METASTABLE SPONTANEOUS BREAKING OF N = 1 AND N = 2 SUPERSYMMETRY

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- Vacuum metastability and sGoldstino masses.
- Metastability constraints in N = 1 models.
- Metastability constraints in N = 2 models.
- Examples within simple classes of models.

Based on works with F. Catino, M. Gomez-Reino, J.-C. Jacot, B. Légeret, J. Louis and P. Smyth

METASTABILITY AND SGOLDSTINO MASSES

Vacuum

Vacua correspond to constant values of the fields with zero kinetic energy T and minimal potential energy V. One has $V' = 0$, whereas $V = \Lambda^4$ defines the vacuum energy and $V'' = m^2$ the fluctuation mass matrix.

In supersymmetric theories, the form of V is constrained. Vacua then display special features. The main issue is to get $\Lambda^4 > 0$ and $m^2 > 0$.

Supersymmetry breaking and metastability

Denef, Douglas 2005
Gomez-Reino, Scrucca 2006

When supersymmetry is broken, there is a Goldstino fermion which has zero mass. Its partners the sGoldstino bosons have masses controlled by breaking effects and difficult to adjust.

Requiring positive sGoldstino masses leads to metastability conditions. Gravity gives only quantitative modifications compared to the rigid case.

N = 1 SUSY WITH CHIRAL MULTIPLETS

General structure of the theory

Zumino 1979
Freedman, Alvarez-Gaumé 1981

In a theory involving n_c chiral multiplets, the n_c complex scalars ϕ^i are described by:

$$T = -g_{i\bar{\jmath}} \partial_\mu \phi^i \partial^\mu \bar{\phi}^{\bar{\jmath}}$$
$$V = g_{i\bar{\jmath}} F^i F^{\bar{\jmath}}$$

The metric $g_{i\bar{\jmath}}$ defines a Kähler geometry admitting a real Kähler potential K and a complex structure $J^i{}_j = i\delta^i{}_j$:

$$g_{i\bar{\jmath}} = K_{i\bar{\jmath}} \text{ : Kähler}$$

The shift F^i is a complex vector set by a holomorphic superpotential W:

$$F^i = g^{i\bar{\jmath}} \bar{W}_{\bar{\jmath}}$$

Supersymmetry breaking

Supersymmetry is broken whenever $F^i \neq 0$, and there are two real scalar sGoldstini:

$$\varphi^{1,2} = {}^{\mathrm{Re}}_{\mathrm{Im}}(\bar{F}_i \phi^i)$$

Metastability

Gomez-Reino, Scrucca 2006

Using the stationarity condition one finds:

$$m^2_{\varphi^{1,2}} = R\, F^i \bar{F}_i \pm \Delta m^2_\varphi$$

where

$$R = -\frac{R_{i\bar{\jmath}k\bar{l}}\, F^i \bar{F}^{\bar{\jmath}} F^k \bar{F}^{\bar{l}}}{(F^n \bar{F}_n)^2}$$

Taking the average of $m^2_{\varphi^{1,2}}$, one finds a degenerate upper and lower bound for the smallest and largest scalar mass eigenvalue:

$$m^2 = R\, F^i \bar{F}_i$$

This gives a sharp constraint for metastability: one needs $R > 0$.

Minimal class of examples

For $n_c = 1$, we have

$$V = \frac{|W_\phi|^2}{K_{\phi\bar{\phi}}}$$

The stationarity condition implies that $W_{\phi\phi} = K^{-1}_{\phi\bar{\phi}} K_{\phi\phi\bar{\phi}} W_\phi$ and the scalar mass matrix is found to be:

$$m^2_{\phi\bar{\phi}} = R\,V \qquad m^2_{\phi\phi} = \Delta\,V$$

where

$$R = -\left(\frac{K_{\phi\phi\bar{\phi}\bar{\phi}}}{K^2_{\phi\bar{\phi}}} - \frac{|K_{\phi\phi\bar{\phi}}|^2}{K^3_{\phi\bar{\phi}}}\right) \qquad \Delta = \left(\frac{W_{\phi\phi\phi}}{W_\phi K_{\phi\bar{\phi}}} - \frac{K_{\phi\phi\bar{\phi}\bar{\phi}}}{K^2_{\phi\bar{\phi}}}\right)$$

The two mass eigenvalues are thus:

$$m^2_{1,2} = \big(R \pm |\Delta|\big)V$$

Supersymmetry is broken since $V \neq 0$, and the average of $m^2_{1,2}$ is

$$m^2 = R\,V$$

N = 1 SUSY WITH CHIRAL AND VECTOR MULTIPLETS

General structure of the theory

Bagger, Witten 1982
Hull, Karlhede, Lindstrom, Rocek 1986

In a theory with n_c chiral and n_v vector multiplets with Abelian gauge symmetries, the n_c complex scalars ϕ^i and n_v real vectors A^a_μ have:

$$T = -g_{i\bar{\jmath}} D_\mu \phi^i D^\mu \bar{\phi}^{\bar{\jmath}} - \frac{1}{4} h_{ab} F^a_{\mu\nu} F^{b\mu\nu} + \frac{1}{4} \theta_{ab} F^a_{\mu\nu} \tilde{F}^{b\mu\nu}$$
$$V = g_{i\bar{\jmath}} F^i F^{\bar{\jmath}} + \frac{1}{2} h_{ab} D^a D^b$$

The metric $g_{i\bar{\jmath}}$ defines a Kähler geometry admitting a real Kähler potential K and a complex structure $J^i{}_j = i\,\delta^i{}_j$, while h_{ab} and θ_{ab} are linked to a holomorphic matrix H_{ab}:

$$g_{i\bar{\jmath}} = K_{i\bar{\jmath}} \qquad h_{ab} = \mathrm{Re}(H_{ab}) \qquad \theta_{ab} = \mathrm{Im}(H_{ab})$$

Moreover $D_\mu \phi^i = \partial_\mu \phi^i + k^i_a A^a_\mu$ where k^i_a are Abelian holomorphic Killing vectors admitting real Killing potentials P_a so that $i\,k^i_a = -g^{i\bar{\jmath}} P_{a\bar{\jmath}}$.

The vector mass matrix is also determined by k_a^i and is given by:

$$M_a^{2\,b} = 2h^{bc} g_{i\bar{\jmath}} k_a^i \bar{k}_c^{\bar{\jmath}}$$

The shift F^i is a complex vector set by a holomorphic superpotential W, while D^a is related to the Killing potentials P_a where Fayet-Iliopoulos are excluded by consistency with gravity:

$$F^i = g^{i\bar{\jmath}} \bar{W}_{\bar{\jmath}} \qquad D^a = h^{ab} P_b$$

Stationarity further implies a relation through the charges $Q_{ai}{}^{j} = ik^j_{a j}$:

$$M^2_{ab} D^b = 2Q_{ai\bar{\jmath}} F^i \bar{F}^{\bar{\jmath}}$$

Supersymmetry breaking

Supersymmetry is broken whenever $F^i, D^a \neq 0$, and there are two real scalar sGoldstini:

$$\varphi^{1,2} = {}^{\mathrm{Re}}_{\mathrm{Im}}(\bar{F}_i \phi^i)$$

Metastability

Gomez-Reino, Scrucca 2007

Using the stationarity condition one finds

$$m^2_{\varphi^{1,2}} = R\, F^i \bar{F}_i + S\, D^a D_a + T\, \frac{(D^a D_a)^2}{4\, F^i \bar{F}_i} + M^2 \frac{D^a D_a}{F^i \bar{F}_i} \pm \Delta m^2_\varphi$$

where

$$R = -\frac{R_{i\bar{\jmath}k\bar{l}}\, F^i \bar{F}^{\bar{\jmath}} F^k \bar{F}^{\bar{l}}}{(F^n \bar{F}_n)^2} \qquad S = \frac{h_{aci} h^{cd} h_{db\bar{\jmath}}\, F^i \bar{F}^{\bar{\jmath}} D^a D^b}{F^n \bar{F}_k\, D^c D_c}$$

$$T = \frac{h_{abi} h_{cb}{}^i D^a D^b D^c D^d}{(D^e D_e)^2} \qquad M^2 = \frac{M^2_{ab} D^a D^b}{D^c D_c}$$

Averaging over $m^2_{\varphi^{1,2}}$, one finds a result with a new semi-positive term in the metastability bound:

$$m^2 = R\, F^i \bar{F}_i + \left(S\, F^i \bar{F}_i + \frac{1}{4} T\, D^a D_a + M^2 \right) \frac{D^b D_b}{F^j \bar{F}_j}$$

This gives a milder and more flexible constraint for metastability.

Minimal class of examples

For $n_c = n_v = 1$, we can take $K = K(\Phi + \bar{\Phi})$, $W = 0$, $h = 1$ and $k^\phi = i\xi$, $P = i\xi K'$. Then:

$$V = \frac{1}{2}\xi^2 |K'|^2 \qquad M^2 = 2\xi^2 K''$$

The stationarity condition fixes $K' = 0$ and the scalar mass matrix is:

$$m^2_{\phi\bar{\phi}} = \frac{1}{2}M^2 \qquad m^2_{\phi\phi} = \frac{1}{2}M^2$$

The two mass eigenvalues are thus:

$$m_0^2 = 0 \qquad m_1^2 = M^2$$

Supersymmetry is unbroken since $V = 0$, and m_1^2 coincides with the vector mass:

$$m^2 = M^2$$

N = 2 SUSY WITH HYPER MULTIPLETS

General structure of the theory

Alvarez-Gaumé, Freedman 1981
Hull, Karlhede, Lindstrom, Rocek 1986

In a theory with n_h hyper multiplets and a global central charge symmetry, the $4n_h$ real scalars q^u are described by:

$$T = -\frac{1}{2} g_{uv} \partial_\mu q^u \partial^\mu q^v$$
$$V = \frac{1}{2} g_{uv} N^u N^v$$

The metric g_{uv} defines a Hyper-Kähler geometry admitting three complex structures $J^{xu}{}_v$ such that $J^{xu}{}_w J^{yw}{}_v = -\delta^{xy}\delta^u{}_v + \epsilon^{xyz} J^{zu}{}_v$:

$$g_{uv} \text{ : Hyper-Kähler}$$

The shift N^u is related to a triholomorphic Killing vector k^u admitting three real Killing potentials P^x such that $(J^x k)^u = -g^{uv} P^x_v$:

$$N^u = 2k^u$$

Supersymmetry breaking

Supersymmetry is broken whenever $N^u \neq 0$, and there are four real scalar sGoldstini:

$$\varphi^0 = N_u q^u \qquad \varphi^x = (J^x N)_u q^u$$

Metastability

Gomez-Reino, Louis, Scrucca 2009
Jacot, Scrucca 2010

Using the stationarity condition one finds:

$$m^2_{\varphi^0} = 0 \quad m^2_{\varphi^x} = -R_x N^u N_u$$

where

$$R_x = \frac{R_{uvrs} N^u (J^x N)^v N^r (J^x N)^s}{(N^t N_t)^2}$$

Averaging of the $m^2_{\varphi^x}$ and using the property $\sum_x R_x = 0$ one finds:

$$m^2 = 0$$

This gives a no-go theorem against metastability.

Minimal class of examples

For $n_h = 1$, one needs a general 4-dimensional Hyper-Kähler manifold with a triholomorphic isometry. Locally, such a space can be described by a metric of the Gibbons-Hawking form, with coordinates $q^u = q^0, \vec{q}$:

$$ds^2 = f\, d\vec{q}^{\,2} + f^{-1}(dq^0 + \vec{\omega} \cdot d\vec{q}\,)^2$$

The real function f can depend only on $\vec{q}$ and must be harmonic:

$$\Delta f = 0$$

The three real functions $\vec{\omega}$ are then determined by the equation:

$$\vec{\nabla} \times \vec{\omega} = \vec{\nabla} f$$

The isometry acts a shift in the coordinate q^0 and is described by

$$k^u = \xi\, \delta^{u0} \qquad \vec{P} = \xi\, \vec{q}$$

The resulting potential depends only on q^x and is given by:

$$V = 2\xi^2 f^{-1}$$

The stationarity condition fixes $f_x = 0$ and the scalar mass matrix is:

$$m^2_{00} = 0 \qquad m^2_{0x} = 0 \qquad m^2_{xy} = -R_{xy} V$$

where

$$R_{xy} = f^{-2} f_{xy}$$

The four mass eigenvalues are thus:

$$m^2_0 = 0 \qquad m^2_x = -\text{eigen}_x(R_{yz})\, V$$

Supersymmetry is broken since $V \neq 0$, and as a consequence of the property $\delta^{xy} R_{xy} = 0$ the average of m^2_x is

$$m^2 = 0$$

N = 2 SUSY WITH VECTOR MULTIPLETS

De Wit, Van Proeyen 1984
Hull, Karlhede, Lindstrom, Rocek 1986
Castellani, D'auria, Frè 1991

General structure of the theory

In a theory with n_v vector multiplets and Abelian gauge symmetries, the n_v complex scalars z^i and n_v real vectors A^a_μ are described by:

$$T = -g_{i\bar{\jmath}}\partial_\mu z^i \partial^\mu \bar{z}^{\bar{\jmath}} - \frac{1}{4}h_{ab}F^a_{\mu\nu}F^{b\mu\nu} + \frac{1}{4}\theta_{ab}F^a_{\mu\nu}\tilde{F}^{b\mu\nu}$$
$$V = g_{i\bar{\jmath}}W^{ix}\bar{W}^{\bar{\jmath}x}$$

The metric $g_{i\bar{\jmath}}$ defines a Special-Kähler metric admitting a holomorphic prepotential F for some sections L^a and a complex structure $J^i{}_j = i\delta^i{}_j$, and h_{ab} and θ_{ab} are also directly related to it through $f^a_i = L^a_i$:

$$g_{i\bar{\jmath}} = \left[\mathrm{Im}\left(F_a \bar{L}^a\right)\right]_{i\bar{\jmath}} : \text{Special-Kähler} \qquad h_{ab} = g_{i\bar{\jmath}} f^i_a f^{\bar{\jmath}}_b$$

The vector mass matrix vanishes:

$$M^{2\,b}_a = 0$$

The shift W^{xi} is set by constant Fayet-Iliopoulos terms ξ_a^x, which must be aligned as $\xi_a^x = \xi_a v^x$ for consistency with gravity:

$$W^{ix} = W^i v^x = (f^{ia} \xi_a) v^x$$

Supersymmetry breaking

Supersymmetry is broken whenever $W^i \neq 0$, and there are two scalar sGoldstini:

$$\varphi^{1,2} = {}^{\mathrm{Re}}_{\mathrm{Im}} (W_i z^i)$$

Metastability

Cremmer, Kounnas, Van Proeyen, Derendinger, Ferrara, de Wit, Girardello 1985
Jacot, Scrucca 2010

Using the stationarity condition one finds:

$$m^2_{\varphi^{1,2}} = \pm \Delta m^2_\varphi$$

Averaging over $m^2_{\varphi^{1,2}}$ one finds a vanishing result:

$$m^2 = 0$$

This gives again a no-go theorem against metastability.

Minimal class of examples

For $n_v = 1$, one needs a general 2-dimensional Special-Kähler manifold. Locally, such a space can be described with special coordinates $z, \bar{z}$ such that $L = z$ and a metric of the form:

$$ds^2 = 2l\,|dz|^2$$

The real function l can depend on $z, \bar{z}$, but since it can be expressed as $l = \mathrm{Im}(F'')$ in terms of the holomorphic prepotential F, it must be harmonic:

$$l_{z\bar{z}} = 0$$

In addition, one has to choose some constant Fayet-Iliopoulos terms:

$$\xi^x = \xi\, v^x$$

The resulting potential depends on $z, \bar{z}$ and one has:

$$V = \xi^2 l^{-1} \qquad M^2 = 0$$

The stationarity condition fixes $l_z = 0$ and the scalar mass matrix is found to be:

$$m^2_{z\bar{z}} = 0 \qquad m^2_{zz} = -\Delta V$$

where

$$\Delta = l^{-2} l_{zz}$$

The two mass eigenvalues are thus:

$$m^2_{1,2} = \pm|\Delta|V$$

Supersymmetry is broken since $V \neq 0$, and the average of $m^2_{1,2}$ is

$$m^2 = 0$$

N = 2 SUSY WITH HYPER AND VECTOR MULTIPLETS

General structure of the theory

Hull, Karlhede, Lindstrom, Rocek 1986
D'auria, Ferrara, Frè 1991

In a theory with n_h hyper and n_v vector multiplets with Abelian gauge symmetries, the $4n_h$ scalars q^u, n_v scalars z^i and n_v vectors A_μ^a have:

$$T = -\frac{1}{2} g_{uv} D_\mu q^u D^\mu q^v - g_{i\bar{\jmath}} \partial_\mu z^i \partial^\mu \bar{z}^{\bar{\jmath}} - \frac{1}{4} h_{ab} F_{\mu\nu}^a F^{b\mu\nu} + \theta\text{-term}$$

$$V = \frac{1}{2} g_{uv} N^u N^v + g_{i\bar{\jmath}} W^{ix} \bar{W}^{\bar{\jmath}x}$$

The metrics g_{uv}, $g_{i\bar{\jmath}}$, h_{ab} and θ_{ab} have the same properties as before:

g_{uv} : Hyper-Kähler $\quad$ $g_{i\bar{\jmath}}$: Special-Kähler $\quad$ $h_{ab} = g_{i\bar{\jmath}} f_a^i \bar{f}_b^{\bar{\jmath}}$

Moreover $D_\mu q^u = \partial_\mu q^u + k_a^u A_\mu^a$ where k_a^u are Abelian triholomorphic Killing vectors with Killing potentials P_a^x so that $(J^x k_a)^u = -g^{uv} P_{av}^x$.

The vector mass matrix is given by:

$$M_a^{2\,b} = h^{bc} g_{uv} k_a^u k_c^v$$

The shift N^u is set by the k^u_a and an extra Abelian triholomorphic Killing vector k^u with Killing potential P^x such that $(J^x k)^u = -g^{uv} P^x_v$, while the W^{ix} are now related to the non-constant and non-aligned P^x_a:

$$N^u = 2\big(k^u + k^u_a L^a\big) \qquad W^{ix} = f^{ia} P^x_a$$

Supersymmetry breaking

Supersymmetry is broken if $N^u, W^{ix} \neq 0$, but the scalar sGoldstini are in this case more subtle to identify.

Metastability

Antoniadis, Buican 2010
Frè, Trigiante, van Proeyen 2002

In theories with an $SU(2)_R$ global symmetry it is impossible to realize supersymmetry non-linearly. The common lore is that Fayet-Iliopoulos terms are needed. This raises two questions:

- Is this no-go theorem again due to a sGoldstino instability ?
- What about theories without any $SU(2)_R$ global symmetry?

Minimal class of examples

Légeret, Scrucca, Smyth 2013

For $n_h = n_v = 1$, we need a 4-dimensional Hyper-Kähler manifold with a triholomorphic isometry and a 2-dimensional Special-Kähler manifold. These can be described with coordinates $q^0, \vec{q}$ and $z, \bar{z}$ and metric:

$$ds^2 = f\, d\vec{q}^{\,2} + f^{-1}(dq^0 + \vec{\omega}\cdot d\vec{q}\,)^2 + 2l\,|dz|^2$$

As before, f and l can depend only on $\vec{q}$ and $z, \bar{z}$ and must be harmonic:

$$\Delta f = 0 \qquad l_{z\bar{z}} = 0$$

The Killing vector and prepotentials associated to the isometry and the symplectic section are given by:

$$X^u = \xi\,\delta^{u0} \qquad \vec{P} = \xi\,\vec{q} \qquad L = z$$

The resulting potential and vector mass depend on $q^x, z, \bar{z}$ and read:

$$V = 2\,\xi^2 f^{-1}|z|^2 + \xi^2 l^{-1}|\vec{q}\,|^2 \qquad M^2 = 2\,\xi^2 f^{-1} l^{-1}$$

The stationarity request sets $f_x = f^2 l^{-1} |z|^{-2} q^x$ and $l_z = 2\, l^2 f^{-1} |\vec{q}\,|^{-2} \bar{z}$ and the scalar mass matrix is found to be:

$$m^2_{00} = 0 \quad m^2_{0x} = 0 \quad m^2_{xy} = \Big[\delta_{xy} + 4 \tan^2\theta\, v_x v_y - \frac{1}{2} \cot^2\theta\, A_{xy}\Big] M^2$$

$$m^2_{z\bar{z}} = \Big[1 + 2\cot^2\theta\Big] M^2 \quad m^2_{zz} = \Big[2\cot^2\theta - \tan^2\theta\, B\Big] e^{-2i\gamma} M^2$$

$$m^2_{0z} = 0 \quad m^2_{xz} = -\Big[\sqrt{2}\,\big(\tan\theta + \cot\theta\big)\, v_x\Big] e^{-i\gamma} M^2$$

where

$$\vec{v} = \frac{\vec{q}}{|\vec{q}\,|} \qquad e^{i\gamma} = \frac{z}{|z|} \qquad \tan^2\theta = \frac{1}{2}\frac{f}{l}\frac{|\vec{q}\,|^2}{|z|^2}$$

and

$$A_{xy} = f^{-1} |\vec{q}\,|^2 f_{xy} \qquad B = l^{-1} z^2 l_{zz}$$

The six mass eigenvalues are thus:

$$m^2_0 = 0 \qquad m^2_{1-5} = (\cdots)\, M^2$$

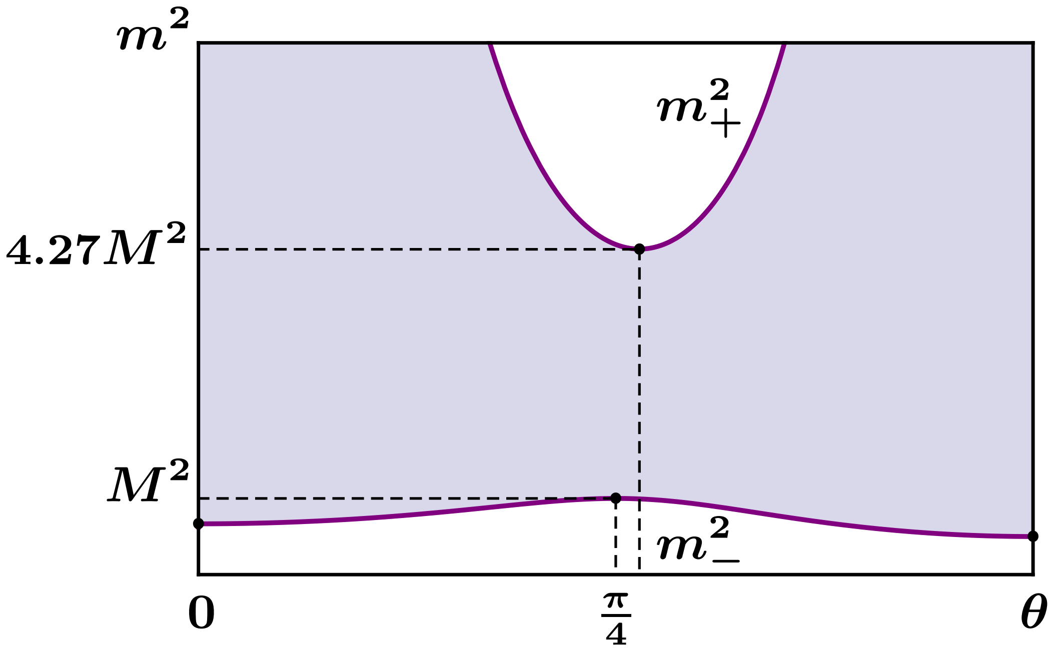
Supersymmetry is broken since $V \neq 0$, in a direction determined by θ. As a consequence of the property $\delta^{xy} A_{xy} = 0$ one finds that the non-trivial part of the mass matrix averaged within each sector reads:

$$m^2_{\mathrm{hh}} = \Big[1 + \frac{4}{3}\tan^2\theta\Big] M^2$$
$$m^2_{\mathrm{vv}} = \Big[1 + 2\cot^2\theta\Big] M^2$$
$$m^2_{\mathrm{hv}} = \Big[\sqrt{\frac{2}{3}}\left(\tan\theta + \cot\theta\right)\Big] M^2$$

The two eigenvalues of this averaged matrix are:

$$m^2_{\pm} = \Bigg[1 + \frac{2}{3}\tan^2\theta + \cot^2\theta \pm \sqrt{\frac{2}{3}\tan^2\theta + \frac{4}{9}\tan^4\theta + \frac{2}{3}\cot^2\theta + \cot^4\theta}\,\Bigg] M^2$$

The m^2_{1-5} must then spread beyond the interval between m^2_- and m^2_+. These thus represent bounds on the possible masses for a given θ.



The absolute upper and lower bounds on the lightest and heaviest masses are therefore found to be:

$$m_{\mathrm{up}}^2 = M^2 \qquad m_{\mathrm{low}}^2 \simeq 4.27\,M^2$$

METASTABILITY IN N = 1 SUGRA

Theories with only chiral multiplets

Gomes-Reino, Scrucca 2006

This case is easy to study in general. The geometry becomes Hodge-Kähler and one finds:

$$V = F^i F_i - 3\, m_{3/2}^2$$
$$m^2 = R\, F^i \bar{F}_i + 2\, m_{3/2}^2$$

This implies that

$$m^2 = \Big[(3+\epsilon)\, R + 2\Big] m_{3/2}^2$$

where

$$\epsilon = \frac{V}{m_{3/2}^2}$$

Metastable de Sitter vacua with $\epsilon > 0$ are therefore possible only when $R > -2/3$.

Theories with chiral and vector multiplets

Gomes-Reino, Scrucca 2007

This case is somewhat more complicated but still easy to study in general. One finds

$$V = F^i F_i + \frac{1}{2} D^a D_a - 3\, m_{3/2}^2$$

$$m^2 = R\, F^i \bar{F}_i + \Big[(S+1)\, F^i \bar{F}_i + \frac{1}{4} T\, D^a D_a + (M^2 - 4\, m_{3/2}^2)\Big] \frac{D^b D_b}{F^j \bar{F}_j} + 2\, m_{3/2}^2$$

This implies that

$$m^2 = \Big[\frac{3+\epsilon}{1+\delta}\big(R + 2\,\delta\,(S+1) + \delta^2 T\big) + \big(2 + 2\,\delta\,(\kappa - 4)\big)\Big] m_{3/2}^2$$

where

$$\epsilon = \frac{V}{m_{3/2}^2} \qquad \kappa = \frac{M^2}{m_{3/2}^2} \qquad \delta = \frac{1}{2} \frac{D^a D_a}{F^i F_i}$$

Metastable de Sitter vacua with $\epsilon > 0$ are now possible in more general situations.

Application to N = 1 strings

Brustein, de Alwis 2003
Gomez-Reino, Scrucca 2006

The universal chiral multiplet of $N = 1$ string models has:

$$\mathcal{M} = \frac{SU(1,1)}{U(1)} \quad \text{deformed by quantum corrections}$$

Metastable de Sitter vacua are possible only at strong coupling or with extra vector multiplet effects, because $R = -2 +$ corrections.

METASTABILITY IN N = 2 SUGRA

Theories with only hyper multiplets

Gomes-Reino, Louis, Scrucca 2009

This case can be studied in general. The geometry becomes Quaternionic-Kähler and one finds:

$$V = \frac{1}{2} N^u N_u - 3 m_{3/2}^2$$

$$m^2 = -\frac{1}{2} N^u N_u + \frac{8}{3} m_{3/2}^2$$

This implies that:

$$m^2 = \left[-\frac{1}{3} - \epsilon \right] m_{3/2}^2$$

where

$$\epsilon = \frac{V}{m_{3/2}^2}$$

Metastable de Sitter vacua with $\epsilon > 0$ are then totally excluded in this framework.

Theories with only vector multiplets Cremmer, Kounnas, Van Proeyen, $\cdots$ 1985

This case can also be studied in general. The geometry becomes Local-Special-Kähler and one finds:

$$V = W^{ix}\bar{W}_i^x - 3\, m_{3/2}^2$$

$$m^2 = -2\, W^{ix}\bar{W}_i^x + 6\, m_{3/2}^2$$

This implies that:

$$m^2 = \Big[-2\,\epsilon\Big] m_{3/2}^2$$

where

$$\epsilon = \frac{V}{m_{3/2}^2}$$

Metastable de Sitter vacua with $\epsilon > 0$ are then again totally excluded in this framework.

Theories with hyper and vector multiplets

Catino, Scrucca, Smyth 2013

This case is more intricate in general but one can study the minimal case with one of each of the multiplets. One finds:

$$V = \frac{1}{2} N^u N_u + W^{ix} \bar{W}_i^x - 3\, m_{3/2}^2$$

$$m_\pm^2 = \text{function of } N^u N_u,\ W^{ix} \bar{W}_i^x \text{ and } m_{3/2}^2$$

One finds:

$$m_\pm^2 = \Big[X \pm \sqrt{Y}\Big] m_{3/2}^2$$

where:

$$X = \frac{2}{3}(3+\epsilon)^2 \cos^4\theta - \frac{1}{3}(3+\epsilon)(6+\epsilon)\cos^2\theta + \frac{1}{3}(4+3\epsilon+2\epsilon^2)$$

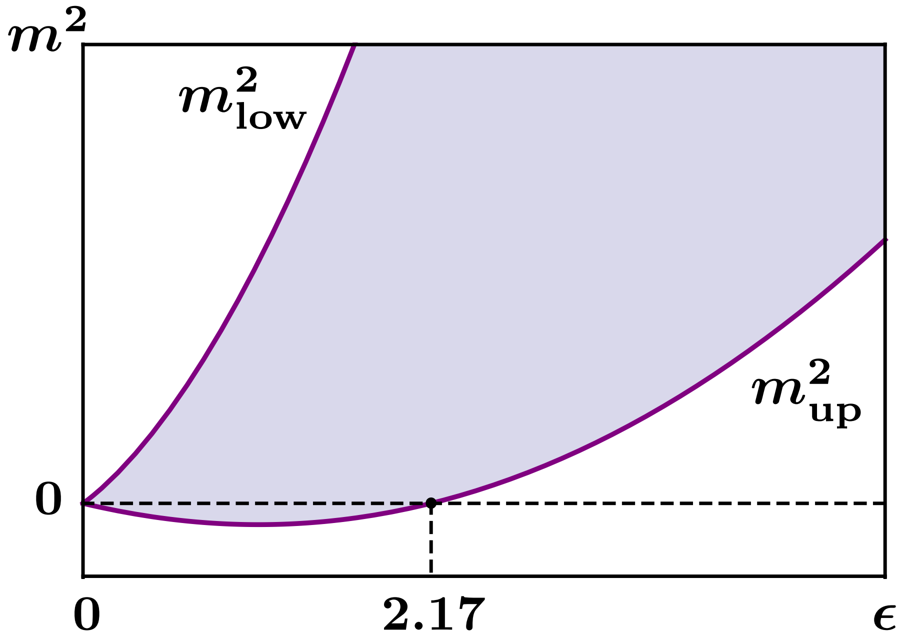
$$Y = \frac{1}{9}(3+\epsilon)^4 \cos^8\theta + \frac{8}{9}(3+\epsilon)^3 \epsilon \cos^6\theta - \frac{2}{9}(3+\epsilon)^2 (4+9\epsilon-3\epsilon^2)\cos^4\theta$$
$$\quad - \frac{2}{9}(3+\epsilon)\,\epsilon\,(16+27\epsilon+5\epsilon^2)\cos^2\theta + \frac{1}{9}(4+9\epsilon+2\epsilon^2)^2$$

and

$$\epsilon = \frac{V}{m_{3/2}^2} \qquad \tan^2\theta = \frac{1}{2}\,\frac{N^u N_u}{W^{ix} \bar{W}_i^x}$$

The absolute upper and lower bounds to the lightest and heaviest masses are obtained by extremizing m_-^2 and m_+^2 over θ. One finds:

$$m^2_{\rm up} = \begin{cases} -\frac{1}{2}\epsilon m^2_{3/2}\,, & \epsilon \ll 1 \\ \frac{1}{4}\epsilon^2 m^2_{3/2}\,, & \epsilon \gg 1 \end{cases} \qquad m^2_{\rm low} = \begin{cases} \frac{3}{2}\epsilon m^2_{3/2}\,, & \epsilon \ll 1 \\ 1.05\,\epsilon^2 m^2_{3/2}\,, & \epsilon \gg 1 \end{cases}$$



Metastable de Sitter vacua with $\epsilon > 0$ are possible but only for $\epsilon \gtrsim 2.17$.

Application to N = 2 strings

Gomez-Reino, Louis, Scrucca 2009
Davidse, Saueressig, Theis, Vandoren 2005

The universal hyper multiplet of $N = 2$ string models has:

$$\mathcal{M} = \frac{SU(1,2)}{U(1) \times SU(2)} \quad \text{deformed by quantum corrections}$$

Metastable de Sitter vacua are in this case not possible even at strong coupling, unless one includes extra vector multiplet effects.

CONCLUSIONS AND OUTLOOK

- In $N = 1$ theories, there exists a sharp necessary condition for the existence of metastable SUSY-breaking vacua, giving constraints. The general case is well understood. There exist many simple classes of examples where viable vacua arise.
- In $N = 2$ theories, there are similar but stronger constraints for metastable SUSY breaking, giving in some cases no-go theorems. The general case is not yet fully understood. But we constructed a novel and simple class of examples where viable vacua arise.