METASTABLE DE SITTER VACUA IN N = 1 AND N = 2 SUPERGRAVITY

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- SUSY breaking in SUGRA.
- N = 1 models with chiral multiplets.
- N = 1 models with chiral and vector multiplets.
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- N = 2 models with abelian vector multiplets.
- N = 2 models with hyper and vector mutliplets.
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SUSY BREAKING IN SUGRA

Constraints on realistic models

In a SUGRA model, the scalar potential V should allow for spontaneous SUSY breaking with certain non-trivial features.

- Phenomenology: To get a viable particle vacuum, need a point where $V \gtrsim 0$, $V' = 0$ and $V'' > 0$.
- Cosmology: To get a viable period of slow-roll inflation, need a region where $V > 0$, $V' \simeq 0$ and $V'' \gtrsim 0$.

The condition on V' can be satisfied by adjusting the values of the fields. But the conditions on V and V'' need an adjustment of parameters.

The natural question is then whether these two conditions can be used to restrict the class of models of potential interest. The answer is yes.

Algebraic formulation of the problem

Consider the critical situation where the scalar fields ϕ take values such that $V' = 0$, leading to broken SUSY and a gravitino mass $m_{3/2}$.

The value of V is linked to SUSY breaking. This gives a first relevant parameter given by:

$$\gamma = \frac{V}{3\, m_{3/2}^2}$$

The value of V'' along a generic direction is not related to SUSY breaking and can be easily adjusted, whereas along the sGoldstino direction η it is related to SUSY breaking. This gives a second relevant parameter:

$$\lambda = \frac{V''(\eta)}{m_{3/2}^2}$$

The structure of SUGRA implies $\gamma \geq -1$ and most importantly that λ is constrained in terms of γ.

Necessary conditions

The requirements coming from phenomenology and cosmology imply that both at the final vacuum and in the rolling region one should have

$$\gamma \gtrsim 0$$

More quantitatively:

$$\gamma_{\rm vac} \ll 1 \,, \quad \gamma_{\rm rol} \gg 1$$

Similarly, since λ defines bounds on the eigenvalues m^2 of V'', namely min$(m^2) \leq \lambda\, m_{3/2}^2$ and max$(m^2) \geq \lambda\, m_{3/2}^2$, one should also have, again both for vacuum metastablity and inflationary slow rolling:

$$\lambda \gtrsim 0$$

More quantitatively:

$$\lambda_{\rm vac} : \text{sizable} \,, \quad \lambda_{\rm rol} : \text{free}$$

N = 1 MODELS WITH CHIRAL MULTIPLETS

Geometric formulation

A model with n_c chiral multiplets $\Phi^i = (\phi^i_{1,2}, \psi^i, F^i_{1,2})$ is specified by a real Kähler potential K and a holomorphic superpotential W. It has a $U(1)$ symmetry under which $e^{K'} = e^{X+\bar{X}} e^K$ and $W' = e^{-X} W$.

The $2n_c$ scalars span a Hodge-Kähler manifold with metric $g_{i\bar{\jmath}} = K_{i\bar{\jmath}}$ and Kähler form $J_{i\bar{\jmath}} = g_{i\bar{\jmath}}$, with a $U(1)$ bundle on it with curvature $J_{i\bar{\jmath}}$. The holonomy is $U(n_c) \times U(1)$. The vielbein has the form e_i^I and $e_{\bar{\imath}}^{\bar{I}}$.

The theory can be described in a $U(1)$ covariant way, with a covariant derivative ∇_i including both the Christoffel and $U(1)$ connections Γ^k_{ij} and $\omega_i = K_i$. On a quantity transforming with weights $(p, \bar{p})$, one has:

$$\nabla_i = D_i(\Gamma) + p\,\omega_i\,, \quad \nabla_{\bar{\jmath}} = D_{\bar{\jmath}}(\Gamma) + \bar{p}\,\omega_{\bar{\jmath}}$$

The gravitino mass is described by a covariantly holomorphic section of weights $(\frac{1}{2},-\frac{1}{2})$:

$$L = e^{K/2} W \qquad \big(m_{3/2} = |L|\big)$$

The auxiliary fields are obtained by taking a covariant derivative, and also have weights $(\frac{1}{2},-\frac{1}{2})$:

$$F_i = e^{K/2}(W_i + K_i W)$$

These quantities satisfy the following relations:

$$\nabla_i L = F_i\,, \quad \nabla_{\bar{\jmath}} L = 0\,, \quad \nabla_{\bar{\jmath}} F_i = g_{i\bar{\jmath}} L$$

Moreover, commutators of covariant derivatives involve generically both the Riemann and the $U(1)$ curvatures. For instance:

$$[\nabla_i, \nabla_{\bar{\jmath}}] L = -g_{i\bar{\jmath}} L$$
$$[\nabla_i, \nabla_{\bar{\jmath}}] F_p = R_{i\bar{\jmath}p\bar{q}} \bar{F}^q - g_{i\bar{\jmath}} F_p$$

Scalar potential

The scalar potential is given by:

$$V = \bar{F}^i F_i - 3|L|^2$$

Its first first derivatives read:

$$\nabla_i V = -2 F_i \bar{L} + \nabla_i F_j \bar{F}^j$$

The second derivatives are also easily calculated, and one finds:

$$\nabla_i \nabla_{\bar{\jmath}} V = -2 g_{i\bar{\jmath}} |L|^2 + \nabla_i F_k \nabla_{\bar{\jmath}} \bar{F}^k - R_{i\bar{\jmath}p\bar{q}} F^p \bar{F}^{\bar{q}} + g_{i\bar{\jmath}} \bar{F}^k F_k - F_i \bar{F}_{\bar{\jmath}}$$
$$\nabla_i \nabla_j V = -\nabla_i F_j \bar{L} + \nabla_i \nabla_j F_k \bar{F}^k$$

Fermions and susy breaking

The n_c chiral fermions ψ^I are naturally defined on the tangent bundle of the scalar manifold, locally defined by the vielbein e_i^I and $\bar{e}_{\bar{\jmath}}^{\bar{J}}$.

The SUSY transformations give $\delta\psi^I \supset -\sqrt{2}\, e^I_i F^i \xi$. At a stationary point, the Goldstino direction in the tangent space is thus:

$$\eta^I = e^I_i F^i$$

The corresponding sGoldstino direction on the scalar manifold is:

$$\eta^i = e^i_I \eta^I = F^i$$

This defines 2 orthogonal directions in the real scalar-field space:

$$\eta^u = (F^i, \bar{F}^{\bar{\imath}})\,, \quad \tilde{\eta}^u = J^u{}_v \eta^v = (iF^i, -i\bar{F}^{\bar{\imath}})$$

SUSY is spontaneously broken whenever $F_i \neq 0$. The $2n_c$ stationarity conditions imply then that

$$\nabla_i F_j \bar{F}^j = 2 F_i \bar{L}$$

Metastability

The strongest constraint on metastability comes from averaging over the $\mathbf{2}$ real sGoldstino directions $\eta, \tilde{\eta}$, and considering:

$$\lambda = \frac{\nabla_i \nabla_{\bar{\jmath}} V \bar{F}^i F^{\bar{\jmath}}}{|L|^2 \bar{F}^k F_k}$$

A simple computation shows that at a stationary point this is given by:

$$\lambda = 2 + R \, \frac{\bar{F}^i F_i}{|L^2|}$$

The quantity R is the holomorphic sectional curvature in the plane $\eta, \tilde{\eta}$:

$$R = -\frac{R_{i\bar{\jmath}p\bar{q}} \bar{F}^i F^{\bar{\jmath}} \bar{F}^p F^{\bar{q}}}{(\bar{F}^k F_k)^2}$$

In terms of the parameter $\gamma = V/(3|L|^2)$, this reads:

$$\lambda = 2 + 3(1 + \gamma) R$$

For a given positive γ, one gets thus a positive λ only if:

$$R \geq -\frac{2}{3}\frac{1}{1+\gamma} = \begin{cases} -\frac{2}{3}\,, & \gamma \ll 1 \\ 0\,, & \gamma \gg 1 \end{cases}$$

This defines a necessary condition for metastability. One can show that if K is kept fixed and W is allowed to be tuned, it becomes also sufficient.

Notice that $\mathcal{M} = \times_x \mathcal{M}_x$ is Hodge-Kähler if each $\mathcal{M}_x$ is Hodge-Kähler. The total R gets then diluted compared to each individual R_x, and:

$$R_{\text{best}} = \Big(\sum_x R_x^{-1}\Big)^{-1}$$

Gomez-Reino, Scrucca 2006

Applications

- $K = \sum_i \Phi^i \bar{\Phi}^i$ $\Rightarrow$ OK
- $K = -\sum_i n_i \log(\Phi^i + \bar{\Phi}^i)$ $\Rightarrow$ OK if $\sum_i n_i > 3(1+\gamma)$
- $K = -\sum_i n_i \log(\Phi^i + \bar{\Phi}^i) + \sum_j \Phi^j \bar{\Phi}^j$ $\Rightarrow$ OK

N = 1 MODELS WITH CHIRAL AND VECTOR MULTIPLETS

Geometric formulation

A model with n_c chiral and n_v vector multiplets $\Phi^i = (\phi^i_{1,2}, \psi^i, F^i_{1,2})$ and $V^a = (\lambda^a, A^a_\mu, D^a)$ is specified by a real Kähler potential K, a holomorphic superpotential W, a holomorphic gauge kinetic function H_{ab} and holomorphic Killing vectors k^i_a. It has again a $U(1)$ symmetry.

The $2n_c$ scalars span a Hodge-Kähler manifold with metric $g_{i\bar{\jmath}} = K_{i\bar{\jmath}}$ and Kähler form $J_{i\bar{\jmath}} = g_{i\bar{\jmath}}$, with a $U(1)$ bundle on it with curvature $J_{i\bar{\jmath}}$. The holonomy is $U(n_c) \times U(1)$. The vielbein is given by e^I_i and $e^{\bar{I}}_{\bar{\imath}}$. In addition, there must exist isometries generated by the k^i_a.

The theory can again be formulated in a $U(1)$ covariant way, with the help of a covariant derivative ∇_i that includes both the Christoffel and the $U(1)$ connections Γ^k_{ij} and $\omega_i = K_i$.

The gravitino mass has weights $(\frac{1}{2},-\frac{1}{2})$ and is covariantly holomorphic:

$$L = e^{K/2} W \qquad \big(m_{3/2} = |L|\big)$$

The auxiliary fields have weights $(\frac{1}{2},-\frac{1}{2})$ and $(0,0)$, and are defined by:

$$F_i = e^{K/2}(W_i + K_i W)$$

$$D_a = i k_a^i \frac{W_i + K_i W}{W} = -i \bar{k}_a^{\bar{\jmath}} \frac{\bar{W}_{\bar{\jmath}} + K_{\bar{\jmath}} \bar{W}}{\bar{W}}$$

They are related by

$$D_a = i k_a^i \frac{F_i}{L} = -i \bar{k}_a^{\bar{\jmath}} \frac{\bar{F}_{\bar{\jmath}}}{\bar{L}}$$

The Killing vectors and the gauge kinetic function have weights $(0,0)$ and are covariantly holomorphic. They define the matter charge matrix, the gauge-boson mass matrix and the inverse coupling matrix:

$$T_{ai\bar{\jmath}} = \frac{i}{2}\big(\nabla_i k_{a\bar{\jmath}} - \nabla_{\bar{\jmath}} \bar{k}_{ai}\big)\,, \quad M^2_{ab} = 2 \bar{k}_{ia} k_b^i\,, \quad h_{ab} = \mathrm{Re}\, H_{ab}$$

These quantities satisfy the following relations:

$$\nabla_i L = F_i\,, \quad \nabla_{\bar{\jmath}} L = 0\,, \quad \nabla_{\bar{\jmath}} F_i = g_{i\bar{\jmath}} L$$
$$\nabla_i D_a = -i\bar{k}_{ai}\,, \quad \nabla_i \nabla_{\bar{\jmath}} D_a = T_{ai\bar{\jmath}}\,, \quad \nabla_i \nabla_j D_a = 0$$
$$\nabla_{\bar{\jmath}} k_a^i = 0\,, \quad \nabla_i k_{a\bar{\jmath}} + \nabla_{\bar{\jmath}} \bar{k}_{ai} = 0\,, \quad \nabla_i \nabla_{\bar{\jmath}} \bar{k}_{ap} = R_{i\bar{\jmath}p\bar{q}} \bar{k}_a^{\bar{q}}$$
$$\nabla_i \nabla_{\bar{\jmath}} h_{ab} = 0$$

From gauge invariance one also deduces the additional relations

$$k_a^i \nabla_i F_j = -\nabla_j k_{a\bar{k}} F^{\bar{k}} - \bar{k}_{aj} L - iF_j D_a$$
$$k_{[a}^i \nabla_i k_{b]}^j = \frac{1}{2} f_{ab}{}^c k_c^j\,, \quad k_a^i \nabla_i h_{bc} = 2 f_{a(b}{}^d h_{dc)}$$

The commutators of covariant derivatives are given by:

$$[\nabla_i, \nabla_{\bar{\jmath}}] L = -g_{i\bar{\jmath}} L$$
$$[\nabla_i, \nabla_{\bar{\jmath}}] F_p = R_{i\bar{\jmath}p\bar{q}} \bar{F}^q - g_{i\bar{\jmath}} F_p$$
$$[\nabla_i, \nabla_{\bar{\jmath}}] D_a = 0$$

Scalar potential

The scalar potential is given by the following expression:

$$V = \bar{F}^i F_i - 3|L|^2 + \frac{1}{2} D^a D_a$$

Its first derivatives read:

$$\nabla_i V = -2 F_i \bar{L} + \nabla_i F_j \bar{F}^j - i \bar{k}_{ai} D^a - \frac{1}{2} \nabla_i h_{ab} D^a D^b$$

Its second derivatives are found to be:

$$\begin{aligned} \nabla_i \nabla_{\bar{\jmath}} V = & -2 g_{i\bar{\jmath}} |L|^2 + \nabla_i F_k \nabla_{\bar{\jmath}} \bar{F}^k - R_{i\bar{\jmath}p\bar{q}} F^p \bar{F}^{\bar{q}} + g_{i\bar{\jmath}} \bar{F}^k F_k - F_i \bar{F}_{\bar{\jmath}} \\ & + \bar{k}_{ia} k^a_{\bar{\jmath}} + T_{ai\bar{\jmath}} D^a + i \big(\bar{k}^a_i \nabla_{\bar{\jmath}} h_{ab} - k^a_{\bar{\jmath}} \nabla_i h_{ab} \big) D^b \\ & + \nabla_i h_{ac} h^{cd} \nabla_{\bar{\jmath}} h_{db} D^a D^b \end{aligned}$$

$$\begin{aligned} \nabla_i \nabla_j V = & -\nabla_i F_j \bar{L} + \nabla_i \nabla_j F_k \bar{F}^k + \bar{k}_{ia} \bar{k}^a_j + 2i \bar{k}^a_{(i} \nabla_{j)} h_{ab} D^b \\ & - \frac{1}{2} \big(\nabla_i \nabla_j h_{ab} - 2 \nabla_i h_{ac} h^{cd} \nabla_j h_{db} \big) D^a D^b \end{aligned}$$

Fermions and susy breaking

The n_c chiralini ψ^I and the n_v gaugini λ^a are naturally defined on the tangent bundle of the scalar manifold, locally defined by e_i^I and $\bar{e}_{\bar{\jmath}}^{\bar{J}}$.

The SUSY transformations give $\delta\psi^I \supset -\sqrt{2}e_i^I F^i \xi$ and $\delta\lambda^a \supset i D^a \xi$.
The Goldstino direction in the tangent and gauge spaces is thus:

$$\eta^I = e_i^I F^i \,, \quad \eta^a = D^a$$

The corresponding sGoldstino direction on the scalar manifold is:

$$\eta^i = e_I^i \eta^I = F^i$$

This defines as before 2 orthogonal directions for scalar fields:

$$\eta^u = (F^i, \bar{F}^{\bar{\imath}}) \,, \quad \tilde{\eta}^u = J^u{}_v \eta^v = (iF^i, -i\bar{F}^{\bar{\imath}})$$

Notice also that the Goldstone directions k_a^i correspond to flat directions of the scalar mass matrix.

SUSY is spontaneously broken when $F_i, D_a \neq 0$. The $2n_c$ stationarity conditions imply then that

$$\nabla_i F_j \bar{F}^j = 2F_i \bar{L} + i\bar{k}_{ai} D^a + \frac{1}{2}\nabla_i h_{ab} D^a D^b$$

At such a point, the values of F_i and D_a get further correlated. Indeed, whereas the vanishing of the real part of $k_a^i \nabla_i V$ is automatic by gauge invariance, the vanishing of its imaginary part implies that:

$$D^a = 2\big[M^2 + 2\big(\bar{F}^k F_k - |L|^2\big)h\big]^{-1ab} T_{ai\bar{\jmath}} \bar{F}^i F^{\bar{\jmath}}$$

Metastability

As before, the strongest constraint on metastability comes from averaging over the 2 real sGoldstino directions $\eta, \tilde{\eta}$, and considering:

$$\lambda = \frac{\nabla_i \nabla_{\bar{\jmath}} V \bar{F}^i F^{\bar{\jmath}}}{|L|^2 \bar{F}^k F_k}$$

After a straightfoward computation, one finds that

$$\lambda = 2 + R\,\frac{\bar{F}^i F_i}{|L^2|} + \big(1 + \Delta_1\big)\frac{D^a D_a}{|L|^2} + \big(-4|L|^2 + M^2\big)\frac{D^a D_a}{|L|^2 \bar{F}^k F_k} + \frac{1}{4}\Delta_2 \frac{(D^a D_a)^2}{|L|^2 \bar{F}^k F_k}$$

where

$$R = -\frac{R_{i\bar{\jmath}p\bar{q}}\bar{F}^i F^{\bar{\jmath}} \bar{F}^p F^{\bar{q}}}{(\bar{F}^k F_k)^2}\,, \quad \Delta_1 = \frac{\nabla_i h_{ac} h^{cd} \nabla_{\bar{\jmath}} h_{bd} \bar{F}^i F^{\bar{\jmath}} D^a D^b}{\bar{F}^k F_k D^c D_c}$$

$$M^2 = \frac{M^2_{ab} D^a D^b}{D^c D_c}\,, \quad \Delta_2 = \frac{\nabla_i h_{ab} \nabla^i h_{cd} D^a D^b D^c D^d}{(D^c D_c)^2}$$

When M^2_{ab} is large, D_a is small. One can then neglect D_a except when multiplied by M^2_{ab}. This corresponds to integrate out the heavy vector multiplets. In terms of $\gamma = V/(3|L|^2)$, one finds then as before:

$$\lambda \simeq 2 + 3(1 + \gamma)\tilde{R}$$

Here $\tilde{R}$ is again the holomorphic sectional curvature in the $\eta, \tilde{\eta}$ plane

$$\tilde{R} = -\frac{\tilde{R}_{i\bar{\jmath}p\bar{q}}\bar{F}^i F^{\bar{\jmath}} \bar{F}^p F^{\bar{q}}}{(\bar{F}^k F_k)^2}$$

However, it involves now the low-energy effective curvature:

$$\tilde{R}_{i\bar{\jmath}p\bar{q}} = R_{i\bar{\jmath}p\bar{q}} - 2\, T_{ai\bar{\jmath}} M^{-2ab} T_{bp\bar{q}} - 2\, T_{ai\bar{q}} M^{-2ab} T_{bp\bar{\jmath}}$$

For a given positive γ the condition for positive λ gets then milder:

$$\tilde{R} \gtrsim -\frac{2}{3}\frac{1}{1+\gamma} = \begin{cases} -\frac{2}{3}\,, & \gamma \ll 1 \\ 0\,, & \gamma \gg 1 \end{cases}$$

Gomez-Reino, Scrucca 2007

Applications

- $K = \sum_i \Phi^i e^{q_{ia} V_a} \bar{\Phi}^i$ $\Rightarrow$ OK
- $K = -\sum_i n_i \log(\Phi^i + \bar{\Phi}^i - \delta_{ia} V_a)$ $\Rightarrow$ OK

N = 2 MODELS WITH HYPER MULTIPLETS

Geometric formulation

A model with $n_{\mathcal{H}}$ hyper multiplets $\mathcal{H}^i = (\phi^i_{1,2,3,4}, \psi^i_{1,2}, N^i_{1,2,3,4})$ is set by a scalar metric h_{uv}, a triplet of Hyperkähler forms J^x_{uv}, and a real Killing vector k^u. The theory also has an $SU(2)$ symmetry.

The $4n_{\mathcal{H}}$ scalars span a Quaternionic-Kähler manifold, with an $SU(2)$ bundle with curvatures J^x_{uv}. The holonomy is $SP(2n_{\mathcal{H}}) \times SU(2)$. The vielbein $U^{\alpha A}_u$ satisfies $U^{\alpha A}_u U^B_{\alpha v} = \frac{1}{2}\epsilon^{AB} h_{uv} + \frac{i}{2}\sigma^{xAB} J^x_{uv}$. Moreover, there should be an isometry associated to k^u.

The theory can be described in an $SU(2)$ covariant way, with a covariant derivative ∇_u involving both the Christoffel and the $SU(2)$ connections Γ^w_{uv} and ω^x_u. On doublets and triplets one has:

$$\nabla_u{}^A{}_B = D_u(\Gamma)\delta^A_B - i\sigma^{xA}{}_B\omega^x_u\,, \quad \nabla_u{}^{xy} = D_u(\Gamma)\delta^{xy} + i\epsilon^{xyz}\omega^z_u$$

The Hyperkähler forms satisfy:

$$\nabla_u J^x_{vw} = 0$$
$$J^x_{uw} J^{yw}{}_v = -h_{uv}\delta^{xy} + \epsilon^{xyz} J^z_{uv}$$

The Riemann tensor is constrained to take the following form:

$$R_{uvrs} = -h_{u[r}h_{vs]} - J^x_{uv}J^x_{rs} - J^x_{u[r}J^x_{vs]} + \Sigma_{uvrs}$$

The tensor Σ_{uvrs} is constructed out of a symmetric $SP(2n_{\mathcal{H}})$ tensor $\Sigma_{\alpha\beta\gamma\delta}$ as $\Sigma_{uvrs} = \epsilon_{AB}\epsilon_{CD}U^{\alpha A}_u U^{\beta B}_v U^{\gamma C}_r U^{\delta D}_s \Sigma_{\alpha\beta\gamma\delta}$. It represents a a Weyl part of the curvature, because

$$g^{ur}\Sigma_{uvrs} = 0$$

The Ricci part of the curvature is instead universal and given by:

$$R_{uv} = -2(n_{\mathcal{H}} + 2)h_{uv}\,, \quad R = -8n_{\mathcal{H}}(n_{\mathcal{H}} + 2)$$

The gravitino masses are described by a triplet of real quantities, which represent Killing potentials for the Killing vector k^u:

$$P^x = \frac{1}{2n_{\mathcal{H}}} J^x_{uv} \nabla^u k^v \qquad \left(m_{3/2}^{AB} = P^x \sigma^{xAB}\,, \;\; m_{3/2} = \sqrt{P^x P^x}\right)$$

The auxiliary fields are obtained by taking a covariant derivative:

$$N_u = 2\, k_u$$

The above quantities satisfy the following relations:

$$\nabla_u P^x = J^x_{uv} N^v\,, \;\; \nabla^2 P^x = 4 n_{\mathcal{H}} P^x$$
$$\nabla_{(u} N_{v)} = 0\,, \;\; \nabla_u \nabla_v N_r = -R_{vrus} N^s$$

For commutators of covariant derivatives, one finds:

$$\left[\nabla_u, \nabla_v\right] P^x = -2\, \epsilon^{xyz} J^y_{uv} P^z$$
$$\left[\nabla_u, \nabla_v\right] N_r = R_{uvrs} N^s$$

Scalar potential

The scalar potential takes the following simple form:

$$V = N^r N_r - 3\, P^x P^x$$

Its first derivatives are given by

$$\nabla_u V = -6 P^x J^x_{ur} N^r + 2 \nabla_u N_r N^r$$

Its second derivatives read instead:

$$\begin{aligned} \nabla_u \nabla_v V = & \; 2 \nabla_u N^r \nabla_v N_r - 2 (R_{urvs} + 3 J^x_{ur} J^x_{vs}) N^r N^s \\ & -6 P^x J^x_{(ur} \nabla_{v)} N^r \end{aligned}$$

Fermions and susy breaking

The $2 n_{\mathcal{H}}$ chiral fermions ψ^α are naturally defined on the tangent space of the scalar manifold, which is locally defined by the vielbein $U_u^{\alpha A}$.

The $\mathbf{2}$ SUSY transformations give $\delta\psi^{\alpha} \supset U^{\alpha A}_{u} N^{u} \xi_{A}$. The $\mathbf{2}$ Goldstino directions are thus described in the tangent space by:

$$\eta^{\alpha A} = U^{\alpha A}_{u} N^{u}$$

The corresponding sGoldstino directions on the scalar manifold are:

$$\eta^{u}_{AB} = U^{u}_{\alpha A} \eta^{\alpha}_{B} = \frac{1}{2} \epsilon_{AB} N^{u} + \frac{i}{2} \sigma^{x}_{AB} J^{xu}{}_{v} N^{v}$$

This defines $\mathbf{4}$ orthogonal directions in scalar-field space:

$$\eta^{u} = N^{u} \,, \quad \tilde{\eta}^{u}_{x} = J^{xu}{}_{v} N^{v}$$

The first corresponds however to the Goldstone flat direction k^{u}.

SUSY is spontaneously broken whenever $N_{u} \neq 0$. The $4n_{\mathcal{H}}$ stationarity conditions imply then that

$$\nabla_{u} N_{r} N^{r} = 3 P^{x} J^{x}_{ur} N^{r}$$

Metastability

The crucial condition on metastability comes in this case from averaging over the 3 non-trivial sGoldstino directions $\tilde{\eta}_x$, and considering:

$$\lambda = \frac{1}{6} \frac{\nabla_u \nabla_v V \, J^{xu}{}_r N^r J^{xv}{}_s N^s}{P^y P^y \, N^w N_w}$$

After a straightfoward but non-trivial computation, one finds a formula that resembles that for $N = 1$ theories with chiral multiplets:

$$\lambda = \frac{8}{3} - (R + 3) \, \frac{N^u N_u}{P^x P^x}$$

The quantity R is now the averaged triholomorphic sectional curvature in the planes $\eta, \tilde{\eta}_x$, namely

$$R = \frac{1}{3} \frac{R_{urvs} N^u J^{xr}{}_p N^p N^v J^{xs}{}_q N^q}{(N^w N_w)^2}$$

But using the constrained form of R_{urvs}, one finds that the Weyl part Σ_{urvs} does not contribute and the Ricci gives a universal answer:

$$R = -2$$

In terms of $\gamma = V/(3P^x P^x)$, it follows then that:

$$\lambda = -\frac{1}{3}(1 + 9\gamma)$$

For any γ that is positive, λ is therefore always negative, and there is unavoidably an instability.

Gomez-Reino, Louis, Scrucca 2009

N = 2 MODELS WITH ABELIAN VECTOR MULTIPLETS

Geometric formulation

A model with $n_\mathcal{V}$ vector multiplets $\mathcal{V}^i = (\phi^i_{1,2}, \lambda^i_{1,2}, A^i_\mu, W^i_{1,2,3})$ is set by a special real Kähler potential K, some holomorphic Killing vectors k^i_Λ and a triplet of constants P^x_Λ. The theory also has an $SU(2)$ symmetry. For Abelian gaugings, $k^i_\Lambda = 0$ and $P^x_\Lambda \to P_\Lambda$ defining $U(1) \in SU(2)$.

The $2n_\mathcal{V}$ scalars span a Special-Kähler manifold with metric $g_{i\bar{\jmath}} = K_{i\bar{\jmath}}$ and Kähler form $J_{i\bar{\jmath}} = g_{i\bar{\jmath}}$, with a $U(1)$ bundle on it of curvature $J_{i\bar{\jmath}}$. The holonomy is $U(n_\mathcal{V}) \times U(1)$. The vielbein has the form e_i^I and $e_{\bar{\imath}}^{\bar{I}}$.

We can use a $U(1)$ covariant formulation, with a covariant derivative ∇_i involving the Christoffel and the $U(1)$ connections Γ^k_{ij} and $\omega_i = K_i$.

The Riemann tensor is constrained to take the following form:

$$R_{i\bar{\jmath}p\bar{q}} = g_{i\bar{\jmath}}g_{p\bar{q}} + g_{i\bar{q}}g_{p\bar{\jmath}} - C_{ijr}g^{r\bar{s}}\bar{C}_{\bar{s}\bar{\jmath}\bar{q}}$$

The symmetric tensor C_{ijk} satisfies the following constraints:

$$\nabla_{\bar{\jmath}} C_{ikl} = 0\,, \quad \nabla_{[i} C_{j]kl} = 0$$

The gravitino masses are degenerate and given by a single covariantly holomorphic section. In special coordinates X^Λ, one has:

$$L = e^{K/2} P_\Lambda X^\Lambda \qquad \big((m_{3/2})_{1,2} = |L|\big)$$

The non-trivial auxiliary fields are defined by taking a covariant derivative:

$$W_i = e^{K/2} P_\Lambda \big(\partial_i X^\Lambda + K_i X^\Lambda\big)$$

These quantities satisfy the following relations:

$$\nabla_i L = W_i\,, \quad \nabla_{\bar{\jmath}} L = 0\,, \quad \nabla_i W_j = C_{ijk} \bar{W}^k\,, \quad \nabla_{\bar{\jmath}} W_i = g_{i\bar{\jmath}} L$$

Moreover, the commutators of covariant derivatives give:

$$[\nabla_i, \nabla_{\bar{\jmath}}] L = -g_{i\bar{\jmath}} L$$
$$[\nabla_i, \nabla_{\bar{\jmath}}] W_p = R_{i\bar{\jmath}p\bar{q}} \bar{W}^q - g_{i\bar{\jmath}} W_p$$

Scalar potential

The scalar potential takes the following simple form:

$$V = \bar{W}^i W_i - 3\,|L|^2$$

Its first derivatives are given by

$$\nabla_i V = -2W_i \bar{L} + C_{ijk} \bar{W}^j \bar{W}^k$$

Its second derivatives read instead:

$$\nabla_i \nabla_{\bar{\jmath}} V = -2\,g_{i\bar{\jmath}} |L|^2 - 2\,W_i \bar{W}_{\bar{\jmath}} + 2\,C_{ijr} g^{r\bar{s}} \bar{C}_{\bar{s}\bar{\jmath}\bar{q}} \bar{W}^p W^{\bar{q}}$$
$$\nabla_i \nabla_j V = \nabla_{(i} C_{j)kl} \bar{W}^k \bar{W}^l$$

Fermions and susy breaking

The $2n_\nu$ chiral fermions $\lambda^I_{1,2}$ are naturally defined on the tangent space of the scalar manifold, locally defined by the vielbein e^I_i and $\bar{e}^{\bar{J}}_{\bar{\jmath}}$.

The $\mathbf{2}$ SUSY transformations give $\delta\psi^I_{1,2} \supset e^I_i W^i \xi_{1,2}$. The $\mathbf{2}$ Goldstino directions are thus degenerate and they are both described in the tangent space by:

$$\eta^I = e^I_i W^i$$

The corresponding sGoldstino direction on the scalar manifold is:

$$\eta^i = e^i_I \eta^I = W^i$$

This defines $\mathbf{2}$ independent directions in the real scalar-field space:

$$\eta^u = (W^i, \bar{W}^{\bar{\imath}})\,, \quad \tilde{\eta}^u = J^u{}_v \eta^v = (iW^i, -i\bar{W}^{\bar{\imath}})$$

SUSY is spontaneously broken whenever $W_i \neq 0$. The $2n_\mathcal{V}$ stationarity conditions imply then that

$$C_{ijk}\bar{W}^j\bar{W}^k = 2W_i\bar{L}$$

Metastability

The crucial constraint on metastability comes in this case by averaging over the $\mathbf{2}$ real sGoldstino directions $\eta, \tilde{\eta}$, and considering:

$$\lambda = \frac{\nabla_i \nabla_{\bar{\jmath}} V \bar{W}^i W^{\bar{\jmath}}}{|L|^2 \bar{W}^k W_k}$$

At a stationary point, this is given by:

$$\lambda = 2 + R \frac{\bar{W}^i W_i}{|L^2|}$$

The quantity R is the holomorphic sectional curvature in the plane $\eta, \tilde{\eta}$:

$$R = -\frac{R_{i\bar{\jmath}p\bar{q}} \bar{W}^i W^{\bar{\jmath}} \bar{W}^p W^{\bar{q}}}{(\bar{W}^k W_k)^2}$$

But using the special form of the curvature and the stationarity condition, one finds that:

$$R = -2 + 4 \frac{|L|^2}{\bar{W}^k W_k}$$

In terms of the parameter $\gamma = V/(3|L|^2)$, one obtains then:

$$\lambda = -6\gamma$$

For γ positive, λ is thus negative and there is an instability.

Cremmer, Kounnas, Van Proeyen, Derendinger, Ferrara, De Wit, Girardello 1985

This result seems to persist in the same form in a large class of N = 4 and N = 8 models with vector multiplets.

Kallosh, Linde, Prokushkin, Shmakova 2001

N = 2 MODELS WITH HYPER AND VECTOR MULTIPLETS

New features

In more general models, there are new possibilities:

- Non-Abelian gaugings
- Fayet-Iliopoulos terms
- Duality twists
- Mixing of hyper and vector multiplets

This allows for models admitting metastable de Sitter vacua.

Fre, Trigiante, Van Proeyen 2002

It would be interesting to generalize our analysis to understand which of these ingredients are really necessary for metastability.

Dall'Agata, Gomez-Reino, Louis, Scrucca WIP

APPLICATIONS IN CALABI-YAU STRING MODELS

Dilaton

In N=1 models, this is a chiral multiplet, with Kähler manifold

$$\mathcal{M} \simeq \frac{SU(1,1)}{U(1)}\,, \quad K \simeq -\log(S+\bar{S})$$

One finds:

$$R \simeq -2$$

To get $\lambda \gtrsim 0$, we need $R \gtrsim -\frac{2}{3}(1+\gamma)$. This could be achieved thanks to corrections, but these should be large.

In N=2 models, this is in a hyper multiplet, with Quaternionic manifold

$$\mathcal{M} \simeq \frac{SU(1,2)}{U(1)\times SU(2)}\,, \quad K \simeq -\log(S+\bar{S}-C\bar{C})$$

In this case $\lambda > 0$, no matter what kind of corrections may appear.

Geometric moduli

In N=1 models, these are chiral multiplets, with Kähler manifold

$$\mathcal{M} \not\simeq \frac{G}{H}\,, \quad K \simeq -\log\left(d_{ijk}(T^i + \bar{T}^i)(T^j + \bar{T}^j)(T^k + \bar{T}^k)\right)$$

The no-scale property $K^i K_i = 3$ implies that $R \simeq -\frac{2}{3}$ along $F^i \propto K^i$.

When $\Delta(d_{ijk}) = 0$, $\mathcal{M}$ becomes a coset and has constant curvature. This happens e.g. for K3-fibrations or orbifolds. One finds then

$$\text{max}(R) \simeq -\frac{2}{3}$$

When $\Delta(d_{ijk}) \neq 0$, the curvature is no longer constant. One finds then:

$$\text{max}(R) \simeq \begin{cases} -\frac{2}{3}\,, & \Delta(d_{ijk}) > 0 \\ -\frac{2}{3} + \text{positive}\,, & \Delta(d_{ijk}) < 0 \end{cases}$$

To get $\lambda \gtrsim 0$, we need $R \gtrsim -\frac{2}{3}(1+\gamma)^{-1}$. This can be achieved with corrections, whose size grow with γ, or without, depending on $\Delta(d_{ijk})$.

In N=2 models, these are in vector multiplets, with Special geometry

$$\mathcal{M} \not\simeq \frac{G}{H}\,, \quad K \simeq -\log\left(d_{ijk}(T^i+\bar{T}^i)(T^j+\bar{T}^j)(T^k+\bar{T}^k)\right)$$

In this case $\lambda > 0$, if the potential comes from an Abelian gauging, no matter what kind of corrections may appear.

Covi, Gomez-Reino, Gross, Louis, Palma, Scrucca 2008

CONCLUSIONS

- In $N = 1$ SUGRA theories, there exist a strong necessary condition on the Kähler potential for the existence of metastable stationary points with broken SUSY, no matter what the superpotential is.
- In $N = 2$ SUGRA theories, there are similar constraints which are even stronger and completely exclude some particular classes of models, like those with only hyper or Abelian vector multiplets.