# SEQUESTERING IN BRANE WORLD SUPERSYMMETRY BREAKING AND ITS REALIZATION IN STRING MODELS

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- Scalar masses in supergravity models
- Phenomenological and cosmological problems
- Full sequestering and mild sequestering
- Minimal and general brane worlds
- String theoretical brane worlds
- Calabi-Yau and orbifold models

# SUPERGRAVITY MODELS

## General structure of the theory

In a supergravity theory with Kähler potential $K$ and superpotential $W$, supersymmetry may be spontaneously broken in a metastable vacuum. The cosmological constant can be adjusted to zero by a tuning and the order parameter is then the norm of the auxiliary fields $|F|$.

The Kähler potential $K$ controls the kinetic terms and the geometry of the scalar manifold:

$$g_{I\bar{J}} = K_{I\bar{J}} \qquad \Gamma^P_{IJ} = K^P{}_{IJ} \qquad R_{I\bar{J}P\bar{Q}} = K_{I\bar{J}P\bar{Q}} - K_{IP\bar{L}}K^{\bar{L}}{}_{\bar{J}\bar{Q}}$$

The superpotential $W$ controls the potential and the direction in field space along which supersymmetry is broken:

$$\bar{F}_I = -\nabla_I W \qquad \mu_{IJ} = \nabla_I \nabla_J W \qquad \lambda_{IJK} = \nabla_I \nabla_J \nabla_K W$$

## General features of mass matrices

There is a general sum rule constraining the average splitting between particles and superparticles:

$$\mathrm{str}[m^2] = -2\,R_{I\bar{J}}F^I\bar{F}^{\bar{J}} + 2(n-1)\,M_{\mathrm{P}}^{-2}|F|^2$$

The are also simple special values for the mass of the gravitino and for the average mass of the two scalar sGoldstini:

$$m_{\psi}^2 = \frac{1}{3}M_{\mathrm{P}}^{-2}|F|^2$$
$$m_{\varphi}^2 = -R_{I\bar{J}P\bar{Q}}\frac{F^I\bar{F}^{\bar{J}}F^P\bar{F}^{\bar{Q}}}{|F|^2} + \frac{2}{3}\,M_{\mathrm{P}}^{-2}|F|^2$$

Superpartner splitting and vacuum metastability both require $R \lesssim M_P^{-2}$.
If $|R| \gg M_{\mathrm{P}}^{-2}$, sigma-model physics dominates and $R$ must be negative.
If $|R| \ll M_{\mathrm{P}}^{-2}$, gravitational physics dominates and $R$ can have any sign.
The simplest and most natural situation is when $|R| \sim M_{\mathrm{P}}^{-2}$.

## General paradigm for models

The general paradigm for model building involves a visible sector with superfields $Q^{\alpha}$ and a hidden sector with superfields $\Sigma^{\Gamma}$, which interact in a suppressed way through physics with typical energy scale $\Lambda$ equal to $R^{-1/2}$ and $M_{\mathrm{P}}$:

$$\text{visible sector}: \;\; Q^{\alpha} \qquad \text{hidden sector}: \;\; \Sigma^{\Gamma}$$

## Delicate issues

The dynamics of the visible sector is parametrized through soft terms. Phenomenological constraints imply that these must have a suitable scale and structure. This restricts the transmission mechanism.

The dynamics of the hidden sector must lead to a metastable vacuum. Cosmological constraints imply that the life-time and fluctuation masses must be sufficiently large. This restricts the breaking mechanism.

## String-derived models

String models admit a low-energy effective description in terms of some supergravity theory. There are however certain peculiarities concerning the field content and the form of $K$ and $W$.

The effective Kähler potential $K$ can usually be derived in a simple way, because it is associated with kinetic terms, which are unavoidably present. It consists of a dominant classical part plus a small quantum correction. It can therefore be considered as an approximately known quantity.

The effective superpotential $W$ is instead more subtle to be determined, because it is related to potential terms, which may arise or may not arise. It can moreover be dominated either by classical or by quantum effects. It may therefore be considered as an essentially unknown quantity.

A conservative strategy is then to consider a fixed $K$ but allow for an a priori arbitrary $W$, and see what can be achieved.

# HIDDEN SECTOR AND COSMOLOGY

## Structure of scalar fluctuation masses

The masses of the scalar components of the hidden sector superfields $\Sigma^{\Gamma}$ have both a supersymmetric part and a splitting part:

$$m^2_{\Gamma\bar{\Delta}} = (\mu\bar{\mu})_{\Gamma\bar{\Delta}} - R_{\Gamma\bar{\Delta}\Sigma\bar{Y}} F^{\Sigma}\bar{F}^{\bar{Y}} + \frac{1}{3} g_{\Gamma\bar{\Delta}} M_{\mathrm{P}}^{-2}|F|^2$$
$$m^2_{\Gamma\Delta} = -\lambda_{\Gamma\Delta\Sigma}F^{\Sigma} + \frac{2}{\sqrt{3}}\mu_{\Gamma\Delta}M_{\mathrm{P}}^{-1}|F|$$

These depend on $K$ through the associated geometry and on $W$ through $F_{\Gamma}$, $\mu_{\Gamma\Delta}$ and $\lambda_{\Gamma\Delta\Sigma}$, but with two important restrictions imposed by the formulae for $m_{\psi}^2$ and $m_{\varphi}^2$, which follow from the conditions of vanishing and stationarity of the vacuum energy:

$$g_{\Gamma\bar{\Delta}} F^{\Gamma}\bar{F}^{\bar{\Delta}} = 3\, m_{\psi}^2 M_{\mathrm{P}}^2$$
$$\Big(R_{\Gamma\bar{\Delta}\Sigma\bar{Y}} - \frac{2}{3} g_{\Gamma[\bar{\Delta}} g_{\Sigma\bar{Y}]} M_{\mathrm{P}}^{-2}\Big) F^{\Gamma}\bar{F}^{\bar{\Delta}} F^{\Sigma}\bar{F}^{\bar{Y}} = -3\, m_{\varphi}^2\, m_{\psi}^2 M_{\mathrm{P}}^2$$

## The cosmological constant problem

The cosmological constant can be adjusted to the tiny observed value only through a tuning of parameters:

$$\Lambda: \quad \text{tuned to approximately zero}$$

A nice idea to make this tuning simple at the practical level is that of subsectors with balancing energies for given value of $m_\psi$.

## Metastability and fluctuation mass problems

The scalar square masses must be positive and sufficiently large, for the vacuum life-time to be long enough and nucleosynthesis to work:

$$m^2_{\Gamma\bar{\Delta}}: \quad \text{positive and sufficiently large}$$

A strong necessary condition is that $m_\varphi^2 > 0$, implying $R(F) < \frac{2}{3} M_{\mathrm{P}}^{-2}$. An obviously safe option is to have $R < \frac{2}{3} M_{\mathrm{P}}^{-2}$ in any direction.

# VISIBLE SECTOR AND PHENOMENOLOGY

## General form of soft scalar masses

The masses that are induced for the scalar components of the visible superfields $Q^{\alpha}$ are entirely due to splitting effects:

$$m^2_{\alpha\bar{\beta}} = -\Big(R_{\alpha\bar{\beta}\Gamma\bar{\Delta}} - \frac{1}{3} g_{\alpha\bar{\beta}} g_{\Gamma\bar{\Delta}} M_{\rm P}^{-2}\Big) F^{\Gamma} \bar{F}^{\bar{\Delta}}$$

This can also be written in a different way in terms of the Kähler function $\Omega = -3M_{\rm P}^2\, e^{-K/(3M_{\rm P}^2)}$ in the form:

$$\begin{aligned} m^2_{\alpha\bar{\beta}} &= 3\, M_{\rm P}^2\, \Omega^{-1} \Big(\Omega_{\alpha\bar{\beta}\Gamma\bar{\Delta}} - \Omega^{-1\bar{\delta}\gamma} \Omega_{\bar{\delta}\alpha\Gamma} \Omega_{\gamma\bar{\beta}\bar{\Delta}}\Big) F^{\Gamma} \bar{F}^{\bar{\Delta}} \\ &= 3\, M_{\rm P}^2\, \Omega^{-1} \Big(\Omega_{\alpha\bar{\beta}}\big|_{D} - \Omega^{-1\bar{\delta}\gamma}\big|\, \Omega_{\bar{\delta}\alpha}\big|_{F}\, \Omega_{\gamma\bar{\beta}}\big|_{\bar{F}}\Big) \end{aligned}$$

The crucial ingredient are thus the operators in $\Omega$ that mix $Q^{\alpha}$ and $\Sigma^{\Gamma}$, and the orientation of the Goldstino direction.

## The supersymmetric flavor problem

The flavor structure of the soft scalar mass matrix $m^2_{\alpha\bar{\beta}}$ is a priori generic, because this is generated at the fundamental scale of the theory where the flavor structure of the ordinary fermion masses must also emerge.

This would however cause a severe phenomenological problem, because it would predict way too large rates for certain flavor-changing processes. One should then find some mechanism that naturally forces $m^2_{\alpha\bar{\beta}}$ to be approximately flavor-universal:

$$m^2_{\alpha\bar{\beta}} \simeq g_{\alpha\bar{\beta}}\, m^2$$

The two most interesting ideas to explain this flavor-universality of soft masses in the context of supergravity models are sector sequestering along extra dimensions and selection rules from global symmetries.

# CRITICAL INGREDIENTS AND HANDLES ON THEM

## Curvature

A first crucial ingredient is the curvature of the scalar manifold, and more precisely its components with non-mixed or mixed indices:

$$\text{curvature tensor}: \; R_{\Gamma\bar{\Delta}\Sigma\bar{Y}}\,,\; R_{\alpha\bar{\beta}\Gamma\bar{\Delta}}$$

Depending on the given form of $K$, these may have a special structure.

## Goldstino direction

A second crucial ingredient is the direction of supersymmetry breaking in field space, given by:

$$\text{breaking vector}: \; F^{\Gamma}$$

Depending on the form that is allowed for $W$, this may be constrained.

## Symmetries

There might be approximate global symmetries, with transformation rules specified by some Killing vectors $k_a^I$ with Killing potentials $D_a$.

This requires a special form of $K$ and the associated curvature, because these symmetries must correspond to isometries.

It also constrains the allowed form of $W$, and in particular the orientation of the Goldstino direction:

$$\bar{k}_{a\Gamma} F^{\Gamma} = -iD_a m_{\psi} \qquad \nabla_{\Gamma} k_{a\bar{\Delta}} F^{\Gamma} \bar{F}^{\bar{\Delta}} = -2iD_a m_{\psi}^2$$

When gravitational effects are negligible, these equations simplify to:

$$\bar{k}_{a\Gamma} F^{\Gamma} \simeq 0 \qquad \nabla_{\Gamma} k_{a\bar{\Delta}} F^{\Delta} \bar{F}^{\bar{\Delta}} \simeq 0$$

In rigid superspace, these follow from the conservation law $\mathcal{D}^2 J_a \simeq 0$ for the Nöther current $J_a \simeq \mathrm{Im}(K_{\Gamma} k_a^{\Gamma})$, and read $J_a|_F \simeq 0$ and $J_a|_D \simeq 0$.

## Vanishing masses starting point

One possible idea is to try to see whether one can find a starting setup which ensures in a robust way that the soft and sGoldstino masses do approximately vanish:

$$m^2_{\alpha\bar{\beta}} \simeq 0 \qquad m^2_{\varphi} \simeq 0$$

On may then look for some additional effects providing corrections that are naturally flavor-universal and positive:

$$\Delta m^2_{\alpha\bar{\beta}} \neq 0 \qquad \Delta m^2_{\varphi} \neq 0$$

For example, these may come from quantum corrections that happen to be dominated by low-energy physics. Alternatively, they may also come from new classical contributions induced by some extra modes of the hidden sector that happen to couple universally.

## Fully sequestered models

Ellis, Kounnas, Nanopoulos 1984
Randal, Sundrum 1999

One way to realize this starting point is to assume that for some reason $\Omega$ has a minimal sequestered form:

$$\Omega = -3M_{\mathrm{P}}^2 + Q^\alpha \bar{Q}^\alpha + \Sigma^\Gamma \bar{\Sigma}^\Gamma$$

This defines a maximally symmetric scalar manifold and the Riemann tensor satisfies the following special property ($I = \alpha, \Gamma$):

$$R_{I\bar{J}P\bar{Q}} = \frac{1}{3}\Big(g_{I\bar{J}}g_{P\bar{Q}} + g_{I\bar{Q}}g_{P\bar{J}}\Big)M_{\mathrm{P}}^{-2}$$

On the vacuum one then finds:

$$R_{\alpha\bar{\beta}\Gamma\bar{\Delta}} = \frac{1}{3}g_{\alpha\bar{\beta}}\,g_{\Gamma\bar{\Delta}}M_{\mathrm{P}}^{-2} \qquad R_{\Gamma\bar{\Delta}\Sigma\bar{Y}} = \frac{2}{3}g_{\Gamma[\bar{\Delta}}\,g_{\Sigma\bar{Y}]}M_{\mathrm{P}}^{-2}$$

It then trivially follows that:

$$m^2_{\alpha\bar{\beta}} = 0 \qquad m^2_{\varphi} = 0$$

## Mildly sequestered models

Schmaltz, Sundrum 2006
Kachru, McAllister, Sundrum 2007

An interesting extension of this is to allow some mixing interactions in $\Omega$ that involve the currents of some approximate symmetries:

$$\Omega \simeq -3\,M_{\rm P}^2 + Q^\alpha \bar{Q}^{\bar{\alpha}} + \Sigma^\Gamma \bar{\Sigma}^{\bar{\Gamma}} + \frac{1}{2} M^{-2} J_Q^a(Q^\alpha, \bar{Q}^\alpha) J_\Sigma^a(\Sigma^\Gamma, \bar{\Sigma}^\Gamma)$$

This no-longer defines a maximally symmetric coset manifold. But if $J_\Sigma^a$ is approximately conserved, so that $J_\Sigma^a|_F \simeq 0$ and $J_\Sigma^a|_D \simeq 0$, and the scalar component of $\Sigma^\Gamma$ is small on the vacuum, so that $\Sigma^\Gamma| \simeq 0$, one nevertheless finds:

$$m^2_{\alpha\bar{\beta}} \simeq 0 \qquad m^2_\varphi \simeq 0$$

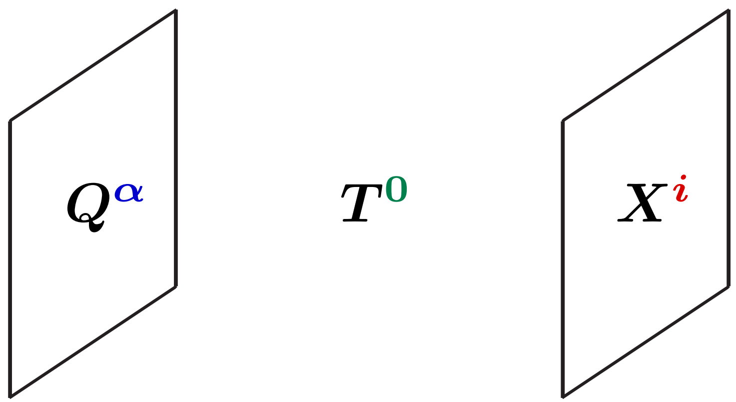
## Concrete models

To build models realizing these two ideas, one may use compact extra dimensions and approximate global symmetries. From now: $M_{\rm P} \to 1$.

# MINIMAL BRANE WORLD

Sequestering along an extra dimension  Randall, Sundrum 1999

Suppose that some matter superfields $Q^\alpha$ and the matter superfields $X^i$ are localized on two branes along an extra dimension $S^1/Z_2$, and that they interact only through the gravity multiplet in the bulk, which provides an extra radion superfield $T^0$ in the low-energy theory:




visible sector: $Q^\alpha$

hidden sector: $X^i$, $T^0$

The effective theory is then strongly constrained by locality.

## Effective Kähler potential and geometry

The effective Kähler potential is derived by reducing the kinetic terms of the 5D theory to 4D. One finds:

$$K = -3\log\left[T^0 + \bar{T}^0 - \frac{1}{3}Q^\alpha\bar{Q}^\alpha - \frac{1}{3}X^i\bar{X}^i\right]$$

This defines a maximally symmetric scalar manifold with fixed curvature scale and diffeomorphic to

$$\mathcal{M} = \frac{SU(1, 1_T + n_Q + n_X)}{U(1) \times SU(1_T + n_Q + n_X)}$$

The Riemann tensor then satisfies the following property ($I = \alpha, 0, i$):

$$R_{I\bar{J}P\bar{Q}} = \frac{1}{3}\left(g_{I\bar{J}}g_{P\bar{Q}} + g_{I\bar{Q}}g_{P\bar{J}}\right)$$

It follows that on the vacuum ($\Gamma = 0, i$):

$$R_{\alpha\bar{\beta}\Gamma\bar{\Delta}} = \frac{1}{3}g_{\alpha\bar{\beta}}\, g_{\Gamma\bar{\Delta}} \qquad R_{\Gamma\bar{\Delta}\Sigma\bar{Y}} = \frac{2}{3}g_{\Gamma[\bar{\Delta}}\, g_{\Sigma\bar{Y}]}$$

## Effective Kähler function

The corresponding effective Kähler function is then separable and takes the following very simple sequestered form:

$$\Omega = -3\left(T^0 + \bar{T}^0\right) + Q^\alpha \bar{Q}^\alpha + X^i \bar{X}^i$$

## Soft scalar masses

There can be two contributions to $m^2_{\alpha\bar{\beta}}$: a brane-mediated effect from the $F^i$ and a bulk-mediated effect from the $F^0$. But they both vanish:

$$m^2_{\alpha\bar{\beta}} = 0$$

## Vacuum metastability

The hidden scalar masses $m^2_{\Gamma\bar{\Delta}}$ cannot be made all large. Indeed, the average sGoldstino mass is found to vanish:

$$m^2_{\varphi} = 0$$

## Quantum corrections to soft scalar masses

Gherghetta, Riotto 2002
Rattazzi, Scrucca, Strumia 2003
Buchbinder et al. 2003

Quantum effects induced by bulk supergravity fields can give corrections to the mixing terms in $\boldsymbol{\Omega}$. They are universal and at one loop one finds:

$$\Delta\Omega = -\frac{9}{\pi^2}\int_0^{+\infty} dx\, x \log\left[1 - \frac{1+|Q^\alpha|^2 x}{1-|Q^\alpha|^2 x}\frac{1+|X^i|^2 x}{1-|X^i|^2 x} e^{-6(T^0+\bar{T}^0)x}\right]$$
$$= \frac{\xi(3)}{6\pi^2}\left[\frac{3/2}{(T^0+\bar{T}^0)^2} + \frac{|Q^\alpha|^2+|X^i|^2}{(T^0+\bar{T}^0)^3} + \frac{(|Q^\alpha|^2+|X^i|^2)^2}{2\,(T^0+\bar{T}^0)^4} + \cdots\right]$$

Then also $m^2_{\alpha\bar{\beta}}$ receives some correction. But unfortunately it is negative. At the reference point where $T^0 \simeq \frac{1}{2}$ and $X^i \simeq 0$ one finds:

$$\Delta m^2_{\alpha\bar{\beta}} \simeq -\frac{\xi(3)}{6\pi^2}\left[|F^i|^2 + 12\,|F^0|^2\right]$$

There exist two ways to make this effect positive. The first is to introduce brane-localized kinetic terms for the bulk supergravity fields. The second is to invoke a $D$-type effect from vector multiplets on the hidden brane.

## Quantum effects on vacuum metastability

Quantum effects induced by hidden brane fields can give some additional corrections to the hidden sector $K$. They are however model-dependent:

$$\Delta K = f(T^0, \bar{T}^0, X^i, \bar{X}^i)$$

The average sGoldstino mass then also acquires a correction:

$$\Delta m_\varphi^2 = -\Delta R_{\Gamma\bar{\Delta}\Sigma\bar{Y}} \frac{F^\Gamma \bar{F}^{\bar{\Delta}} F^\Sigma \bar{F}^{\bar{Y}}}{|F|^2}$$

This can be have either sign, and in suitable circumstances it can thus stabilize the sGoldstini.

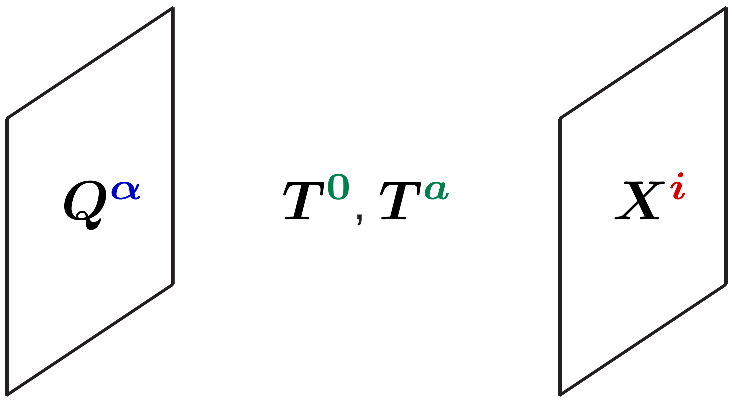
## Tuning of the cosmological constant

The tuning of the cosmological constant can be realized by a balancing of energy between the $T^0$ bulk sector and the $X^i$ brane sector.

# MORE GENERAL BRANE WORLDS

Extra vector multiplets in the bulk — Anisimov, Dine, Graesser, Thomas 2002

In more general setups based on the space $S^1/Z_2$, one may have two brane sectors with superfields $Q^\alpha$ and $X^i$, and a bulk sector with some vector multiplets besides the gravity multiplet, which provide extra moduli superfields $T^a$ besides the radion $T^0$ in the low-energy theory.




Visible sector: $Q^\alpha$

Hidden sector: $X^i$, $T^0$, $T^a$

There are now new effects mediated by the vector multiplet KK modes.

## Effective Kähler function

The effective Kähler function is now expected to get extra contributions, which can be determined by properly integrating out the heavy vector multiplets. The precise form of the result depends on the brane couplings, but we expect something like

$$\Omega = -3\,J^0 + \frac{1}{2}\,(J^0)^{-1} J^a J^a + \cdots$$

where

$$J^0 = T^0 + \bar{T}^0 - \frac{1}{3} Q^\alpha \bar{Q}^\alpha - \frac{1}{3} X^i \bar{X}^i$$
$$J^a = J^a(T^a, \bar{T}^a, Q^\alpha, \bar{Q}^\alpha, X^i, \bar{X}^i)$$

There is thus no longer a full sequestering. But one may try to implement a mild sequestering by looking for some approximate global symmetries ensuring the approximate conservation of the currents $J^a$.

# STRING BRANE WORLDS

## Heterotic M-theory on a Calabi-Yau

Horava,Witten 1996
Lukas, Ovrut, Stelle, Waldram 1999

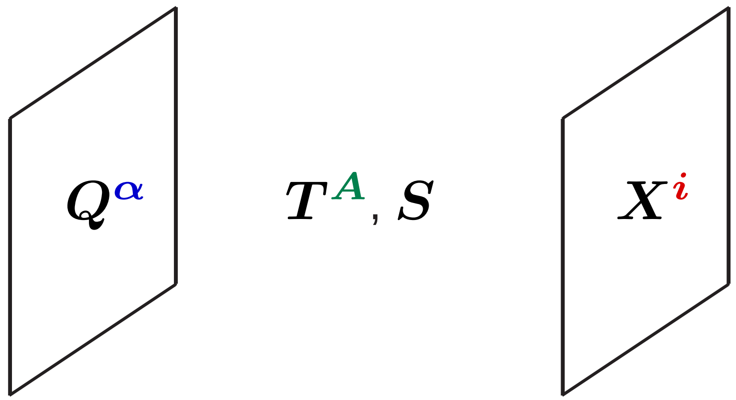
Let us consider a generic heterotic string model based on a Calabi-Yau manifold $M$ and a stable holomorphic vector bundle $E_{\mathbf{v}} \times E_{\mathbf{h}}$ over it. This also arises from M-theory on $M \times S^1/Z_2$ with two sequestered branes, in the weekly coupled limit where the size of $S^1/Z_2$ is small.

The 4D effective theory can be lifted to a 5D theory with two brane sectors containing matter superfields $Q^\alpha$ and $X^i$, and a bulk sector containing in particular some Kähler moduli superfields $T^A$ and the dilaton $S$.

The non-minimal Kähler moduli $T^a$ come along with heavy vectors $V^a$, which arise from the M-theory $3$-form and couple to $Q^\alpha$, $X^i$ and $T^A$ in a way dictated by the non-trivial Bianchi identity for this. When integrated out, these induce contact terms in the effective Kähler function $\Omega$.

## Geometric picture

From the M-theory viewpoint, the picture is that of a generic brane world, where at most a mild sequestering could perhaps occur:




Visible sector: $Q^{\alpha}$

Hidden sector: $X^{i}$, $T^{A}$, $S$

The general structure of the effective Kähler potential is the following:

$$K = -\log\left(S + \bar{S}\right) - \log Y(Q^{\alpha}, \bar{Q}^{\alpha}, X^{i}, \bar{X}^{i}, T^{A}, \bar{T}^{A})$$

Interestingly, the dilaton enters in a universal way. But unfortunately it cannot dominate supersymmetry breaking, because this would lead to a tachyonic sGoldstino. One then has to involve the other fields.

## Viable possibilities

The couplings among the fields $Q^\alpha$, $X^i$, $T^A$ are a priori expected to be non-universal. One may then try to realize in this sector a starting point with vanishing masses, using approximate global symmetries.

If this can be done and the vacuum energy is non-zero, one may then rely on the extra universal effect of $S$ to go in business. The fields $X^i$ and $T^A$ would then play the role of an uplifting sector.

One may also rely on quantum corrections, and try to reach a situation similar to the one discussed for minimal brane worlds.

The general problem is then to determine the full dependence of the Kähler potential $K$ on the matter and moduli fields $Q^\alpha$, $X^i$, $T^A$, and study its properties.

# DERIVATION OF THE EFFECTIVE THEORY

Witten 1985
Ferrara, Kounnas, Porrati 1986
Candelas, de la Ossa 1990

## Reduction of the standard heterotic string

The light 4D fields arise from the possible zero-modes of the 10D fields. The $Q^\alpha$, $X^i$ come from harmonic $1$-forms in $H^1(M, E_{\rm v})$, $H^1(M, E_h)$, while the $T^A$ come from harmonic $(1,1)$-forms in $H^{1,1}(M)$:

$$Q^\alpha \;\Leftrightarrow\; u_\alpha \qquad X^i \;\Leftrightarrow\; u_i \qquad T^A \;\Leftrightarrow\; \omega_A$$

The effective $K$ for the light fields may be derived by working out their kinetic terms by reduction on $M$ and comparing with the general structure of supergravity theories.

Discarding rather than integrating out heavy non-zero modes associated to non-harmonic forms is justified only whenever:

$$\mathrm{tr}(u_\alpha \wedge \bar u_{\bar\beta}) \;\text{ and }\; \mathrm{tr}(u_i \wedge \bar u_{\bar\jmath}) \;\text{ harmonic } \;\Leftrightarrow\; \omega_A$$

General result for matter fields and Kähler moduli

Paccetti Correia, Schmidt 2008
Andrey, Scrucca 2011

The effective Kahler potential is found to be

$$K = -\log\left(d_{ABC} J^A J^B J^C\right)$$

where

$$J^A = T^A + \bar{T}^A - c^A_{\alpha\bar{\beta}} Q^\alpha \bar{Q}^{\bar{\beta}} - c^A_{i\bar{\jmath}} X^i \bar{X}^{\bar{\jmath}}$$

The numerical quantities defining this result are:

$$d_{ABC} = \int \omega_A \wedge \omega_B \wedge \omega_C$$

$$c^A_{\alpha\bar{\beta}} = \int \omega^A \wedge \mathrm{tr}(u_\alpha \wedge \bar{u}_{\bar{\beta}}) \qquad c^A_{i\bar{\jmath}} = \int \omega^A \wedge \mathrm{tr}(u_i \wedge \bar{u}_{\bar{\jmath}})$$

This extends the results for the special cases of the untwisted sector of orbifolds, where harmonic forms are covariantly constant, to a larger class of cases, where harmonic forms close under multiplication.

## Canonical parametrization

With a appropriate parametrization of the fields, which corresponds to a suitable basis for the harmonic forms, where the moduli fields are split into an overall modulus $T^0$ and some relative moduli $T^a$, one may rewrite $K$ in the form:

$$K = -\log\left(J^{03} - \frac{1}{2}J^0 J^a J^a + \frac{1}{6}d_{abc}J^a J^b J^c\right)$$

where

$$J^0 = T^0 + \bar{T}^0 - \frac{1}{3}Q^\alpha \bar{Q}^{\bar{\alpha}} - \frac{1}{3}X^i \bar{X}^{\bar{\imath}}$$
$$J^a = T^a + \bar{T}^a - c^a_{\alpha\bar{\beta}}Q^\alpha \bar{Q}^{\bar{\beta}} - c^a_{i\bar{\jmath}}X^i \bar{X}^{\bar{\jmath}}$$

## Contact terms

The leading terms in the Kähler function for $J^a \ll J^0$ are

$$\Omega \simeq -3\,J^0 + \frac{1}{2}(J^0)^{-1}J^a J^a - \frac{1}{6}d_{abc}(J^0)^{-2}J^a J^b J^c$$

## Effect of heavy vector multiplets in the M-theory picture

In the M-theory picture, the contact terms in $\boldsymbol{\Omega}$ are induced by the heavy vectors $V^a$ coming with the light moduli $T^a$ in $N = 2$ vector multiplets. In terms of 5D $N = 1$ superfields, the Lagrangian for these modes is:

$$\mathcal{L} = \Big[-\frac{1}{4}\mathcal{N}_{ab}(T^0, T^e)W^a W^b + \frac{1}{48}\mathcal{N}_{abc}\bar{\mathcal{D}}^2(V^a \overleftrightarrow{\mathcal{D}}\partial_y V^b)W^c\Big]_F + \text{c.c.}$$
$$+\Big[-3\,\mathcal{N}^{1/3}(J_y^0, J_y^e)\Big]_D$$

with prepotential $\mathcal{N}(Z, Z^e) = Z^3 - \frac{1}{2}Z Z^a Z^a + \frac{1}{6}d^{abc}Z^a Z^b Z^c$ and

$$J_y^0 = T^0 + \bar{T}^0 - \frac{1}{3}Q^\alpha \bar{Q}^{\bar{\alpha}}\delta_{\mathrm{v}}(y) - \frac{1}{3}X^i \bar{X}^{\bar{\imath}}\delta_{\mathrm{h}}(y)$$
$$J_y^a = -\partial_y V^a + T^a + \bar{T}^a - c^a_{\alpha\bar{\beta}}Q^\alpha \bar{Q}^{\bar{\beta}}\delta_{\mathrm{v}}(y) - c^a_{i\bar{\jmath}}X^i \bar{X}^{\bar{\jmath}}\delta_{\mathrm{h}}(y)$$

Integrating out $V^a$ effective sets $(J_y^0, J_y^a, W^a) \to (J^0, J^a, 0)$ and gives $\mathcal{L} = \Omega|_D$, where $\boldsymbol{\Omega}$ corresponds to the previous result for $K$.

## Geometry of the scalar manifold

The scalar manifold is in general not a coset manifold, and its curvature depends on the point. One may however consider the following particular reference point:

$$T^0 \simeq \frac{1}{2} \qquad T^a \simeq 0 \qquad Q^\alpha \simeq 0 \qquad X^i \simeq 0$$

At this point, the metric is diagonal:

$$g_{0\bar{0}} = 3 \qquad g_{a\bar{b}} = \delta_{ab} \qquad g_{\alpha\bar{\beta}} = \delta_{\alpha\bar{\beta}} \qquad g_{i\bar{\jmath}} = \delta_{i\bar{\jmath}}$$

and the relevant components of the curvature tensor read:

$$R_{0\bar{0}0\bar{0}} = 6 \quad R_{a\bar{b}0\bar{0}} = 2\delta_{ab} \qquad R_{a\bar{b}c\bar{d}} = \delta_{ab}\delta_{cd} + \delta_{ad}\delta_{cb} - \frac{1}{3}\delta_{ac}\delta_{bd}$$

$$R_{\alpha\bar{\beta}i\bar{\jmath}} = \frac{1}{3}g_{\alpha\bar{\beta}}g_{i\bar{\jmath}} + c^a_{\alpha\bar{\beta}}c^a_{i\bar{\jmath}}$$

$$R_{\alpha\bar{\beta}0\bar{0}} = \delta_{\alpha\bar{\beta}} \quad R_{\alpha\bar{\beta}a\bar{b}} = \Big(\frac{2}{3}\delta_{ab}\delta + d_{abc}c^c - c^a c^b\Big)_{\alpha\bar{\beta}} \quad R_{\alpha\bar{\beta}0\bar{b}} = c^b_{\alpha\bar{\beta}}$$

# STRUCTURE OF SOFT SCALAR MASSES

## General structure of soft scalar masses

The general structure taken by soft scalar masses in these models can be studied by restricting to the previously defined reference point, around which the canonical parametrizaton is particularly convenient.

Using the general result that has been derived for $K$ and imagining an arbitrary form for $W$, one obtains:

$$m^2_{\alpha\bar\beta} \simeq -c^a_{\alpha\bar\beta} c^a_{i\bar\jmath} F^i \bar F^{\bar\jmath} - \Big(\frac{1}{3}\delta_{ab}\delta + d_{abc}c^c - c^a c^b\Big)_{\alpha\bar\beta} F^a \bar F^{\bar b} \\ - c^a_{\alpha\bar\beta} F^a \bar F^0 + \mathrm{c.c.}$$

This vanishes identically if the Goldstino direction is suitably constrained:

$$m^2_{\alpha\bar\beta} \simeq 0 \;\Leftrightarrow\; F^a \simeq 0 \;\text{ and }\; c^a_{i\bar\jmath} F^i \bar F^{\bar\jmath} \simeq 0$$

## Smooth Calabi-Yau models

Andrey, Scrucca 2011

The Goldstino direction can be guaranteed to point in a direction for which $m^2_{\alpha\bar{\beta}} \simeq 0$ by postulating that the following transformations represent two approximate symmetries not only of $K$ but also of $W$:

$$\delta^1_a T^b = i\delta^b_a \;\Leftrightarrow\; F^a \simeq 0$$
$$\delta^2_a X^i = -ic^a_{\bar{\jmath}i} X^j \;\Leftrightarrow\; c^a_{i\bar{\jmath}} F^i \bar{F}^{\bar{\jmath}} \simeq 0$$

Clearly $\delta^1_a$ always form a group $U(1)^{\#}$ and give exact symmetries of $K$. However $\delta^2_a$ only form a group $H$ if $c^a_{i\bar{\jmath}}$ generate a closed algebra and only extends to exact symmetries of $K$ if $d_{abc}$ is a symmetric invariant of this algebra.

We conclude that a mild sequestering relying on symmetries is possible only for certain very specific models:

Mild sequestering possible only for some Calabi-Yau models

## Untwisted sector of orbifolds

Li, Peschanski, Savoy 1987
Andrey, Scrucca 2010

One special class of models where one is automatically in business is provided by orbifold constructions. In the untwisted sector, the formula for $K$ that has been obtained applies, with:

$$c^a_{\alpha\bar{\beta}}, c^a_{i\bar{\jmath}}: \quad \text{generators of some } H \subset SU(3)$$
$$d_{abc}: \quad \text{symmetric invariant of this } H \subset SU(3)$$

The scalar manifold is always a symmetric coset manifold, and $H$ belongs to the stability group. As a result, $U(1)^{\#} \times H$ is an exact symmetry of $K$, and imposing it also to $W$ leads to vanishing masses.

We conclude that a mild sequestering relying on symmetries is possible for any such model:

Mild sequestering possible for all orbifold models

# LARGEST VALUE OF THE SGOLDSTINO MASS

**Bound on the sGoldstino mass**

Gomez-Reino, Scrucca 2006
Covi, Gomez-Reino, Groos, Louis, Palma, Scrucca 2008

The involved scalar manifolds satisfy the no-scale property $K^I K_I = 3$.
This implies that $R(K^I) = \frac{2}{3}$ and therefore that $m_\varphi^2 = 0$ for $F^\Gamma \propto K^\Gamma$.
The question is then whether one can get $m_\varphi^2 > 0$ for some $F^\Gamma \not\propto K^\Gamma$.

**Smooth Calabi-Yau models**

For smooth Calabi-Yau models, one finds:

$$m_\varphi^2 > 0 \quad \text{for some} \quad F^\Gamma \not\propto K^\Gamma \quad \text{in some models}$$

**Orbifold untwisted sector**

In the untwisted sector of orbifold models, one finds:

$$m_\varphi^2 \leq 0 \quad \text{for any} \quad F^\Gamma \not\propto K^\Gamma \quad \text{in all models}$$

# CONCLUSIONS

- Under some assumptions, the Kähler potential of heterotic models can be fully computed. The resulting soft scalar masses are found to vanish for suitably oriented Goldstino directions.
- The Goldstino direction can be forced to align along such special directions by relying on some global symmetries, but this appears to be possible only under some extra assumptions.
- A special class of models where this mechanism can always work is that of orbifold models. But it might be possible to put it at work also for other special classes of Calabi-Yau models.