INTERACTION OF D-BRANES ON ORBIFOLDS AND MASSLESS PARTICLE EMISSION

F. Hussain, R. Iengo, C. Núñez, C. A. Scrucca

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- VARIOUS SPACE-TIME 0-BRANES IN 4D ORBIFOLD COMPACTIFICATIONS.

  TWO INTERESTING CASES: 0-BRANE OF TYPE IIA AND 3-BRANE OF TYPE IIB.
- DYNAMICS AND BOUNDARY STATE; FORCE.

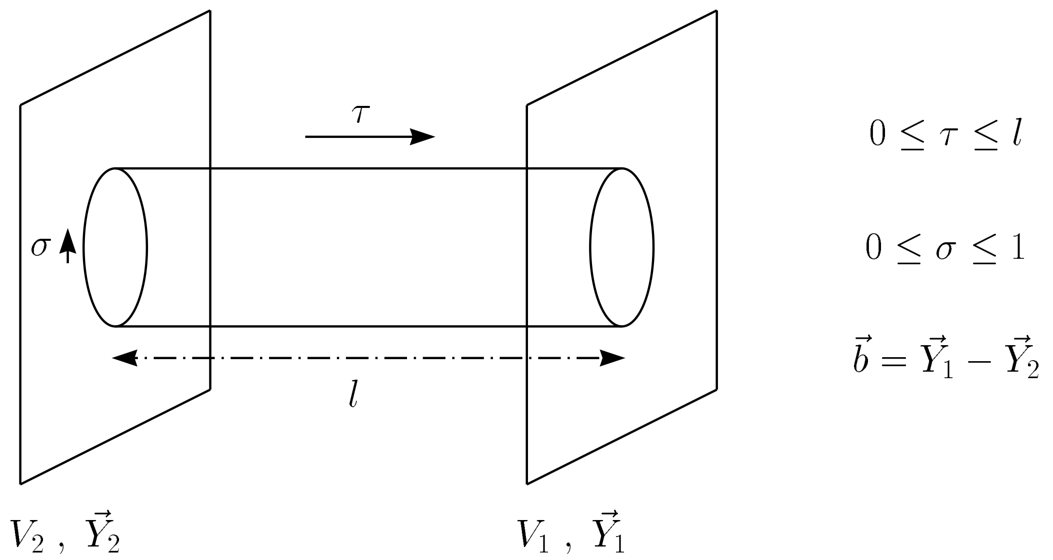
  LARGE DISTANCE INTERACTIONS AND RELATION WITH SOLUTIONS OF 4D EFFECTIVE SUGRA.
- EMISSION OF MASSLESS NSNS STATES FROM TWO INTERACTING SPACE-TIME 0-BRANES.

  CORRELATORS WITH TWISTED BOUNDARY CONDITIONS.

  LARGE DISTANCE BEHAVIOR AND FIELD THEORY INTERPRETATION.

INTERACTIONS ON ORBIFOLDS

Consider two 0-branes moving with velocities $V_1 = \tanh v_1$, $V_2 = \tanh v_2$ (along 1) and transverse positions $\vec{Y}_1$, $\vec{Y}_2$ (along 2-3).



The amplitude in the closed string channel is

$$\mathcal{A} = \int_0^\infty dl \sum_s < B, V_1, \vec{Y}_1 | e^{-lH} | B, V_2, \vec{Y}_2 >_s$$

There are two sectors, RR and NSNS, and after the GSO projection four spin structures contribute, R$\pm$ and NS$\pm$.

In the static case, one has Neumann b.c. in time and Dirichlet b.c. in space. The velocity twist the 0-1 directions.

The moving boundary state is obtained by boosting the static one with $v = v_1 - v_2$ (Billó, Di Vecchia, Cangemi)

$$|B, V, \vec{Y} >= e^{-ivJ^{01}} |B, \vec{Y} >$$

In the large distance limit $b \to \infty$ only world-sheets with $l \to \infty$ will contribute.

Momentum or winding in the compact directions can be neglected since they correspond to massive components.

The moving boundary states

$$|B,V_1,\vec{Y}_1> \;=\; \int \frac{d^3\vec{k}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{Y}_1}|B,V_1>\otimes|k_B>$$
$$|B,V_2,\vec{Y}_2> \;=\; \int \frac{d^3\vec{q}}{(2\pi)^3} e^{i\vec{q}\cdot\vec{Y}_2}|B,V_2>\otimes|q_B>$$

can only carry the boosted space-time momenta

$$k_B^\mu \;=\; (V_1\gamma_1 k^1, \gamma_1 k^1, \vec{k}_T) = (\sinh v_1 k^1, \cosh v_1 k^1, \vec{k}_T)$$
$$q_B^\mu \;=\; (V_2\gamma_2 q^1, \gamma_2 q^1, \vec{q}_T) = (\sinh v_2 q^1, \cosh v_2 q^1, \vec{q}_T)$$

Taking into account momentum conservation ($k_B^\mu = q_B^\mu$), the amplitude factorizes

$$\mathcal{A} \;=\; \frac{1}{\sinh v}\int_0^\infty dl \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} e^{-\frac{q_B^2}{2}} \sum_s Z_B Z_F^s$$
$$\;=\; \frac{1}{\sinh v}\int_0^\infty \frac{dl}{2\pi l} e^{-\frac{b^2}{2l}} \sum_s Z_B Z_F^s$$

with (from now on $X^\mu \equiv X^\mu_{osc}$)

$$Z_{B,F} =< B,V_1|e^{-lH}|B,V_2>^s_{B,F}$$

Group the fields into pairs

$$X^\pm = X^0 \pm X^1 \;\rightarrow\; \alpha_n, \beta_n = a_n^0 \pm a_n^1$$
$$X^i, X^{i*} = X^i \pm iX^{i+1} \;\rightarrow\; \beta_n^i, \beta_n^{i*} = a_n^i \pm i a_n^{i+1} \quad , \quad i = 2,4,6,8$$
$$\chi^{A,B} = \psi^0 \pm \psi^1 \;\rightarrow\; \chi_n^{A,B} = \psi_n^0 \pm \psi_n^1$$
$$\chi^i, \chi^{i*} = \psi^i \pm i\psi^{i+1} \;\rightarrow\; \chi_n^i, \chi_n^{i*} = \psi_n^i \pm i\psi_n^{i+1} \quad , \quad i = 2,4,6,8$$

with

$$[\alpha_m, \beta_{-n}] = -2\delta_{mn} \quad , \quad [\beta_m^i, \beta_{-n}^{i*}] = 2\delta_{mn}$$
$$\{\chi_m^A, \chi_{-n}^B\} = -2\delta_{mn} \quad , \quad \{\chi_m^i, \chi_n^{i*}\} = 2\delta_{mn}$$

Orbifold construction

Identify points connected by discrete rotations $g = e^{2\pi i \sum_a z_a J_{aa+1}}$ on some of the compact pairs X^a,χ^a, a=4,6,8.

In order to preserve at least on SUSY: $\Sigma_a \, z_a = 0$.

- For T_6/Z_3 ($N = 2$ SUSY) take $z_4, z_6 = \frac{1}{3}, \frac{2}{3}$, $z_8 = -z_4 - z_6$
- For $T_2 \otimes T_4/Z_2$ ($N = 4$ SUSY) take $z_4 = -z_6 = \frac{1}{2}$, $z_8 = 0$
- For T_6 ($N = 8$ SUSY) take $z_4 = z_6 = z_8 = 0$

There can be additional twisted sectors. One can diagonalize the fields such that ($g_a = e^{2\pi i z_a}$)

$$X^a(\sigma + 1) = g_a X^a(\sigma) \;\; , \;\; X^{*a}(\sigma + 1) = g_a^* X^{*a}(\sigma)$$

and similarly for fermions. This leads to fractional moding.

The twisted states become massless only at fixed points of the orbifold.

In all sectors, one has to project onto invariant states to get the physical spectrum.

The physical boundary state is

$$|B_{phys} >= \frac{1}{N}(|B, 1 > +|B, g > +... + |B, g^{N-1} >)$$

in terms of the twisted boundary states

$$|B, g^k >= g^k |B >$$

0-brane: untwisted sector

Consider first the static case. The b.c. are Neumann for time and Dirichlet for all other directions (i=2,4,6,8 and a=2,4,6).

For the bosons, the b.c. are

$$(\alpha_n + \tilde{\beta}_{-n})|B>_B = 0 \quad , \quad (\beta_n + \tilde{\alpha}_{-n})|B>_B = 0$$

$$(\beta^i_n - \tilde{\beta}^i_{-n})|B>_B = 0 \quad , \quad (\beta^{i*}_n - \tilde{\beta}^{i*}_{-n})|B>_B = 0$$

They are solved by the following boundary state

$$|B>_B = \exp \frac{1}{2} \sum_{n=1}^{\infty} (\alpha_{-n}\tilde{\alpha}_{-n} + \beta_{-n}\tilde{\beta}_{-n} + \beta^i_{-n}\tilde{\beta}^{i*}_{-n} + \beta^{i*}_{-n}\tilde{\beta}^i_{-n})|0>$$

For the fermions, one has integer or half-integer moding in the RR and NSNS sectors respectively. The b.c are

$$(\chi^A_n + i\eta\tilde{\chi}^B_{-n})|B,\eta>_F = 0 \quad , \quad (\chi^B_n + i\eta\tilde{\chi}^A_{-n})|B,\eta>_F = 0$$

$$(\chi^i_n - i\eta\tilde{\chi}^i_{-n})|B,\eta>_F = 0 \quad , \quad (\chi^{i*}_n - i\eta\tilde{\chi}^{i*}_{-n})|B,\eta>_F = 0$$

where $\eta = \pm 1$ to deal with the GSO projection.

The corresponding boundary state can be factorized into zero mode and oscillator parts:

$$|B,\eta>_F = |B_o>_F \otimes |B_{osc}>_F$$

The oscillator part is the same for both sectors, with appropriate moding

$$|B_{osc},\eta>_F = \exp \frac{i\eta}{2} \sum_{n>0} (\chi^A_{-n}\tilde{\chi}^A_{-n} + \chi^B_{-n}\tilde{\chi}^B_{-n} - \chi^i_{-n}\tilde{\chi}^{i*}_{-n} - \chi^{i*}_{-n}\tilde{\chi}^i_{-n})|0>$$

The zero mode part exist only in the RR sector.

The zero modes are proportional to Γ-matrices

$$\psi_o^\mu = \frac{i}{\sqrt{2}}\Gamma^\mu \quad , \quad \tilde{\psi}_o^\mu = \frac{i}{\sqrt{2}}\tilde{\Gamma}^\mu$$

One can construct

$$a, a^* = \frac{1}{2}(\Gamma^0 \pm \Gamma^1)$$
$$b^i, b^{i*} = \frac{1}{2}(-i\Gamma^i \pm \Gamma^{i+1})$$

and similar for tilded, satisfying

$$\{a, a^*\} = \{b^i, b^{i*}\} = 1$$

The b.c. for the zero modes can be rewritten as

$$(a + i\eta\tilde{a}^*)|B_o, \eta >_F = 0 \quad , \quad (a^* + i\eta\tilde{a})|B_o, \eta >_F = 0$$

$$(b^i - i\eta\tilde{b}^i)|B_o, \eta >_F = 0 \quad , \quad (b^{i*} - i\eta\tilde{b}^{i*})|B_o, \eta >_F = 0$$

Defining the spinor vacuum $|0> \otimes |\tilde{0}>$ such that

$$a|0>= 0 \quad , \quad \tilde{a}|\tilde{0}>= 0$$

$$b^i|0>= 0 \quad , \quad \tilde{b}^{i*}|\tilde{0}>= 0$$

the zero mode part of the boundary state can be written as

$$|B_o, \eta >_{RR} = \exp -i\eta(a^*\tilde{a}^* - b^{i*}\tilde{b}^i)|0> \otimes |\tilde{0}>$$

The complete boundary state is already invariant under orbifold rotations, for which

$$\beta_n^a \to g_a\beta_n^a \quad , \quad \beta_n^{a*} \to g_a^*\beta_n^{a*}$$

$$\chi_n^a \to g_a\chi_n^a \quad , \quad \chi_n^{a*} \to g_a^*\chi_n^{a*}$$

$$b^a \to g_a b^a \quad , \quad b^{a*} \to g_a^* b^{a*}$$

For a boost of rapidity v:

$$\alpha_n \to e^{-v}\alpha_n \quad , \quad \beta_n \to e^{v}\beta_n$$

$$\chi_n^A \to e^{-v}\chi_n^A \quad , \quad \chi_n^B \to e^{v}\chi_n^B$$

$$a \to e^{-v}a \quad , \quad a^* \to e^{v}a^*$$

The spinor vacuum is not invariant, but transforms as

$$|0>\otimes|\tilde{0}>\to e^{-v}|0>\otimes|\tilde{0}>$$

Finally, the complete boosted boundary state is

$$|B,V>_B = \exp\frac{1}{2}\sum_{n>0}(e^{-2v}\alpha_{-n}\tilde{\alpha}_{-n} + e^{2v}\beta_{-n}\tilde{\beta}_{-n} + \beta^i_{-n}\tilde{\beta}^{i*}_{-n} + \beta^{i*}_{-n}\tilde{\beta}^i_{-n})|0>$$

$$|B_{osc},V,\eta>_F = \exp\frac{i\eta}{2}\sum_{n>0}(e^{-2v}\chi^A_{-n}\tilde{\chi}^A_{-n} + e^{2v}\chi^B_{-n}\tilde{\chi}^B_{-n}$$
$$-\chi^i_{-n}\tilde{\chi}^{i*}_{-n} - \chi^{i*}_{-n}\tilde{\chi}^i_{-n})|0>$$

$$|B_o,V,\eta>_{RR} = e^{-v}\exp -i\eta(e^{2v}a^*\tilde{a}^* - b^{i*}\tilde{b}^i)|0>\otimes|\tilde{0}>$$

In both sectors one gets

$$(-1)^F|B,V,\eta> = -|B,V,-\eta>$$

and the GSO-projected boundary state is

$$|B,V> = \frac{1}{2}(|B,V,+> - |B,V,->)$$

In the partition function, the ghosts cancel one untwisted pair, say 2-3, and the result is the product of the contributions of the 0-1 pair and the 3 compact pairs.

For the bosons, one finds ($q = e^{-2\pi l}$)

$$<B,V_1|e^{-lH}|B,V_2>_B^{(0,1)} = \prod_{n=1}^{\infty}\frac{1}{(1-e^{-2v}q^{2n})(1-e^{2v}q^{2n})}$$

$$<B,V_1|e^{-lH}|B,V_2>_B^{(a,a+1)} = \prod_{n=1}^{\infty}\frac{1}{(1-q^{2n})^2}$$

The total bosonic partition function is (zero-point energy $q^{-\frac{2}{3}}$)

$$Z_B = \frac{i}{\pi} \sinh v q^{-\frac{2}{3}} f(q^2)^{-8} \frac{\vartheta_1'(0|2il)}{\vartheta_1(i\frac{v}{\pi}|2il)}$$

For the fermions, the 0-1 pair gives

$$< B, V_1, \eta | e^{-lH} | B, V_2, \eta' >_F^{s(0,1)} = Z_o^s(\eta\eta') \prod_{n>0} (1 + \eta\eta' e^{-2v} q^{2n})(1 + \eta\eta' e^{2v} q^{2n})$$

with $\eta\eta' = \pm 1$ and

$$Z_o^R(+) = 2\cosh v \quad , \quad Z_o^R(-) = 2\sinh v$$
$$Z_o^{NS}(\pm) = 1$$

Each compact pair gives instead

$$< B, V_1, \eta | e^{-lH} | B, V_2, \eta' >_F^{s(a,a+1)} = Z_o^s(\eta\eta') \prod_{n>0} (1 + \eta\eta' q^{2n})^2$$

with

$$Z_o^R(+) = 2 \quad , \quad Z_o^R(-) = 0$$
$$Z_o^{NS}(\pm) = 1$$

After the GSO projection, only the three even spin structures R+ and NS± contribute, and (zero-point energy $q^{-\frac{1}{3}}$ for NSNS and $q^{\frac{2}{3}}$ for RR)

$$Z_F = q^{-\frac{1}{3}} f(q^2)^{-4} \Big\{ \vartheta_2(i\frac{v}{\pi}|2il)\vartheta_2(0|2il)^3 - \vartheta_3(i\frac{v}{\pi}|2il)\vartheta_3(0|2il)^3 + \vartheta_4(i\frac{v}{\pi}|2il)\vartheta_4(0|2il)^3 \Big\}$$
$$\sim V^4$$

corresponding to the usual SUSY cancellation of the force (Bachas).

0-brane: twisted sector

The boundary state is similar to the one of the untwisted sector, with fractional moding.

In the Z_3 case, each pair of compact bosons gives

$$< B, V_1|e^{-lH}|B, V_2 >_B^{(a,a+1)} = \prod_{n=1}^{\infty} \frac{1}{(1-q^{2(n-\frac{1}{3})})(1-q^{2(n-\frac{2}{3})})}$$

For a pair of compact fermions (no zero modes)

$$< B, V_1, \eta|e^{-lH}|B, V_2, \eta' >_R^{s(a,a+1)} = \prod_{n=1}^{\infty} (1+\eta\eta' q^{2(n-\frac{1}{3})})(1+\eta\eta' q^{2(n-\frac{2}{3})})$$

$$< B, V_1, \eta|e^{-lH}|B, V_2, \eta' >_{NS}^{s(a,a+1)} = \prod_{n=1}^{\infty} (1+\eta\eta' q^{2(n-\frac{1}{6})})(1+\eta\eta' q^{2(n-\frac{5}{6})})$$

The total partition functions after the GSO projection are (the zero-point energies add to zero)

$$Z_B = 2i\sinh v f(q^2)^4 \frac{1}{\vartheta_1(i\frac{v}{\pi}|2il)\vartheta_1(-\frac{2}{3}il|2il)^3}$$

$$Z_F = f(q^2)^{-4}\left\{\vartheta_2(i\frac{v}{\pi}|2il)\vartheta_2(-\frac{2}{3}il|2il)^3 \right.$$
$$\left. -\vartheta_3(i\frac{v}{\pi}|2il)\vartheta_3(-\frac{2}{3}il|2il)^3 - \vartheta_4(i\frac{v}{\pi}|2il)\vartheta_4(-\frac{2}{3}il|2il)^3\right\}$$
$$\sim V^2$$

In the Z_2 case, the analysis is similar and the results are

$$Z_B = 2i\sinh v q^{-\frac{1}{6}} f(q^2)^4 \frac{1}{\vartheta_1(i\frac{v}{\pi}|2il)\vartheta_1(0|2il)\vartheta_1(-il|2il)^2}$$

$$Z_F = q^{\frac{1}{6}} f(q^2)^{-4}\left\{\vartheta_2(i\frac{v}{\pi}|2il)\vartheta_2(0|2il)\vartheta_2(-il|2il)^2 \right.$$
$$-\vartheta_3(i\frac{v}{\pi}|2il)\vartheta_3(0|2il)\vartheta_3(-il|2il)^2$$
$$\left. -\vartheta_4(i\frac{v}{\pi}|2il)\vartheta_4(0|2il)\vartheta_4(-il|2il)^2\right\}$$
$$\sim V^2$$

3-brane

In the static case, take Neumann b.c. for time, Dirichlet b.c. for space and mixed b.c. for each pair of compact directions, say Neumann for the a directions and Dirichlet for the a+1 directions.

The new b.c. for the compact directions are

$$(\beta^a_n + \tilde\beta^{a*}_{-n})|B>_B = 0 \quad , \quad (\beta^{a*}_n + \tilde\beta^a_{-n})|B>_B = 0$$
$$(\chi^a_n + i\eta\tilde\chi^{a*}_{-n})|B_{osc},\eta>_F = 0 \quad , \quad (\chi^{a*}_n + i\eta\tilde\chi^a_{-n})|B_{osc},\eta>_F = 0$$
$$(b^a + i\eta\tilde b^{a*})|B_o,\eta>_F = 0 \quad , \quad (b^{a*} + i\eta\tilde b^a)|B_o,\eta>_F = 0$$

Defining a new spinor vacuum $|0> \otimes|\tilde 0>$ such that

$$b^a|0>=0 \ , \ \tilde b^a|\tilde 0>=0$$

the compact part of the boundary state is

$$|B>_B = \exp -\frac{1}{2}\sum_{n>0}(\beta^a_{-n}\tilde\beta^a_{-n} + \beta^{a*}_{-n}\tilde\beta^{a*}_{-n})|0>$$
$$|B_{osc},\eta>_F = \exp\frac{i\eta}{2}\sum_{n>0}(\chi^a_{-n}\tilde\chi^a_{-n} + \chi^{a*}_{-n}\tilde\chi^{a*}_{-n})|0>$$
$$|B_o,\eta>_{RR} = \exp -i\eta b^{a*}\tilde b^{a*}|0>\otimes|\tilde 0>$$

In this case, the boundary state is not invariant under orbifold rotations. Recall that ($g_a = e^{2\pi i z_a}$)

$$\beta^a_n \to g_a\beta^a_n \quad , \quad \beta^{a*}_n \to g^*_a\beta^{a*}_n$$
$$\chi^a_n \to g_a\chi^a_n \quad , \quad \chi^{a*}_n \to g^*_a\chi^{a*}_n$$
$$b^a \to g_a b^a \quad , \quad b^{a*} \to g^*_a b^{a*}$$

Moreover, the spinor vacuum now transform

$$|0>\otimes|\tilde 0> \to g_a|0>\otimes|\tilde 0>$$

The compact part of the twisted boundary state is

$$|B, V, g_a>_B = \exp -\frac{1}{2}\sum_{n>0}(g_a^2\beta^a_{-n}\tilde{\beta}^a_{-n} + g_a^{*2}\beta^{a*}_{-n}\tilde{\beta}^{a*}_{-n})|0>$$

$$|B_{osc}, V, g_a, \eta>_F = \exp\frac{i\eta}{2}\sum_{n>0}(g_a^2\chi^a_{-n}\tilde{\chi}^a_{-n} + g_a^{*2}\chi^{a*}_{-n}\tilde{\chi}^{a*}_{-n})|0>$$

$$|B_o, V, g_a, \eta>_{RR} = g_a \exp -i\eta g_a^{*2} b^{a*}\tilde{b}^{a*}|0> \otimes|\tilde{0}>$$

A pair of compact bosons gives ($(g_a^* g_a')^2 = e^{2\pi i w_a}$)

$$< B, V_1, g_a|e^{-lH}|B, V_2, g_a' >_B^{(a,a+1)} = \prod_{n=1}^{\infty}\left|\frac{1}{1+\eta\eta' e^{2\pi i w_a}q^{2n}}\right|^2$$

For fermions

$$< B, V_1, g_a, \eta|e^{-lH}|B, V_2, g_a', \eta' >_F^{s(a,a+1)} = Z_o^s(\eta\eta')\prod_{n>0}\left|1+\eta\eta' e^{2\pi i w_a}q^{2n}\right|^2$$

where

$$Z_o^R(+) = 2\cos\pi w_a \quad , \quad Z_o^R(-) = 2i\sin\pi w_a$$

$$Z_o^{NS}(\pm) = 1$$

After the GSO projection, the total partition functions for a given relative twist are

$$Z_B = 16i\sinh v q^{\frac{1}{3}} f(q^2)^4 \frac{1}{\vartheta_1(i\frac{v}{\pi}|2il)}\prod_a \frac{\sin\pi w_a}{\vartheta_1(w_a|2il)}$$

$$Z_F = q^{-\frac{1}{3}} f(q^2)^{-4}\Big\{\vartheta_2(i\frac{v}{\pi}|2il)\prod_a \vartheta_2(w_a|2il)$$

$$-\vartheta_3(i\frac{v}{\pi}|2il)\prod_a\vartheta_3(w_a|2il) + \vartheta_4(i\frac{v}{\pi}|2il)\prod_a\vartheta_4(w_a|2il)\Big\}$$

$$\sim \begin{cases} V^4 & , \quad w_a = 0 \\ V^2 & , \quad w_a \neq 0 \end{cases}$$

To obtain the invariant amplitude, one has to average over all possible orbifold twists. There is no twisted sector.

Large distance limit

Explicit expressions for $l \to \infty$

0-brane

a) Untwisted sector

$$\mathcal{A} \sim 4\cosh v - \cosh 2v - 3 \sim V^4$$

b) Twisted sector

$$\mathcal{A} \sim \cosh v - 1 \sim V^2$$

3-brane

$$\mathcal{A}(w_a) \sim 4\prod_a \cos \pi w_a \cosh v - \cosh 2v - \sum_a \cos 2\pi w_a$$

$$\mathcal{A} \sim \begin{cases} \cosh v - \cosh 2v \sim V^2 \quad , \quad T_6/Z_3 \\ 4\cosh v - \cosh 2v - 3 \sim V^4 \quad , \quad T_2 \otimes T_4/Z_2 \; , \; T_6 \end{cases}$$

Field theory interpretation

The possible contributions in the eikonal approximation are

Dilaton: $-a^2$

Vector: $e^2 \cosh v$

Graviton: $-M^2 \cosh 2v$

Thus

$4\cosh v - \cosh 2v - 3 \quad \Leftrightarrow \quad$ N=8 Grav. multiplet

$\cosh v - \cosh 2v \quad \Leftrightarrow \quad$ N=2 Grav. multiplet

$\cosh v - 1 \quad \Leftrightarrow \quad$ Vec. multiplet

V^2 terms

In the dual open string channel, V corresponds to an electric field E, and V^2 terms correspond to a renormalization of the Maxwell term E^2. This can not happen for the maximally supersymmetric theory.

The V^2 behavior is thus forbidden for $N = 8$ and allowed for $N < 8$; our results are compatible with this.

Spin effects

The 0-brane belongs to a BPS multiplet realizing half of the SUSY. Its $2^{N/2}$ components ($N = \#$ of SUSY generators) have different spins.

The annulus vacuum amplitude with Polchinski's prescription for the b.c gives only the universal spin-independent potential. The additional spin-dependent interactions are obtained by applying the broken SUSY.

In the closed string channel, one can construct higher spin boundary states in the G-S formalism by acting with the supercharges on the scalar one (Morales, Scrucca, Serone). On T_{10-D}, the complete potential is generically

$$V(r) = \sum_{k=0}^{4} \frac{V^{4-k}}{r^{D-3+k}}$$

In the dual open string channel, this structure is obvious. The path integral gives a vanishing result unless the 8 fermionic zero modes are soaked up.

The insertion of $2k$ supercharges provides $2k$ fermionic zero modes. The remaining $8 - 2k$ come from the expansion of the interaction

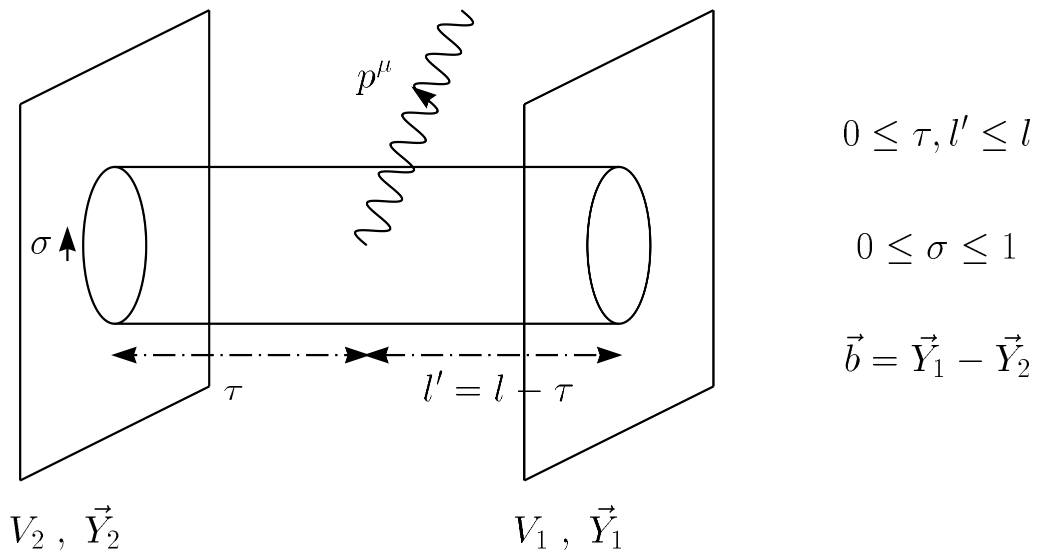
$$\exp \frac{V}{2} \int d\tau \left[X^0 \partial_\sigma X^1 - iS\gamma^{01} S\right]$$

yielding a factor $V^{(4-k)}$.

This result is consitent with string duality (Harvey).

EMISSION OF MASSLESS NSNS BOSONS

Consider two moving 0-branes in interaction emitting a massless NSNS boson.



The amplitude is computed inserting the vertex operator ($z = \sigma + i\tau$)

$$V(z,\bar{z}) = G_{ij}(\partial X^i - \frac{1}{2}p\cdot\psi\psi^i)(\bar{\partial}X^j + \frac{1}{2}p\cdot\bar{\psi}\bar{\psi}^j)e^{ip\cdot X}$$

between the two boundary states

$$\begin{aligned}\mathcal{A} &= \int_0^\infty dl \int_0^l d\tau \sum_s < B, V_1, \vec{Y}_1 | e^{-lH} V(z,\bar{z}) | B, V_2, \vec{Y}_2 >_s \\ &= \int_0^\infty d\tau \int_0^\infty dl' \sum_s < V(z,\bar{z}) >_s\end{aligned}$$

Split the bosons into zero mode and oscillators to be treated separately (again $X^\mu \equiv X^\mu_{osc}$).

The zero mode part gives the kinematics ($p^\mu = k^\mu_B - q^\mu_B$).

The energies and longitudinal momenta are completely fixed ($\cos\theta = \frac{p^1}{p}$, $p = p^0$)

$$k^0_B = V_1 k^1_B \quad , \quad k^1_B = \frac{p}{V_1 - V_2}(1 - V_2\cos\theta)$$

$$q^0_B = V_2 q^1_B \quad , \quad q^1_B = \frac{p}{V_1 - V_2}(1 - V_1\cos\theta)$$

The zero mode contribution is ($v = v_1 - v_2$)

$$< e^{ip\cdot X} >_o = \frac{1}{\sinh v} \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} e^{-\frac{q^2}{2}\tau} e^{-\frac{k^2}{2}l'}$$

Further zero mode insertions give

$$\begin{aligned} \partial X_o^i &\Rightarrow -\frac{1}{2}k_B^i \\ \bar{\partial} X_o^j &\Rightarrow \frac{1}{2}k_B^j \\ \partial X_o^i \bar{\partial} X_o^j &\Rightarrow -\frac{1}{4}k_B^i k_B^j \end{aligned}$$

Finally, the amplitude is (from now on $q^\mu \equiv q_B^\mu$ and $k^\mu \equiv k_B^\mu$)

$$\mathcal{A} = \frac{1}{\sinh v} \int_0^\infty d\tau \int_0^\infty dl' \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} e^{-\frac{q^2}{2}\tau} e^{-\frac{k^2}{2}l'} < e^{ip\cdot X} > \sum_s Z_B Z_F^s \mathcal{M}_s$$

with

$$\begin{aligned} \mathcal{M}^s = G_{ij} \Big\{ &< \partial X^i \bar{\partial} X^j > - < \partial X^i p\cdot X >< \bar{\partial} X^j p\cdot X > \\ &+\frac{1}{4}\Big(< p\cdot\psi p\cdot\bar{\psi} >_s < \psi^i \bar{\psi}^j >_s - < p\cdot\psi\psi^i >_s < p\cdot\bar{\psi}\bar{\psi}^j >_s \\ &\qquad + < p\cdot\bar{\psi}\psi^i >_s < p\cdot\psi\bar{\psi}^j >_s \Big) \\ &+\frac{i}{2}\Big(< \partial X^i p\cdot X >< p\cdot\bar{\psi}\bar{\psi}^j >_s - < \bar{\partial} X^j p\cdot X >< p\cdot\psi\psi^i >_s \Big) \\ &-\frac{1}{2}k^i \left(i < \bar{\partial} X^j p\cdot X > + \frac{1}{2} < p\cdot\bar{\psi}\bar{\psi}^j >_s \right) \\ &+\frac{1}{2}k^j \left(i < \partial X^i p\cdot X > - \frac{1}{2} < p\cdot\psi\psi^i >_s \right) \\ &-\frac{1}{4}k^i k^j \Big\} \end{aligned}$$

Obviously, the partition function factorizes, leaving connected correlators. In the odd spin structure, appropriate zero modes insertion is understood.

Correlators

The boundary state provides a systematic way of computing correlators with non trivial b.c.

$$< X^\mu X^\nu >= \frac{< B_1, V_1 | e^{-lH} X^\mu X^\nu | B_2, V_2 >_B}{< B_1, V_1 | e^{-lH} | B_2, V_2 >_B}$$

$$< \psi^\mu \psi^\nu >_s= \frac{< B_1, V_1, \eta | e^{-lH} \psi^\mu \psi^\nu | B_2, V_2, \eta' >_F^s}{< B_1, V_1, \eta | e^{-lH} | B_2, V_2, \eta' >_F^s}$$

For the bosons, one obtains ($q = e^{-2\pi\tau}$)

$$\begin{aligned} &< X^0(z)\bar{X}^0(\bar{z}) >=< X^1(z)\bar{X}^1(\bar{z}) >= \\ &\quad = \frac{1}{4\pi}\sum_{n=0}^{\infty}\Big\{\cosh 2[(v_1 - v_2)n - v_2]\ln(1 - q^{2n}e^{-4\pi\tau}) \\ &\qquad\qquad - \cosh 2[(v_2 - v_1)n - v_1]\ln(1 - q^{2n}e^{-4\pi l'})\Big\} \end{aligned}$$

$$\begin{aligned} &< X^0(z)\bar{X}^1(\bar{z}) >=< X^1(z)\bar{X}^0(\bar{z}) >= \\ &\quad = -\frac{1}{4\pi}\sum_{n=0}^{\infty}\Big\{\sinh 2[(v_1 - v_2)n - v_2]\ln(1 - q^{2n}e^{-4\pi\tau}) \\ &\qquad\qquad + \sinh 2[(v_2 - v_1)n - v_1]\ln(1 - q^{2n}e^{-4\pi l'})\Big\} \end{aligned}$$

For the fermions in the NS± sectors, the results are

$$\begin{aligned} &< \psi^0(z)\bar{\psi}^0(\bar{z}) >_{NS\pm}=< \psi^1(z)\bar{\psi}^1(\bar{z}) >_{NS\pm}= \\ &\quad = -i\sum_{n=0}^{\infty}(\mp)^n\left\{\cosh 2[(v_1 - v_2)n - v_2]\frac{q^n e^{-2\pi\tau}}{1 - q^{2n}e^{-4\pi\tau}}\right. \\ &\qquad\qquad \left.\pm \cosh 2[(v_2 - v_1)n - v_1]\frac{q^n e^{-2\pi l'}}{1 - q^{2n}e^{-4\pi l'}}\right\} \end{aligned}$$

$$\begin{aligned} &< \psi^0(z)\bar{\psi}^1(\bar{z}) >_{NS\pm}=< \psi^1(z)\bar{\psi}^0(\bar{z}) >_{NS\pm}= \\ &\quad = i\sum_{n=0}^{\infty}(\mp)^n\left\{\sinh 2[(v_1 - v_2)n - v_2]\frac{q^n e^{-2\pi\tau}}{1 - q^{2n}e^{-4\pi\tau}}\right. \\ &\qquad\qquad \left.\pm \sinh 2[(v_2 - v_1)n - v_1]\frac{q^n e^{-2\pi l'}}{1 - q^{2n}e^{-4\pi l'}}\right\} \end{aligned}$$

For the fermions in the R$\pm$ sectors, the results are similar, with a zero mode contribution

$$
\begin{aligned}
&< \psi^0(z)\bar{\psi}^0(\bar{z}) >_{R\pm}=< \psi^1(z)\bar{\psi}^1(\bar{z}) >_{R\pm}= \\
&\quad = F_o^R(\pm) - i\sum_{n=0}^{\infty}(\mp)^n \left\{ \cosh 2[(v_1-v_2)n - v_2]\frac{q^{2n}e^{-4\pi\tau}}{1-q^{2n}e^{-4\pi\tau}} \right. \\
&\qquad\qquad \left. \pm \cosh 2[(v_2-v_1)n - v_1]\frac{q^{2n}e^{-4\pi l'}}{1-q^{2n}e^{-4\pi l'}} \right\} \\
&\quad F_o^R(+) = -\frac{i}{2}\frac{\cosh(v_1+v_2)}{\cosh(v_1-v_2)} \;\; , \;\; F_o^R(-) = -\frac{i}{2}\frac{\sinh(v_1+v_2)}{\sinh(v_1-v_2)}
\end{aligned}
$$

$$
\begin{aligned}
&< \psi^0(z)\bar{\psi}^1(\bar{z}) >_{R\pm}=< \psi^1(z)\bar{\psi}^0(\bar{z}) >_{R\pm}= \\
&\quad = G_o^R(\pm) + i\sum_{n=0}^{\infty}(\mp)^n \left\{ \sinh 2[(v_1-v_2)n - v_2]\frac{q^{2n}e^{-4\pi\tau}}{1-q^{2n}e^{-4\pi\tau}} \right. \\
&\qquad\qquad \left. \pm \sinh 2[(v_2-v_1)n - v_1]\frac{q^{2n}e^{-4\pi l'}}{1-q^{2n}e^{-4\pi l'}} \right\} \\
&\quad G_o^R(+) = -\frac{i}{2}\frac{\sinh(v_1+v_2)}{\cosh(v_1-v_2)} \;\; , \;\; G_o^R(-) = -\frac{i}{2}\frac{\cosh(v_1+v_2)}{\sinh(v_1-v_2)}
\end{aligned}
$$

World-sheet SUSY means (for osc.)

$$
< \partial X^\mu(z)\bar{X}^\nu(\bar{z}) >= \frac{1}{2} < \psi^\mu(z)\bar{\psi}^\nu(\bar{z}) >_{R-}
$$

There are also non vanishing equal-point correlators, which can be computed in the same way. They can also be deduced from the previous ones using the b.c.

The correlators can be expressed in terms of twisted ϑ-functions.

Form the combinations $\psi^{\pm} = e^{\mp v_2}(\psi^0 \pm \psi^1)$, satisfying the b.c.

$$\psi^{\pm}(z) = -i\bar{\psi}^{\mp}(\bar{z}) \quad , \quad \tau = 0 \;\Leftrightarrow\; z = \bar{z}$$

$$\psi^{\pm}(z) = -ie^{\pm 2v}\bar{\psi}^{\mp}(\bar{z}) \quad , \quad \tau = l \;\Leftrightarrow\; z = \bar{z} + 2il$$

The propagators

$$P^{s}_{(\pm)}(z - \bar{z}) =< \psi^{\pm}(z)\bar{\psi}^{\pm}(\bar{z}) >_s$$

should have appropriate periodicity conditions on the covering torus with modulus 2il from which the cylinder can be obtained by the involution $z \doteq \bar{z} + 2il$.

In fact, under

$$w \to w + m + 2iln$$

the propagators transform as

$$P^{R+}_{(\pm)}(w + m + 2iln) = e^{i\pi n}e^{\pm 2nv}P^{R+}_{(\pm)}(w)$$

$$P^{R-}_{(\pm)}(w + m + 2iln) = e^{\pm 2nv}P^{R-}_{(\pm)}(w)$$

$$P^{NS+}_{(\pm)}(w + m + 2iln) = e^{i\pi m}e^{i\pi n}e^{\pm 2nv}P^{NS+}_{(\pm)}(w)$$

$$P^{NS-}_{(\pm)}(w + m + 2iln) = e^{i\pi m}e^{\pm 2nv}P^{NS-}_{(\pm)}(w)$$

These properties, together with the universal local behavior

$$P^{s}_{(\pm)}(w) \to \frac{1}{4\pi w}$$

imply for the even spin structures:

$$P^{s}_{(\pm)}(w) = \frac{1}{4\pi}\frac{\vartheta_s(w \pm i\frac{v}{\pi}|2il)\vartheta_1'(0|2il)}{\vartheta_s(\pm i\frac{v}{\pi}|2il)\vartheta_1(w|2il)}$$

Results

Axion

$$G_{ij} = \frac{1}{2}\epsilon_{ijk}\frac{p^k}{p}$$

Only the odd spin structure can contribute because of the antisymmetry of G_{ij}. In the twisted sector of the Z_3 case, there are only two fermionic zero modes in the 2-3 pair, and the amplitude could be non vanishing.

After integrating by parts the two-derivative bosonic term, world-sheet SUSY leads to

$$\mathcal{M}^{R-}_{ax} = \frac{i}{8}\cos\theta\left[-\partial_\tau < p\cdot X(z) p\cdot \bar{X}(\bar{z}) > +\frac{1}{2}(k^2-q^2)\right]$$

Since $\partial_\tau|_l = \partial_\tau|_{l'} - \partial_{l'}|_\tau$ the final amplitude is a total derivative ($Z_B Z_F^{R-} = 2\sinh v$ for the twisted sector of Z_3)

$$\begin{aligned}\mathcal{A}_{ax} &= \frac{i}{4}\cos\theta\int_0^\infty d\tau\int_0^\infty dl'\int\frac{d^2\vec{k}_T}{(2\pi)^2}e^{i\vec{k}\cdot\vec{b}}(\partial_\tau-\partial_{l'})\left\{e^{-\frac{q^2}{2}\tau}e^{-\frac{k^2}{2}l'} < e^{ip\cdot X} >\right\} \\ &= 0\end{aligned}$$

Dilaton

$$G_{ij} = \delta_{ij} - \frac{p^i p^j}{p^2}$$

Only the even spin structures contribute, because of the symmetry of G_{ij}. Again, the two-derivative bosonic term is integrated by parts.

In the large distance limit, one keep only leading terms for $l \to \infty$ in the propagators, and

$$< e^{ip\cdot X} >= \left(1-e^{-4\pi\tau}\right)^{-\frac{p^{(2)2}}{2\pi}}\left(1-e^{-4\pi l'}\right)^{-\frac{p^{(1)2}}{2\pi}}$$

with the boosted energies

$$p^{(1,2)} = p\gamma_{1,2}(1-V_{1,2}\cos\theta) = p(\cosh v_{1,2} - \sinh v_{1,2}\cos\theta)$$

One finds for the contractions

$$\mathcal{M}_{dil}^{R+} = \frac{1}{4p^2}\left[(k^2-q^2) - 2p^2\cos\theta\tanh v\right] \times$$
$$\left\{\frac{1}{4}(k^2-q^2) - p^{(2)2}\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}} + p^{(1)2}\frac{e^{-4\pi l'}}{1-e^{-4\pi l'}}\right\}$$
$$-\frac{k^0}{p}\left(\frac{q^2}{4} + p^{(2)2}\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}}\right) + \frac{q^0}{p}\left(\frac{k^2}{4} + p^{(1)2}\frac{e^{-4\pi l'}}{1-e^{-4\pi l'}}\right)$$

$$\mathcal{M}_{dil}^{NS\pm} = \frac{1}{4p^2}\left[(k^2-q^2) \mp 8e^{-2\pi l}p^2\cos\theta\sinh v\right] \times$$
$$\left\{\frac{1}{4}(k^2-q^2) - p^{(2)2}\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}} + p^{(1)2}\frac{e^{-4\pi l'}}{1-e^{-4\pi l'}}\right\}$$
$$-\frac{k^0}{p}\left(\frac{q^2}{4} + p^{(2)2}\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}}\right) + \frac{q^0}{p}\left(\frac{k^2}{4} + p^{(1)2}\frac{e^{-4\pi l'}}{1-e^{-4\pi l'}}\right)$$

Taking into account $< e^{ip\cdot X} >$ and integrating by parts in the final amplitude, one gets the rules

$$\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}} \doteq -\frac{1}{4}\frac{q^2}{p^{(2)2}} \quad , \quad \frac{e^{-4\pi l'}}{1-e^{-4\pi l'}} \doteq -\frac{1}{4}\frac{k^2}{p^{(1)2}}$$

Using these equivalence relations, one finds

$$\mathcal{M}_{dil}^{R+} = \mathcal{M}_{dil}^{NS\pm} = 0$$

This means that the amplitude is a total derivative and

$$\mathcal{A}_{dil} = 0$$

Graviton

$$G_{ij} = h_{ij} = h_{ji} \quad , \quad p^i h_{ij} = h^i_i = 0$$

Proceeding as for the dilaton, one obtains for $l \to \infty$

$$\begin{aligned}
&\mathcal{M}^{R+}_{grav} = \\
&\quad -\frac{1}{4}\left[h_{ij}k^i k^j - p\tanh v h_{i1}k^i\right] \\
&\quad -V_2\gamma_2\left[p^{(2)}\left(h_{i1}k^i - \frac{p}{2}\tanh v h_{11}\right) + \frac{1}{4}(k^2-q^2)V_2\gamma_2 h_{11}\right]\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}} \\
&\quad +V_1\gamma_1\left[p^{(1)}\left(h_{i1}k^i - \frac{p}{2}\tanh v h_{11}\right) + \frac{1}{4}(k^2-q^2)V_1\gamma_1 h_{11}\right]\frac{e^{-4\pi l'}}{1-e^{-4\pi l'}}
\end{aligned}$$

$$\begin{aligned}
&\mathcal{M}^{NS\pm}_{grav} = \\
&\quad -\frac{1}{4}\left[h_{ij}k^i k^j \mp 4e^{-2\pi l}\left(p\sinh 2v h_{i1}k^i - p^2\sinh^2 v h_{11}\right)\right] \\
&\quad -V_2\gamma_2\left[p^{(2)}\left(h_{i1}k^i \mp 2e^{-2\pi l}p\sinh v h_{11}\right) + \frac{1}{4}(k^2-q^2)V_2\gamma_2 h_{11}\right]\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}} \\
&\quad +V_1\gamma_1\left[p^{(1)}\left(h_{i1}k^i \mp 2e^{-2\pi l}p\sinh v h_{11}\right) + \frac{1}{4}(k^2-q^2)V_1\gamma_1 h_{11}\right]\frac{e^{-4\pi l'}}{1-e^{-4\pi l'}}
\end{aligned}$$

One can use the same equivalence relations as before to write $\mathcal{M}^s_{grav}$ in a τ, l'-independent form. Anyway:

$$\mathcal{A}_{grav} \neq 0$$

The general structure of the amplitude is

$$\mathcal{A}_{grav} = \frac{1}{\sinh v} \int_0^\infty d\tau \int_0^\infty dl' \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} e^{-\frac{q^2}{2}\tau} e^{-\frac{k^2}{2}l'} \times$$
$$\left(1 - e^{-4\pi\tau}\right)^{-\frac{p^{(2)2}}{2\pi}} \left(1 - e^{-4\pi l'}\right)^{-\frac{p^{(1)2}}{2\pi}} \sum_s Z_B Z_F^s \mathcal{M}^s_{grav}$$
$$= \frac{1}{\sinh v} \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} I_1 I_2 \sum_s Z_B Z_F^s \mathcal{M}^s_{grav}$$

with

$$\mathcal{M}^s_{grav} = B^s(p,k,q) + q^2 C_1^s(p,k,q) + k^2 C_2^s(p,k,q)$$

The kinematical integrals over the two proper times τ, l' give

$$I_1 = \int_0^\infty d\tau e^{-\frac{q^2}{2}\tau} \left(1 - e^{-4\pi\tau}\right)^{-\frac{p^{(2)2}}{2\pi}} = -\frac{1}{4\pi} \frac{\Gamma[\frac{q^2}{8\pi}]\Gamma[-\frac{p^{(2)2}}{2\pi}+1]}{\Gamma[\frac{q^2}{8\pi} - \frac{p^{(2)2}}{2\pi}+1]}$$
$$I_2 = \int_0^\infty dl' e^{-\frac{k^2}{2}l'} \left(1 - e^{-4\pi l'}\right)^{-\frac{p^{(1)2}}{2\pi}} = -\frac{1}{4\pi} \frac{\Gamma[\frac{k^2}{8\pi}]\Gamma[-\frac{p^{(1)2}}{2\pi}+1]}{\Gamma[\frac{k^2}{8\pi} - \frac{p^{(1)2}}{2\pi}+1]}$$

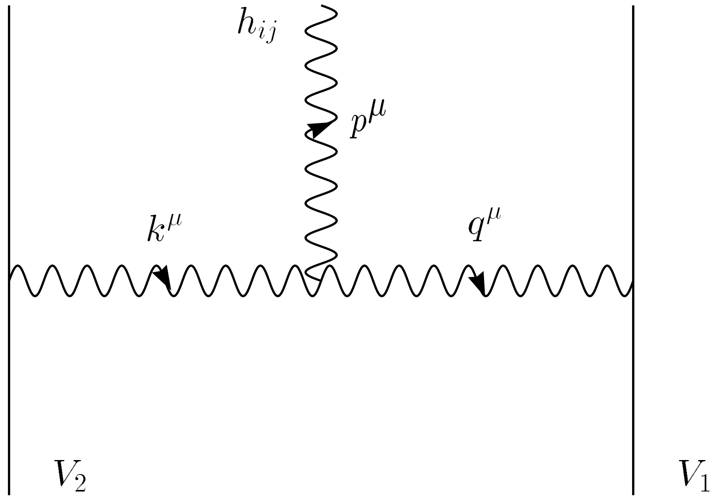
One finds the usual dual structure with a double serie of poles. However, in the eikonal approximation $p \ll M = 1$ and

$$I_1 \to -\frac{2}{q^2} \quad , \quad I_2 \to -\frac{2}{k^2}$$

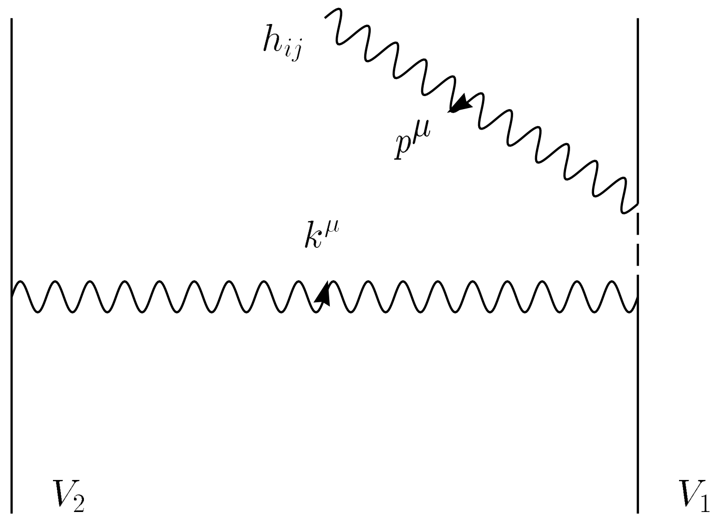
Finally, the amplitude becomes

$$\mathcal{A}_{grav} = \frac{4}{\sinh v} \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} \left\{ B^s \frac{1}{q^2k^2} + C_1^s \frac{1}{k^2} + C_2^s \frac{1}{q^2} \right\}$$

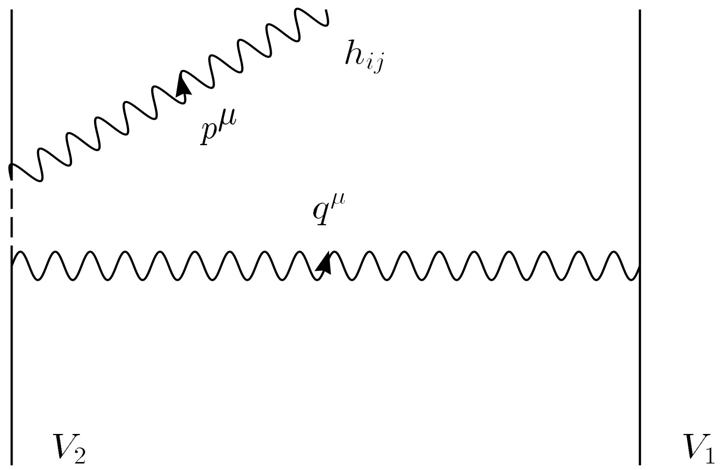
The graphical interpretation is the following



$$\Rightarrow \quad B^s \frac{1}{q^2 k^2} \quad (\tau, l' \neq 0)$$



$$\Rightarrow \quad C_1^s \frac{1}{k^2} \quad (\tau = l, l' = 0)$$



$$\Rightarrow \quad C_2^s \frac{1}{q^2} \quad (\tau = 0, l' = l)$$

Annihilation term

Field theory results

For the axion and the dilaton, there is no coupling in SUGRA allowing the emission process.

For the annihilation term of the graviton, there are three possible diagrams in SUGRA, involving the exchange of R vectors and NS dilatons and gravitons. Their respective contributions in the eikonal approximation are

$$
\begin{aligned}
B^R_{V_\mu} &= e^2 \left[\cosh v h_{ij} k^i k^j - p \sinh v h_{i1} k^i\right] \\
B^{NS}_{\phi} &= -a^2 h_{ij} k^i k^j \\
B^{NS}_{g_{\mu\nu}} &= -M^2 \left[\cosh 2v h_{ij} k^i k^j - 2p \sinh 2v h_{i1} k^i + 2p^2 \sinh^2 v h_{11}\right]
\end{aligned}
$$

String results

The string results in the various compactification schemes are the following

0-brane: untwisted sector & **3-brane on** $T_2 \otimes T_4/Z_2, T_6$

$$
\begin{aligned}
& Z^{R+} - Z^{NS+} + Z^{NS-} \rightarrow 16 \cosh v - 4 \cosh 2v - 12 \\
& Z^{NS+} + Z^{NS-} \rightarrow 2 e^{2\pi l}
\end{aligned}
$$

and

$$
\begin{aligned}
B^R_{grav} &= 4 \left[\cosh v h_{ij} k^i k^j - p \sinh v h_{i1} k^i\right] \\
B^{NS}_{grav} &= - \left[\cosh 2v h_{ij} k^i k^j - 2p \sinh 2v h_{i1} k^i + 2p^2 \sinh^2 v h_{11}\right] \\
&\qquad -3 h_{ij} k^i k^j \\
\Rightarrow B_{grav} &\sim V^4 h_{ij} k^i k^j + V^3 p h_{i1} k^i + V^2 p^2 h_{11}
\end{aligned}
$$

0-brane: twisted sector

$$Z^{R+} - Z^{NS+} - Z^{NS-} \to 4\cosh v - 4$$

$$Z^{NS+} - Z^{NS-} \to 0$$

and

$$B^R_{grav} = \left[\cosh v h_{ij} k^i k^j - p \sinh v h_{i1} k^i\right]$$

$$B^{NS}_{grav} = -h_{ij} k^i k^j$$

$$\Rightarrow B_{grav} \sim V^2 h_{ij} k^i k^j + V p h_{i1} k^i + V^2 p^2 h_{11}$$

3-brane on T_6/Z_3

$$Z^{R+} - Z^{NS+} + Z^{NS-} \to 4\cosh v - 4\cosh 2v$$

$$Z^{NS+} + Z^{NS-} \to 2e^{2\pi l}$$

and

$$B^R_{grav} = \left[\cosh v h_{ij} k^i k^j - p \sinh v h_{i1} k^i\right]$$

$$B^{NS}_{grav} = -\left[\cosh 2v h_{ij} k^i k^j - 2p \sinh 2v h_{i1} k^i + 2p^2 \sinh^2 v h_{11}\right]$$

$$\Rightarrow B_{grav} \sim V^2 h_{ij} k^i k^j + V p h_{i1} k^i + V^2 p^2 h_{11}$$

Colinear emission

At $\theta = 0$ one finds

$$B_{grav} \sim V^n h_{ij} k^i k^j$$

$$C_{1grav} = C_{2grav} = 0$$

with $n = 2, 4$ depending on the amount of SUSY.

Radiated energy

The average energy radiated when the two branes pass each other at impact parameter $\vec{b}$ is

$$< p >\sim \int \frac{d^3\vec{p}}{p} p \left|\mathcal{A}\right|^2$$

For $\theta = 0$ and $V \ll 1$ one has

$$\mathcal{A} \sim V^{n-1} g_s l_s f(\frac{p \cdot b}{V}) e^{-\frac{p \cdot b}{V}}$$

where is a slowly varying function and $n = 2, 4$. Notice that the emission is exponentially suppressed for $p \sim p_{max} = V/b$.

By dimensional analysis one finds

$$< p >\sim g_s^2 l_s^2 \frac{V^{1+2n}}{b^3}$$

Extrapolating down to the $b \sim l_{11} = g_s^{1/3} l_s$ one would find

$$< p >_{11}\sim \frac{g_s}{l_s} V^{1+2n}$$

This small distance extrapolation could be invalidated by additional dynamical effects, like open strings pair creation for $b \sim \sqrt{V} l_s$ (Bachas) or massive states exchange if $p_{max} \sim l_s^{-1}$.

There could be also a kinematical breakdown of the eikonal approximation if $p_{max} \sim p_{br} = V/(g_s l_s)$. However, even for $b \sim l_{11}$, all these effects are negligible for small relative velocity $V \ll g_s^{2/3}$.