omega = -3\, e^{-K/3}$ as:

$$\begin{aligned} m^2_{\alpha\bar\beta} &= 3\,\Omega^{-1}\Big(\Omega_{\alpha\bar\beta i\bar\jmath} - \Omega^{-1\bar\delta\gamma}\Omega_{\bar\delta\alpha i}\Omega_{\gamma\bar\beta\bar\jmath}\Big) F^i \bar F^{\bar\jmath} \\ &= 3\,\Omega^{-1}\Big(\Omega_{\alpha\bar\beta}\big|_D - \Omega^{-1\bar\delta\gamma}\big|\,\Omega_{\bar\delta\alpha}\big|_F\,\Omega_{\gamma\bar\beta}\big|_{\bar F}\Big) \end{aligned}$$

The crucial ingredient are thus the operators in Ω that mix Q^α and X^i, and the orientation of the Goldstino direction.

THE SUPERSYMMETRIC FLAVOR PROBLEM

Flavor structure of soft scalar masses

The flavor structure of the soft scalar mass matrix $m^2_{\alpha\bar{\beta}}$ is a priori generic, because this is generated at the fundamental scale of the theory where the flavor structure of the ordinary fermion masses must also emerge.

This causes severe problems at the phenomenological level. One should then find some mechanism that naturally forces $m^2_{\alpha\bar{\beta}}$ to be approximately flavor-universal.

Possible explanations for universality

The two most interesting ideas to explain flavor-universality of soft scalar masses are mode sequestering along extra dimensions and selection rules from global symmetries.

SEQUESTERING ALONG EXTRA DIMENSIONS

Randall, Sundrum 1999
Anisimov, Dine, Graesser, Thomas 2002

Localization along an extra dimension

If the visible and hidden sectors are separated along an extra dimension, as it happens quite naturally in string models, the classical contribution to $m^2_{\alpha\bar{\beta}}$ is very restricted by locality.

In the minimal case with only the gravity multiplet in the bulk $m^2_{\alpha\bar{\beta}} = 0$. In the general case involving also vector multiplets in the bulk $m^2_{\alpha\bar{\beta}} \neq 0$, but its form is very special and one may force it to vanish by other means.

When $m^2_{\alpha\bar{\beta}} = 0$ at the classical level, one may get a viable $m^2_{\alpha\bar{\beta}} \neq 0$ from quantum corrections, which are approximately flavor-universal and insensitive to high-energy physics.

It would be interesting to understand whether and how this situation can be implemented in generic string models.

HIDDEN GLOBAL SYMETRIES

Constraints from global symmetries

Schmaltz, Sundrum 2006
Andrey, Scrucca 2010

If the hidden sector possesses global symmetries generated by some Killing vectors k_a^i with potentials D_a, the form of $m^2_{\alpha\bar{\beta}}$ gets constrained. Most importantly, the Goldstino direction is restricted to satisfy:

$$\bar{k}_{ai} F^i = -i D_a m_{3/2} \qquad \nabla_i k_{a\bar{\jmath}} F^i \bar{F}^{\bar{\jmath}} = -2 i D_a m^2_{3/2}$$

In situations where $D_a \simeq 0$ on the vacuum, the gravity effects on the right-hand sides trivialize and one then finds:

$$\bar{k}_{ai} F^i \simeq 0 \qquad \nabla_i k_{a\bar{\jmath}} F^i \bar{F}^{\bar{\jmath}} \simeq 0$$

In rigid superspace, these follow from the conservation law $\mathcal{D}^2 J_a \simeq 0$ for the Nöther current $J_a \simeq \mathrm{Im}(K_i k_a^i)$, and read $J_a|_F \simeq 0$ and $J_a|_D \simeq 0$. It follows that contact terms in Ω that involve J_a do not contribute to $m^2_{\alpha\bar{\beta}}$.

MILD SEQUESTERING BY GLOBAL SYMMETRIES

Contact terms and global symmetries

Kachru, McAllister, Sundrum 2007

For generic sequestered models, we expect Ω to have the form

$$\Omega \simeq Q^{\alpha}\bar{Q}^{\bar{\alpha}} + X^{i}\bar{X}^{\bar{\imath}} + \frac{1}{2}M^{-2}\sum_{a}\left(c^{a}_{\alpha\bar{\beta}}Q^{\alpha}\bar{Q}^{\bar{\beta}} + c^{a}_{i\bar{\jmath}}X^{i}\bar{X}^{\bar{\jmath}}\right)^{2}$$

The soft scalar masses induced by a generic W are then given by

$$m^{2}_{\alpha\bar{\beta}} \simeq M^{-2}\Big[- c^{a}_{\alpha\bar{\beta}}c^{a}_{i\bar{\jmath}}F^{i}\bar{F}^{\bar{\jmath}} + (c^{a}c^{b})_{\alpha\bar{\beta}}c^{a}_{i\bar{\jmath}}F^{i}X^{\bar{\jmath}}c^{b}_{p\bar{q}}X^{p}\bar{F}^{\bar{q}}\Big]$$

In presence of global symmetries with Nöther currents $J^{a}_{X} \simeq c^{a}_{i\bar{\jmath}}X^{i}\bar{X}^{\bar{\jmath}}$, the Ward identities $J^{a}_{X}|_{F} \simeq 0$ and $J^{a}_{X}|_{D} \simeq 0$ imply:

$$c^{a}_{i\bar{\jmath}}F^{i}\bar{X}^{\bar{\jmath}} \simeq 0 \qquad c^{a}_{i\bar{\jmath}}F^{i}\bar{F}^{\bar{\jmath}} \simeq 0$$

In such a situation one then finds:

$$m^{2}_{\alpha\bar{\beta}} \simeq 0$$

HETEROTIC STRING MODELS

Heterotic M-theory on a Calabi-Yau

Horava,Witten 1996
Lukas, Ovrut, Stelle, Waldram 1999

Let us consider a generic heterotic string model based on a Calabi-Yau manifold M and a stable holomorphic vector bundle $E_{\mathbf{v}} \times E_{\mathbf{h}}$ over it. This also arises from M-theory on $M \times S^1/Z_2$ with two sequestered branes, in the weekly coupled limit where the size of S^1/Z_2 is small.

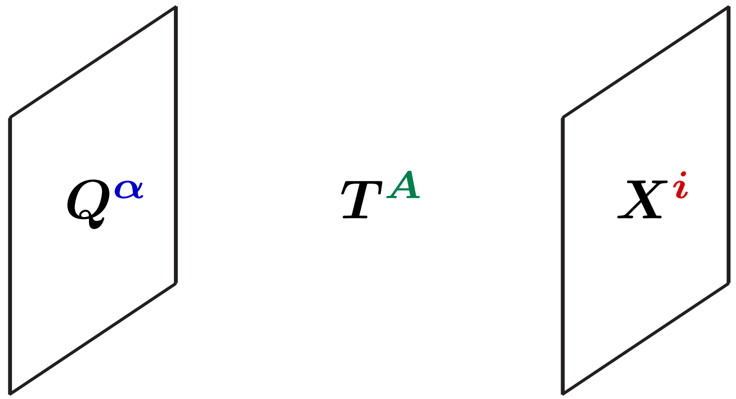
The 4D effective theory can be lifted to a 5D theory with localized brane sectors containing visible and hidden matter superfields Q^α and X^i, and a bulk sector containing in particular Kähler moduli superfields T^A.

The non-minimal Kähler moduli T^a come along with heavy vectors V^a, which couple non-trivially to Q^α, X^i and T^A. When integrated out, these induce contact terms in the effective Kähler function Ω.

BRANE WORLD INTERPRETATION

Geometric picture

The brane sectors contain only the light modes Q^α and X^i, while the bulk sector contains the light modes T^A plus heavy Kaluza-Klein modes.



Visible sector: Q^α
Hidden sector: X^i, T^A

The masses $m^2_{\alpha\bar\beta}$ get two kinds of contributions: one from operators in Ω mixing Q^α and X^i, and one from operators in Ω mixing Q^α and T^A. We thus need to known the full dependence of K on Q^α, X^i and T^A.

DERIVATION OF THE EFFECTIVE THEORY

Reduction of the standard heterotic string — Witten 1985

The light 4D fields arise from the possible zero-modes of the 10D fields. The Q^α, X^i come from harmonic 1-forms in $H^1(M, E_{\rm v})$, $H^1(M, E_h)$, while the T^A come from harmonic $(1,1)$-forms in $H^{1,1}(M)$:

$$Q^\alpha \;\Leftrightarrow\; u_\alpha \qquad X^i \;\Leftrightarrow\; u_i \qquad T^A \;\Leftrightarrow\; \omega_A$$

The effective K for the light fields may be derived by working out their kinetic terms by reduction on M and comparing with the general structure of supergravity theories. The effective W can be imagined to be arbitrary.

Discarding rather than integrating out heavy non-zero modes associated to non-harmonic forms is justified only whenever:

$$\mathrm{tr}(u_\alpha \wedge \bar u_{\bar\beta}) \;\text{ and }\; \mathrm{tr}(u_i \wedge \bar u_{\bar\jmath}) \;\text{ harmonic } \;\Leftrightarrow\; \omega_A$$

EFFECTIVE KAHLER POTENTIAL

Candelas, de la Ossa 1990
Paccetti Correia, Schmidt 2008
Andrey, Scrucca 2011

General result for matter fields and Kähler moduli

The effective Kahler potential is found to be

$$K = -\log\left(d_{ABC} J^A J^B J^C\right)$$

where

$$J^A = T^A + \bar{T}^A - c^A_{\alpha\bar{\beta}} Q^\alpha \bar{Q}^{\bar{\beta}} - c^A_{i\bar{\jmath}} X^i \bar{X}^{\bar{\jmath}}$$

The numerical quantities defining this result are:

$$d_{ABC} = \int \omega_A \wedge \omega_B \wedge \omega_C$$

$$c^A_{\alpha\bar{\beta}} = \int \omega^A \wedge \mathrm{tr}(u_\alpha \wedge \bar{u}_{\bar{\beta}}) \qquad c^A_{i\bar{\jmath}} = \int \omega^A \wedge \mathrm{tr}(u_i \wedge \bar{u}_{\bar{\jmath}})$$

This extends the results for the special cases of one-modulus Calabi-Yaus and untwisted sectors of orbifolds, where harmonic forms are covariantly constant, to a larger class of cases, where harmonic forms close.

CANONICAL PARAMETRIZATON

Canonical basis

With a suitable basis for the harmonic forms and fields, which defines an overall modulus T and some relative moduli T^a, one may rewrite K as:

$$K = -\log\left(J^3 - \frac{1}{2}JJ^aJ^a + \frac{1}{6}d_{abc}J^aJ^bJ^c\right)$$

where

$$J = T + \bar{T} - \frac{1}{3}Q^\alpha\bar{Q}^{\bar{\alpha}} - \frac{1}{3}X^i\bar{X}^{\bar{\imath}}$$

$$J^a = T^a + \bar{T}^a - c^a_{\alpha\bar{\beta}}Q^\alpha\bar{Q}^{\bar{\beta}} - c^a_{i\bar{\jmath}}X^i\bar{X}^{\bar{\jmath}}$$

Contact terms

The leading terms in the Kähler function for $J^a \ll J$ are

$$\Omega \simeq -3\,J + \frac{1}{2}J^{-1}J^aJ^a - \frac{1}{6}d_{abc}J^{-2}J^aJ^bJ^c$$

INTERPRETATION OF THE CONTACT TERMS

Effect of heavy vector multiplets in the M-theory picture

In the M-theory picture, the contact terms in Ω are induced by the heavy vectors V^a coming with the light moduli T^a in $N = 2$ vector multiplets. In terms of 5D $N = 1$ superfields, the Lagrangian for these modes is:

$$\mathcal{L} = \Big[-\frac{1}{4}\mathcal{N}_{ab}(T, T^e) W^a W^b + \frac{1}{48}\mathcal{N}_{abc}\bar{\mathcal{D}}^2 (V^a \overleftrightarrow{\mathcal{D}} \partial_y V^b) W^c\Big]_F + \text{c.c.}$$
$$+\Big[-3\,\mathcal{N}^{1/3}(J_y, J_y^e)\Big]_D$$

with prepotential $\mathcal{N}(Z, Z^e) = Z^3 - \frac{1}{2} Z Z^a Z^a + \frac{1}{6} d^{abc} Z^a Z^b Z^c$ and

$$J_y = T + \bar{T} - \frac{1}{3} Q^\alpha \bar{Q}^{\bar{\alpha}} \delta_{\mathrm{v}}(y) - \frac{1}{3} X^i \bar{X}^{\bar{\imath}} \delta_{\mathrm{h}}(y)$$
$$J_y^a = -\partial_y V^a + T^a + \bar{T}^a - c^a_{\alpha\bar{\beta}} Q^\alpha \bar{Q}^{\bar{\beta}} \delta_{\mathrm{v}}(y) - c^a_{i\bar{\jmath}} X^i \bar{X}^{\bar{\jmath}} \delta_{\mathrm{h}}(y)$$

Integrating out V^a sets $(J_y, J_y^a, W^a) \to (J, J^a, 0)$ and gives $\mathcal{L} = \Omega|_D$.

STRUCTURE OF SOFT SCALAR MASSES

Structure of soft terms

One may study the following reference point, around which the canonical parametrizaton is particularly convenient:

$$T \simeq \frac{1}{2} \qquad T^a \simeq 0 \qquad Q^\alpha \simeq 0 \qquad X^i \simeq 0$$

Using the computed K and imagining a generic W, one obtains:

$$m^2_{\alpha\bar\beta} \simeq -c^a_{\alpha\bar\beta} c^a_{i\bar\jmath} F^i \bar F^{\bar\jmath} - \Big(\frac{1}{3}\delta_{\alpha\bar\beta}\delta_{ab} + (d_{abc}c^c - c^a c^b)_{\alpha\bar\beta}\Big) F^a \bar F^{\bar b} - c^a_{\alpha\bar\beta} F^a \bar F + \text{c.c.}$$

This vanishes identically if the Goldstino direction is suitably constrained:

$$m^2_{\alpha\bar\beta} \simeq 0 \;\Leftrightarrow\; F^a \simeq 0 \;\text{ and }\; c^a_{i\bar\jmath} F^i \bar F^{\bar\jmath} \simeq 0$$

SEQUESTERING BY GLOBAL SYMMETRIES

Approximate global symmetries

Andrey, Scrucca 2011

The Goldstino direction can be guaranteed to point in a direction for which $m^2_{\alpha\bar{\beta}} \simeq 0$ by postulating that the following transformations represent two approximate symmetries not only of K but also of W:

$$\delta^1_a T^b = i\delta^b_a \;\Leftrightarrow\; F^a \simeq 0$$
$$\delta^2_a X^i = -ic^a_{\bar{\jmath}i} X^j \;\Leftrightarrow\; c^a_{i\bar{\jmath}} F^i \bar{F}^{\bar{\jmath}} \simeq 0$$

Clearly δ^1_a always form a group $U(1)^\#$ and give exact symmetries of K. However δ^2_a only form a group H if $c^a_{i\bar{\jmath}}$ generate a closed algebra and only extends to exact symmetries of K if d_{abc} is a symmetric invariant of this algebra. We conclude that:

Sequestering by symmetries possible for some Calabi-Yau models

PARTICULAR CASE OF ORBIFOLDS

Ferrara, Kounnas, Porrati 1986
Li, Peschanski, Savoy 1987
Andrey, Scrucca 2010

Symmetric structure in orbifolds

One special class of models where one is automatically in business is provided by orbifold constructions. In the untwisted sector, the formula for K that has been obtained applies, with:

$$c^a_{\alpha\bar\beta}, c^a_{i\bar\jmath}: \quad \text{generators of some } H \subset SU(3)$$
$$d_{abc}: \quad \text{symmetric invariant of this } H \subset SU(3)$$

The scalar manifold is always a symmetric cost manifold, and H belongs to the stability group. As a result, $U(1)^{\#} \times H$ is an exact symmetry of K, and imposing it also to W leads to vanishing masses. We thus conclude that:

Sequestering by symmetries possible for all orbifold models

CONCLUSIONS

- Under some assumptions, the Kähler potential of heterotic models can be fully computed. The resulting soft scalar masses are found to vanish for suitably oriented Goldstino directions.
- The Goldstino direction can be forced to align along such special directions by relying on some global symmetries, but this appears to be possible only under some extra assumptions.
- A special class of models where this mechanism can always work is that of orbifold models. But it might be possible to put it at work also for other special classes of Calabi-Yau models.