MEDIATION OF SUPERSYMMETRY BREAKING IN EXTRA DIMENSIONS

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- Mechanisms for the transmission of SUSY breaking.
  Generic problems and possible solutions.
- Geometrical sequestering through extra dimensions.
  Soft masses and relevance of quantum corrections.
- Brane worlds from orbifold models.
  Microscopic and low energy effective theories.
  Loop corrections and SUSY cancellations.
- Radius-dependent loop effects in sequestered models.
  Universal correction to soft masses.
- Prototype of viable model.

Rattazzi, Scrucca, Strumia (hep-th/0305184)

SUSY BREAKING

The standard scenario is that SUSY breaking occurs at a scale M in a hidden sector and is transmitted to the visible sector through some interactions, generating soft breaking terms.

There are two main delicate points for phenomenology:

- Sflavour: m_0^2 must be positive and nearly universal.
- Shierarchy: must have $\mu \sim m_0 \sim m_{1/2} \ll \Lambda_{\rm UV}$.

Gauge mediation

SUSY breaking at $M < M_{\rm M}$ mediated by gauge interactions:

- $m_{1/2} \sim \dfrac{g^2}{16\pi^2} \dfrac{M^2}{M_M}$, $m_0^2 \sim \Big(\dfrac{g^2}{16\pi^2}\Big)^2 \Big(\dfrac{M^2}{M_M}\Big)^2$.
- m_0^2 universal ($M_{\rm F} \gg M_M$) and positive.
- $\mu \sim M_{\rm M}$ from interactions.

Gravity mediation

SUSY breaking at $M \ll M_{\rm P}$ mediated by gravity interactions:

- $m_{1/2}, m_{3/2} \sim \dfrac{M^2}{M_{\rm P}}$, $m_0^2 \sim \Big(\dfrac{M^2}{M_{\rm P}}\Big)^2$.
- m_0^2 generic ($M_{\rm F} \sim M_{\rm P}$).
- $\mu \sim \dfrac{M^2}{M_{\rm P}}$ from interactions.

Symmetries

To naturally solve both problems, one can try to introduce new symmetries. Main options:

- Gauge mediation + complications for Shierarchy.
  No simple and compelling model so far.
- Gravity mediation + constraints for Sflavour.
  Difficult to forbid mixing of the two sectors at $M_{\rm P}$.

Geometry

An interesting possibility, natural in string theory, is to separate the visible and the hidden sectors along an extra dimension. This framework has very peculiar characteristics going beyond symmetries:

- Geometric distinction between visible sector, hidden sector, and mediating interactions.
- New physical scale $M_{\rm C}$ acting as cut-off for the mixing between the two sectors.

GEOMETRICAL SEQUESTERING IN SUGRA

Consider a general SUGRA theory with:

Visible: $\Phi_0 = (\phi_0, \chi_0; F_0)$, $V_0 = (A_0^\mu, \lambda_0; D_0)$.

Hidden: $\Phi_\pi = (\phi_\pi, \chi_\pi; F_\pi)$, $V_\pi = (A_\pi^\mu, \lambda_\pi; D_\pi)$.

Interactions: $C = (e_\mu^a, \psi_\mu; a_\mu, b_\mu)$, $S = (\phi_S, \psi_S; F_S)$.

After superconformal gauge-fixing, $b_\mu = 0$, $\phi_S = 1$, $\psi_S = 0$, and the structure of the matter action reads:

$$\begin{aligned}\mathcal{L}_{\rm mat} &= \Big[\Omega(\Phi, \Phi^\dagger) S S^\dagger\Big]_D + \Big[P(\Phi) S^3\Big]_F + \Big[P(\Phi) S^3\Big]_F^\dagger \\ &\quad + \Big[\tau(\Phi) \mathcal{W}^2\Big]_F + \Big[\tau(\Phi) \mathcal{W}^2\Big]_F^\dagger\end{aligned}$$

The functions Ω, $\tau\mathcal{W}^2$ and P have expansions of the type:

$$\begin{aligned}\Omega &= -3M_{\rm P}^2 + \Phi_0\Phi_0^\dagger + \Phi_\pi\Phi_\pi^\dagger + \frac{h}{M_{\rm P}^2}\Phi_0\Phi_0^\dagger\Phi_\pi\Phi_\pi^\dagger + \dots \\ P &= \Lambda^3 + M_\pi^2\Phi_\pi + \dots \\ \tau\mathcal{W}^2 &= \frac{1}{g_0^2}\mathcal{W}_0^2 + \frac{1}{g_\pi^2}\mathcal{W}_\pi^2 + \frac{k}{M_{\rm P}}\Phi_\pi\mathcal{W}_0^2 + \dots\end{aligned}$$

For a vanishing cosmological constant, we tune $\Lambda^3 \sim M_\pi^2 M_{\rm P}$.

The SUSY breaking VEVs are then:

$$|F_\pi| \sim M_\pi^2\,, \quad |F_S| \sim \frac{\Lambda^3}{M_{\rm P}^2} \sim \frac{M_\pi^2}{M_{\rm P}}$$

Classical theory

Leading soft masses at classical level:

$$m_{3/2} \sim |F_S| \sim \frac{M_\pi^2}{M_\mathrm{P}}$$

$$m_{1/2} \sim k\,\frac{|F_\pi|}{M_\mathrm{P}} \sim k\,\frac{M_\pi^2}{M_\mathrm{P}}$$

$$m_0^2 \sim h\,\frac{|F_\pi|^2}{M_\mathrm{P}^2} \sim h\,\frac{M_\pi^4}{M_\mathrm{P}^2}$$

Non-universal; separating visible and hidden sectors in an extra dimension, $h = k = 0$ $\Rightarrow$ quantum corrections important.

Quantum corrections

Corrections from gauge loops due to superconformal anomaly:

$$\delta m_{1/2} \sim \frac{g^2}{16\pi^2}\,|F_S| \sim \frac{g^2}{16\pi^2}\,\frac{M_\pi^2}{M_\mathrm{P}}$$

$$\delta m_0^2 \sim \Big(\frac{g^2}{16\pi^2}\Big)^2\,|F_S|^2 \sim \Big(\frac{g^2}{16\pi^2}\Big)^2\,\frac{M_\pi^4}{M_\mathrm{P}^2}$$

Universal; positive for squarks and negative for sleptons !

Randall, Sundrum;

Giudice, Luty, Murayama, Rattazzi

With an extra dimension, corrections from gravity loops are cut off at $M_\mathrm{C} = (\pi R)^{-1}$ and computable:

$$\delta m_0^2 \sim \frac{M_\mathrm{C}^2}{16\pi^2 M_\mathrm{P}^2}\,\frac{|F_\pi|^2}{M_\mathrm{P}^2} \sim \frac{M_\mathrm{C}^2}{16\pi^2 M_\mathrm{P}^2}\,\frac{M_\pi^4}{M_\mathrm{P}^2}$$

Universal; positive or negative ?

Gauge and gravitational quantum corrections can compete if (gravity loop at M_C) $\sim$ (gauge loop)^2, that is:

$$\frac{M_\mathrm{C}^2}{16\pi^2 M_\mathrm{P}^2} \sim \left(\frac{g^2}{16\pi^2}\right)^2 \Rightarrow \frac{M_\mathrm{C}}{M_\mathrm{P}} \sim \frac{g^2}{4\pi}$$

This is reasonable $\Rightarrow$ possible very interesting hybrid models of SUSY breaking.

Dynamics of extra dimensions

In the 4D effective theory for $E \ll M_\mathrm{C}$, the dynamics of an extra dimension is described by a chiral multiplet:

Radion: $T = (T, \psi_T; F_T)$.

The VEV of T controls the radius $(\mathrm{Re}\, T = \pi R)$, whereas a VEV for F_T gives additional SUSY-breaking effects.

There are various ways to get a satisfactory radion dynamics.
F-terms: e.g. strong coupling condensation of bulk gaugino.
D-terms: e.g. Casimir energy with localized kinetic terms.

Luty, Sundrum;
Ponton, Poppitz

To compute radiative effects involving the radion multiplet, one needs a full 5D supergravity description. All these effects are non-local and therefore finite and insensitive to UV physics.

$S^1/\mathbf{Z_2}$ ORBIFOLD MODELS

The extra dimension is a circle with coordinate $x^5 \in [0, 2\pi]$ and gauged parity $\mathbf{Z_2} : \mathbf{x^5} \to -\mathbf{x^5}$. The radius is $e_{\dot{5}}^{5} = R$.

The visible and hidden sectors are located at the fixed-points at 0 and π, and have $N = 1$ SUSY with $U(1)$ R-symmetry (bosons: q, fermions: $q-1$, aux: $q-2$):

Visible: $\Phi_0 = (\phi_0, \chi_0; F_0)$, $V_0 = (A_0^\mu, \lambda_0; D_0)$.

Hidden: $\Phi_\pi = (\phi_\pi, \chi_\pi; F_\pi)$, $V_\pi = (A_\pi^\mu, \lambda_\pi; D_\pi)$.

The interactions are in the bulk, and have $N = 2$ SUSY with $SU(2)$ R-symmetry (bosons: $\mathbf{1}$, fermions: $\mathbf{2}$, aux: $\mathbf{1}$ or $\mathbf{3}$):

Gauge: $\mathcal{V} = (A_M, \lambda, \Sigma; \vec{X})$.

Gravity: $\mathcal{M} = (e_M^A, \psi_M, A_M; \vec{V}_M, \vec{t}, v_{AB}, \lambda, C)$,
$\mathcal{T} = (\vec{Y}, B_{MNP}, \rho; N)$.

Bulk and boundary theories: fixed by $N = 2$ and $N = 1$ SUSY. Bulk-boundary couplings: fixed by $N = 1$ SUSY with $N = 2$ bulk multiplets decomposed into $N = 1$ boundary multiplets.

The Lagrangian (with e factored out) has the form:

$$\mathcal{L} = \mathcal{L}_5 + e_{\dot{5}}^{5}\, \delta(x^5 - 0)\mathcal{L}_{4,0} + e_{\dot{5}}^{5}\, \delta(x^5 - \pi)\mathcal{L}_{4,\pi}$$

Singularities

Auxiliary fields have a dimensionless propagator and could give divergences in the sums over KK modes with $m_n = n/R$.

In the natural formulation, auxiliary and odd fields mix through $\partial_5 \Rightarrow$ propagators $\Box_4/\Box_5$ and $1/\Box_5$. Matter couples to auxiliary fields $\Rightarrow$ no singularities.

Making a shift, auxiliary and odd fields can be decoupled $\Rightarrow$ propagators 1 and $1/\Box_5$. Matter couples to odd fields through $\partial_5 \Rightarrow$ singularities cancelled by contact terms proportional to

$$\delta(0) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} 1 = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \frac{p^2 - m_n^2}{p^2 - m_n^2}$$

Gauge interactions

In this case, the off-shell formulation of the bulk theory is simple. The bulk-to-boundary couplings are well understood.

Mirabelli, Peskin

Gravity interactions

In this case, the off-shell formulation is rather involved and has been formulated only recently.

Zucker

The bulk-to-boundary couplings have been only partly studied.

Gherghetta, Riotto

GAUGE INTERACTIONS

The Lagrangian for the $N = 2$ bulk vector mult. $\mathcal{V}$ is $(g_5 \to 1)$:

$$\mathcal{L}_5 = -\frac{1}{4}F_{MN}^2 + \frac{i}{2}\bar{\lambda}\partial\!\!\!/\lambda + \frac{1}{2}|\partial_M \Sigma|^2 + \frac{1}{2}\vec{X}^2$$

The $\mathbf{Z_2}$ parities of $\mathcal{V}$ are:

$\mathcal{V}$	A_M	λ	Σ	$\vec{X}$
$+$	A_μ	λ^1		X^3
$-$	A_5	λ^2	Σ	$X^{1,2}$

At the fixed-points, the non-vanishing components of $\mathcal{V}$ from an $N = 1$ vector multiplet $V = (A_\mu, \lambda^1; D)$ with

$$D = X^3 - \partial_{\dot{5}}\Sigma$$

The Lagrangian for a charged $N = 1$ boundary chiral multiplet $\Phi = (\phi, \chi; F)$ is then:

$$\mathcal{L}_4^\Phi = |D_\mu \phi|^2 + i\bar{\chi}D\!\!\!\!/\,\chi + |F|^2 + |\phi|^2 D + \dots$$

After integrating out $X^{1,2}$ and F, the total Lagrangian reads:

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F_{MN}^2 + \frac{i}{2}\bar{\lambda}\partial\!\!\!/\lambda + \frac{1}{2}|\partial_\mu \Sigma|^2 + \frac{1}{2}D^2 \\ &\quad + e_{\dot{5}}^5\,\delta(x^5)\Big[|D_\mu\phi|^2 + i\bar{\chi}D\!\!\!\!/\,\chi\Big] + \Big(\partial_{\dot{5}}\Sigma + \rho_{\dot{5}}(x^5)\Big)D + \dots\end{aligned}$$

The density which couples to D is given by:

$$\rho_{\dot{5}}(x^5) = e_{\dot{5}}^5\, \delta(x^5)\, |\phi|^2$$

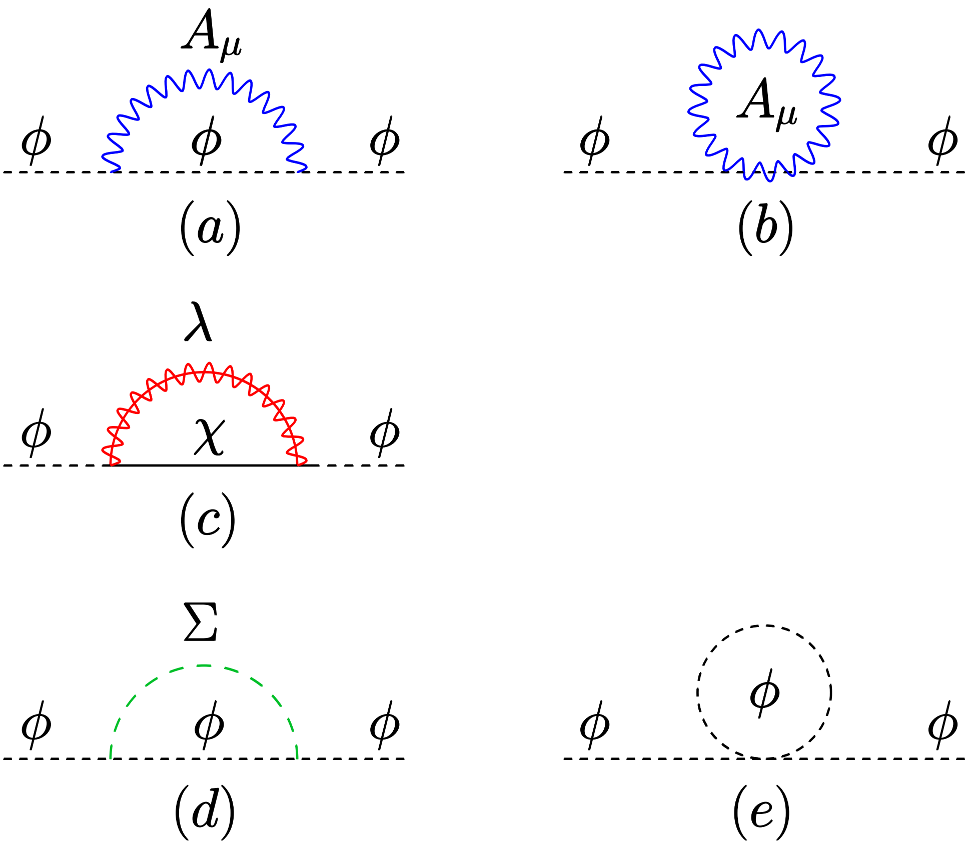
Redefining D through a shift to complete squares, one gets:

$$\begin{aligned}\mathcal{L} \;=\; & -\frac{1}{4}F_{MN}^2 + \frac{i}{2}\bar{\lambda}\not{\partial}\lambda + \frac{1}{2}|\partial_\mu \Sigma|^2 + \frac{1}{2}\tilde{D}^2 \\ & + e_{\dot{5}}^5\, \delta(x^5)\Big[|D_\mu\phi|^2 + i\bar{\chi}\not{D}\chi\Big] \\ & - \frac{1}{2}\Big(\partial_{\dot{5}}\Sigma + \rho_{\dot{5}}(x^5)\Big)^2 + \dots\end{aligned}$$

Loop corrections

Consider for example the 1-loop correction to the mass of ϕ.

This must vanish by SUSY non-renormalization theorem.



The result is:

$$\Delta m^2 = \frac{i}{2\pi R} \sum_\alpha \sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \frac{N_{\alpha,n}}{p^2 - m_n^2}$$

with

$$\begin{aligned} N_{a,n} &= -p^2 & \qquad N_{b,n} &= 4\,p^2 \\ N_{c,n} &= -4\,p^2 \\ N_{d,n} &= m_n^2 & \qquad N_{e,n} &= p^2 - m_n^2 \end{aligned}$$

Low-energy theory

The low energy theory for $E \ll M_{\rm C}$ is obtained by integrating out Σ. Neglecting $\partial_\mu \sim E$ with respect to $\partial_5 \sim M_{\rm C}$, its equation of motion is:

$$\partial_{\dot{5}} \Big(\partial_{\dot{5}} \Sigma + \rho_{\dot{5}}(x^5) \Big) = 0$$

The solution is

$$\partial_{\dot{5}} \Sigma = - \Big(\rho_{\dot{5}}(x^5) - \frac{1}{2\pi R} \rho \Big)$$

with

$$\rho = \int_0^{2\pi} dx^5 e_5^{\dot{5}} \, \rho_{\dot{5}}(x^5) = |\phi|^2$$

Substituting back in the Lagrangian and integrating over x^5, one finds:

$$\begin{aligned} \mathcal{L}^{\rm eff} \;=\; & -\frac{1}{4} F_{\mu\nu}^2 + \frac{i}{2} \bar{\lambda}^1 \not{\partial} \lambda^1 \\ & + |D_\mu \phi|^2 + i \bar{\chi} \not{D} \chi - \frac{1}{2} \frac{\rho^2}{2\pi R} + \ldots \end{aligned}$$

GRAVITY INTERACTIONS

The Lagrangians for the $N=2$ bulk minimal multiplet $\mathcal{M}$ and tensor multiplet $\mathcal{T}$ are $(M_5 \to 1)$:

$$
\begin{aligned}
\mathcal{L}_5^{\mathcal{M}} = &-32\,\vec{t}^{\,2} - \frac{1}{\sqrt{3}} F_{AB} v^{AB} + \bar{\psi}_M \vec{\tau} \gamma^{MN} \psi_N \vec{t} \\
&- \frac{1}{6\sqrt{3}} \varepsilon^{MNPQR} \left(A_M F_{NP} - \frac{3}{2} \bar{\psi}_M \gamma_N \psi_P\right) F_{QR} \\
&- 4C - 2i\bar{\lambda}\gamma^M \psi_M
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_5^{\mathcal{T}} = \; & Y^{-1}\Big(-\frac{1}{4} |\mathcal{D}_M \vec{Y}|^2 + W_A^2 - \frac{i}{2} \bar{\rho} \not{\mathcal{D}} \rho - (N + 6\,\vec{t}\vec{Y})^2 \\
&- \frac{1}{24} \varepsilon^{MNPQR} \vec{Y} (\vec{H}_{MN} - Y^{-2} \mathcal{D}_M \vec{Y} \times \mathcal{D}_N \vec{Y}) B_{PQR} \\
&- \frac{1}{4} \bar{\psi}_M \vec{\tau} \gamma^{MNP} \psi_N (\vec{Y} \times \mathcal{D}_P \vec{Y}) + 4\,\bar{\rho}\,\vec{\tau} \lambda\, \vec{Y} Y \Big) \\
&+ Y \Big(-\frac{1}{4} \mathcal{R}(\widehat{\omega}) - \frac{i}{2} \bar{\psi}_M \gamma^{MNP} \mathcal{D}_N \psi_P - \frac{1}{6} \widehat{F}_{MN}^2 \\
&+ 20\,\vec{t}^{\,2} + v_{AB}^2 - \frac{i}{2} \bar{\psi}_A \psi_B v^{AB} - \bar{\psi}_M \vec{\tau}\, \gamma^{MN} \psi_N\, \vec{t} \\
&- \frac{i}{4\sqrt{3}} \bar{\psi}_P \gamma^{MNPQ} \psi_Q \widehat{F}_{MN} + 4C + 2i\bar{\lambda}\gamma^M \psi_M \Big)
\end{aligned}
$$

$+\ \rho$-dep. λ-indep.

Notation:

$$W^M = \frac{1}{12}\epsilon^{MNPQR}\partial_N B_{PQR} + \frac{1}{4}\bar\psi_P\vec\tau\gamma^{PMQ}\psi_Q\vec Y$$

$$\widehat{F}_{MN} = \partial_M A_N - \partial_N A_M + i(\sqrt{3}/2)\,\bar\psi_M\psi_N$$

$$\vec H_{MN} = \mathcal{D}_M\vec V_N - \mathcal{D}_N\vec V_M$$

The derivatives $\mathcal{D}_M$ are $SU(2)_R$ and super-Lorentz covariant.

In particular:

$$\mathcal{D}_M\vec Y = \partial_M\vec Y + \vec V_M\times\vec Y$$

$$\mathcal{D}_M\vec V_N = D_M\vec V_N + \vec V_M\times\vec V_N$$

$$\mathcal{D}_M\psi_N = D_M\psi_N - \frac{i}{2}\vec V_M\vec\tau\,\psi_N$$

The $\mathbf{Z_2}$ parities of $\mathcal{M}$ and $\mathcal{T}$ are:

$\mathcal{M}$	e_M^A	ψ_M	A_M	$\vec t$	v_{AB}	$\vec V_M$	λ	C
$+$	$e_\mu^a, e_5^{\dot 5}$	ψ_μ^1, ψ_5^2	A_5	$t^{1,2}$	$v_{a\dot 5}$	$V_\mu^3, V_5^{1,2}$	λ^1	C
$-$	$e_\mu^{\dot 5}, e_5^a$	ψ_μ^2, ψ_5^1	A_μ	t^3	v_{ab}	$V_\mu^{1,2}, V_5^3$	λ^2	

$\mathcal{T}$	$\vec Y$	B_{MNP}	ρ	N
$+$	$Y^{1,2}$	$B_{\mu\nu\rho}$	ρ^1	N
$-$	Y^3	$B_{\mu\nu 5}$	ρ^2	

At the fixed-points, the non-vanishing space-time components of $\mathcal{M}$ lead to an $N=1$ intermediate gravitational multiplet $I=(e_\mu^a,\psi_\mu^1;a_\mu,b_a,t,\lambda,S)_0$ with

$$
\begin{aligned}
a_\mu &= -\frac{1}{2}\Big(V_\mu^3+4\,v_{\mu\dot5}\Big)-\frac{2}{\sqrt3}\,e_{\dot5}^5\,\widehat{F}_{\mu5} \\
b_a &= v_{a\dot5}\ ,\quad t=t^2+it^1\ ,\quad \lambda=\lambda^1 \\
S &= C+\frac{1}{2}e_{\dot5}^5\bar\lambda^1\psi_5^2-\frac{1}{2}\mathcal{D}_{\dot5}\,t^3
\end{aligned}
$$

The internal components of $\mathcal{M}$ yield instead an $N=1$ chiral radion multiplet $T=(\pi e_5^{\dot5}+i(2\pi/\sqrt3)A_5,\pi\psi_5^2;F_T)_0$ with

$$
F_T=\pi\Big(V_5^1-4\,e_5^{\dot5}\,t^2\Big)+i\pi\Big(V_5^2+4\,e_5^{\dot5}\,t^1\Big)
$$

Finally, the components of $\mathcal{T}$ give rise to an $N=1$ chiral compensator multiplet $S=(Y^2+iY^1,\rho^1;F_S)_2$ with

$$
F_S=\Big(-2N+\mathcal{D}_{\dot5}Y^3\Big)+i\Big(-2W_{\dot5}+12(Y^2t^1-Y^1t^2)\Big)
$$

The appropriate Lagrangians for $N=1$ boundary chiral and vector multiplets $\Phi=(\phi,\chi;F)_{2/3}$ and $V=(A_\mu,\lambda;D)_0$ are in this formalism:

$$
\begin{aligned}
\mathcal{L}_4^\Phi &= |\mathcal{D}_\mu\phi|^2+i\bar\chi\not{\mathcal{D}}\chi+|F-4\,\phi\,t^*|^2 \\
&\qquad +\frac{1}{6}\,|\phi|^2\Big(\mathcal{R}+2i\bar\psi_\mu^1\gamma^{\mu\nu\rho}D_\nu\psi_\rho^1\Big)+\cdots \\
\mathcal{L}_4^V &= -\frac{1}{4}G_{\mu\nu}^2+i\bar\lambda\not{\mathcal{D}}\lambda+\frac{1}{2}D^2+\cdots
\end{aligned}
$$

The chiral $U(1)_R$-covariant derivative on bosons $(F=0)$ and fermions $(F=1)$ is given by

$$\mathcal{D}_\mu = D_\mu + i\,q\,\Big(a_\mu + 3\,b_\mu\Big)(i\gamma^{\dot{5}})^F$$

with

$$\begin{aligned} q_\phi &= 2/3 \qquad & q_\chi &= -1/3 \\ q_{A_\mu} &= 0 \qquad & q_\lambda &= -1 \end{aligned}$$

The only combination of auxiliary fields coupling non-trivially to the boundary fields is therfore:

$$V_\mu = -2(a_\mu + 3\,b_\mu) = V^3_\mu - 2\,v_{\mu\dot{5}} - \frac{2}{\sqrt{3}}\,e^5_{\dot{5}}\,\widehat{F}_{\mu 5}$$

All the other auxiliary fields can be integrated out through their equations of motion, leaving a partially off-shell formulation that is still powerful enough for our purposes.

The fields C and λ act as Lagrangian multipliers and enforce the contraints $Y=1$ and $\rho=0$. After gauge-fixing $\vec{Y}=(0,1,0)^T$, and integrating out the other auxiliary fields, one finds finally the following total Lagrangian:

$$\begin{aligned} \mathcal{L} \;=\; & \frac{1}{6}\Omega_{\dot{5}}(x^5)\Big[\mathcal{R} + 2i\bar{\psi}_M\gamma^{MNP}D_N\psi_P + \frac{2}{3}V^2_\mu\Big] - \frac{1}{4}\widehat{F}^2_{\mu\nu} \\ & + e^5_{\dot{5}}\,\delta(x^5)\Big[|\partial_\mu\phi|^2 + i\bar{\chi}\not{D}\chi - \frac{1}{4}G^2_{\mu\nu} + i\bar{\lambda}\not{D}\lambda\Big] \\ & + \frac{1}{\sqrt{3}}\Big(\partial_{\dot{5}}A_\mu + \frac{1}{\sqrt{3}}J_{\mu\dot{5}}(x^5)\Big)V^\mu + \cdots \end{aligned}$$

The Kähler kinetic function is defined as

$$\Omega_{\dot{5}}(x^5) = -\frac{3}{2} + e_{\dot{5}}^{5}\,\delta(x^5)|\phi|^2$$

The current which couples to V_μ is the sum of

$$J^{\Phi}_{\mu\dot{5}}(x^5) \;=\; e_{\dot{5}}^{5}\,\delta(x^5)\Big[i(\phi^*\partial_\mu\phi - \text{c.c.}) - \frac{i}{2}\bar{\chi}\gamma_\mu\gamma^{\dot{5}}\chi + \cdots\Big]$$

$$J^{V}_{\mu\dot{5}}(x^5) \;=\; e_{\dot{5}}^{5}\,\delta(x^5)\Big[-\frac{3i}{2}\bar{\psi}\gamma_\mu\gamma^{\dot{5}}\psi + \cdots\Big]$$

$$J^{T}_{\mu\dot{5}}(x^5) \;=\; -\sqrt{3}\,e_{\dot{5}}^{5}\,\partial_\mu A_5 + \cdots$$

Note that:

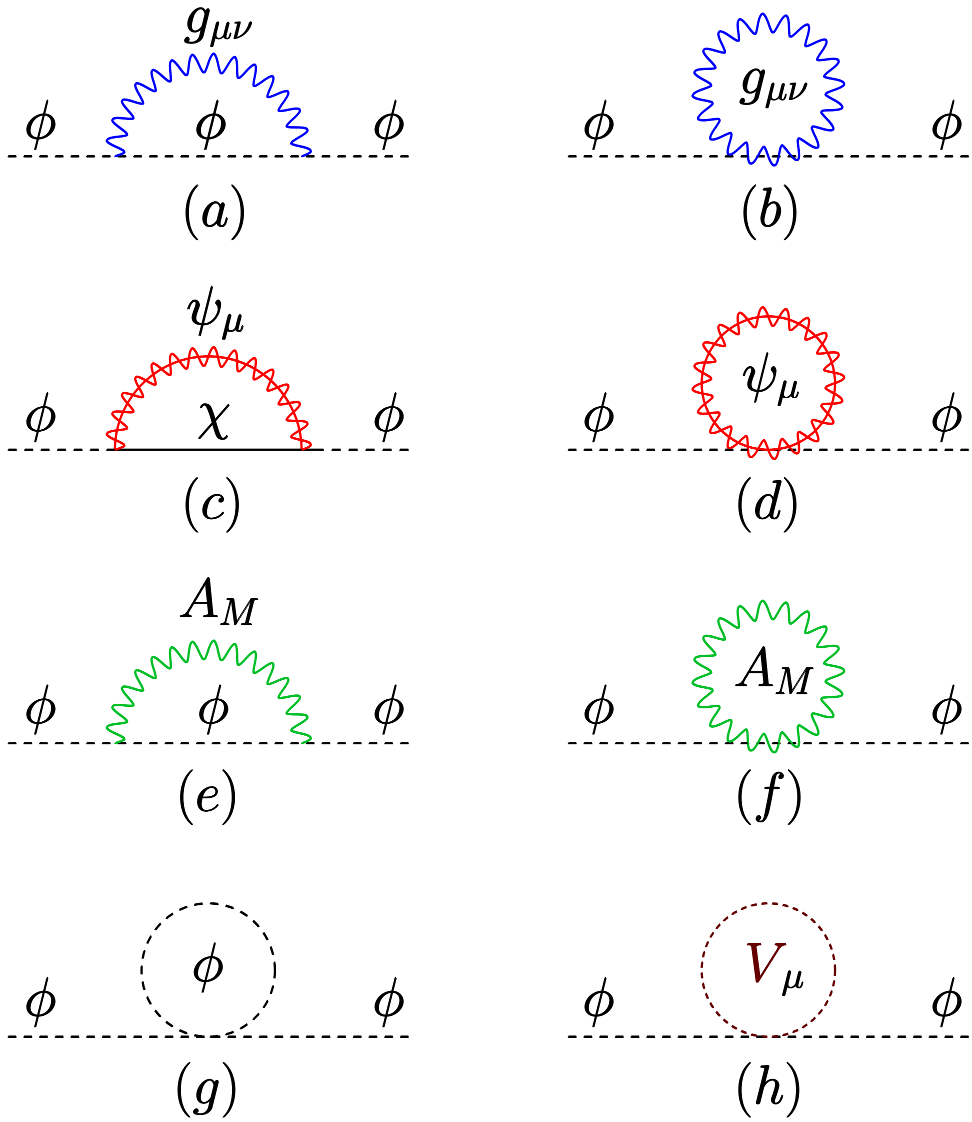
$$\partial_{\dot{5}}A_\mu + \frac{1}{\sqrt{3}}J^{T}_{\mu\dot{5}}(x^5) = -\,e_{\dot{5}}^{5}\widehat{F}_{\mu5}$$

Redefining V_μ through a shift to complete the squares, one finds finally:

$$\begin{aligned}\mathcal{L} \;=\; & \frac{1}{6}\Omega_{\dot{5}}(x^5)\Big[\mathcal{R} + 2i\bar{\psi}_M\gamma^{MNP}D_N\psi_P + \frac{2}{3}\tilde{V}_\mu^2\Big] - \frac{1}{4}\widehat{F}^2_{\mu\nu} \\ & + e_{\dot{5}}^{5}\,\delta(x^5)\Big[|\partial_\mu\phi|^2 + i\bar{\chi}\not{D}\chi - \frac{1}{4}G^2_{\mu\nu} + i\bar{\lambda}\not{D}\lambda\Big] \\ & - \frac{3}{4\,\Omega_{\dot{5}}(x^5)}\Big(\partial_{\dot{5}}A_\mu + \frac{1}{\sqrt{3}}J_{\mu\dot{5}}(x^5)\Big)^2 + \cdots\end{aligned}$$

Loop corrections

Consider as before the 1-loop correction to the mass of ϕ, which must vanish by SUSY non-renormalization theorem.



The result is:

$$\Delta m^2 = \frac{i}{6\pi R} \sum_\alpha \sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \frac{N_{\alpha,n}}{p^2 - m_n^2}$$

with

$$N_{a,n} = 0 \qquad N_{b,n} = 5\,p^2$$
$$N_{c,n} = 0 \qquad N_{d,n} = -8\,p^2$$
$$N_{e,n} = p^2 - m_n^2 \qquad N_{f,n} = -p^2 + 4\,m_n^2$$
$$N_{g,n} = -p^2 + m_n^2 \qquad N_{h,n} = 4\,p^2 - 4\,m_n^2$$

Low-energy theory

The low energy theory for $E \ll M_{\rm C}$ is obtained by integrating out A_μ. Neglecting $\partial_\mu \sim E$ with respect to $\partial_5 \sim M_{\rm C}$, its equation of motion is:

$$\partial_{\dot 5}\Big[\frac{1}{\Omega_{\dot 5}(x^5)}\Big(\partial_{\dot 5}A_\mu + \frac{1}{\sqrt{3}}J_{\mu\dot 5}(x^5)\Big)\Big] = 0$$

The solution is

$$\partial_{\dot 5}A_\mu = -\frac{1}{\sqrt{3}}\Big(J_{\mu\dot 5}(x^5) - \frac{\Omega_{\dot 5}(x^5)}{\Omega}J_\mu\Big)$$

with

$$\Omega = \int_0^{2\pi} dx^5 e_5^{\dot 5}\,\Omega_{\dot 5}(x^5) = -\frac{3}{2}(T+T^*) + |\phi|^2$$

and

$$J_\mu^\Phi = \int_0^{2\pi} dx^5 e_5^{\dot 5} J^\Phi_{\mu\dot 5}(x^5) = i(\Omega_\phi\partial_\mu\phi - \text{c.c.}) - \frac{i}{2}\Omega_{\phi\phi^*}\bar\chi\gamma_\mu\gamma^{\dot 5}\chi + \cdots$$

$$J_\mu^V = \int_0^{2\pi} dx^5 e_5^{\dot 5} J^V_{\mu\dot 5}(x^5) = -\frac{3i}{2}\bar\lambda\gamma_\mu\gamma^{\dot 5}\lambda + \cdots$$

$$J_\mu^T = \int_0^{2\pi} dx^5 e_5^{\dot 5} J^T_{\mu\dot 5}(x^5) = i(\Omega_T\partial_\mu T - \text{c.c.}) + \cdots$$

Substituting back in the Lagrangian and integrating over x^5, one finds:

$$\begin{aligned}\mathcal{L}^{\rm eff} &= \frac{1}{6}\Omega\Big[\mathcal{R} + 2i\bar\psi^1_\mu\gamma^{\mu\nu\rho}D_\nu\psi^1_\rho\Big] - \frac{1}{4\,\Omega}J_\mu^2 \\ &\quad + \Omega_{\phi\phi^*}\Big[|\partial_\mu\phi|^2 + \bar\chi\not{D}\chi\Big] + \Big[-\frac{1}{4}G_{\mu\nu}^2 + i\bar\lambda\not{D}\lambda\Big] + \cdots\end{aligned}$$

LOOP EFFECTS IN SEQUESTERED MODELS

A generic sequestered model is defined by:

$$\Omega_{\dot{5}}(x^5) = -\frac{3}{2}M_5^3 + \Omega_0\, e_{\dot{5}}^5\, \delta(x^5 - 0) + \Omega_\pi\, e_{\dot{5}}^5\, \delta(x^5 - \pi)$$

We take:

$$\Omega_{0,\pi} = -3L_{0,\pi}M_5^3 + \Phi_{0,\pi}\Phi_{0,\pi}^\dagger$$

The kinetic function of the effective theory is then

$$\Omega(\Omega_{0,\pi}, T + T^\dagger) = -\frac{3}{2}(T + T^\dagger)M_5^3 + \Omega_0 + \Omega_\pi$$

and

$$M_{\mathrm{P}}^2 = \Big(\mathrm{Re}\,T + L_0 + L_\pi\Big)M_5^3$$

The 1-loop correction to this has a divergent T-indep. (local) plus a finite T-dep. (non-local) parts. The relevant part is:

$$\Delta\Omega(\Omega_{0,\pi}, T + T^\dagger) = \sum_{m,n=0}^{\infty} \frac{c_{m,n}\,\Omega_0^m\,\Omega_\pi^n}{M_5^{3(m+n)}(T + T^\dagger)^{2+m+n}}$$

The corresponding component effective action is $\Delta\Gamma = \Big[\Delta\Omega\Big]_D$.
In particular, when $F_\pi \neq 0$ and/or $F_T \neq 0$:

Vac. energy: $c_{m,n}L_0^m L_\pi^n |F_T|^2$, $c_{m,n}L_0^m L_\pi^{n-1}|F_\pi|^2$

Soft masses: $c_{m,n}L_0^{m-1} L_\pi^n |F_T|^2$, $c_{m,n}L_0^{m-1} L_\pi^{n-1}|F_\pi|^2$

To derive the $c_{m,n}$s, one chooses one operator for each superspace term in $\Delta\Omega$, and computes its induced coefficient.

Strategy

Crucial trick: use non-SUSY background with $F_T = 2\pi\epsilon \neq 0$. This corresponds to a $SU(2)_R$ Wilson line, and can be achieved in two ways: SS twist or constant boundary superpotentials. Only the gravitino KK modes are affected: $m_n = (n+\epsilon)/R$.

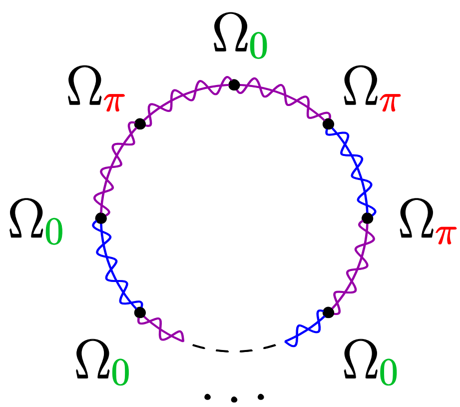
von Gersdorff, Quiros, Riotto;

Bagger, Feruglio, Zwirner

This leads to huge simplifications:

- One can use operators with scalars and no derivatives
  $\Rightarrow$ Few diagrams, mostly with quartic couplings.
- The amplitudes must vanish in the SUSY limit $\epsilon \to 0$
  $\Rightarrow$ All the information is in the gravitino diagrams.

In the end, there is a single type of diagram for each $c_{m,n}$:



The set of operator that we want to compute is given by the effective potential $\Delta V = -\partial_T \partial_{T^*} \Delta\Omega |F_T|^2$, function of R and

$$\alpha_{0,\pi} = \frac{\Omega_{0,\pi}}{6\pi R M_5^3} = -\frac{L_{0,\pi}}{2\pi R} + \frac{|\phi_{0,\pi}|^2}{6\pi R M_5^3} = -r_{0,\pi} + |\varphi_{0,\pi}|^2$$

The precise expression of the operators to be matched is:

$$\Delta V(\alpha_{0,\pi}, R, \epsilon) = \frac{-\epsilon^2}{4\pi^2 R^4} \sum_{m,n=0}^{\infty} c_{m,n}(2+m+n)(3+m+n)\alpha_0^m \alpha_\pi^n$$

Computation

The gravitino contribution to the full effective potential is:

$$\Delta W_\psi(\alpha_{0,\pi}, R, \epsilon) = -\frac{1}{2} \ln\det\left[\Box_5 + \left(\alpha_0\,\delta_0 + \alpha_\pi\,\delta_\pi\right)\Box_4\right]$$

The $\alpha_{0,\pi}$-independent part is

$$\ln\det\left[\Box_5\right] = 8\,\mathrm{Re}\int \frac{d^4p}{(2\pi)^4} \ln\left[F(pR,\epsilon)\right]$$

with

$$F(pR,\epsilon) = \prod_{n=-\infty}^{+\infty} \left(p + i\, m_n\right) = (\text{Div.}) \sinh \pi(pR + i\epsilon)$$

The $\alpha_{0,\pi}$-dependent part is

$$\ln\det\left[1 + \left(\alpha_0\,\delta_0 + \alpha_\pi\,\delta_\pi\right)\frac{\Box_4}{\Box_5}\right]$$

$$= 8\,\mathrm{Re}\int \frac{d^4p}{(2\pi)^4} \ln\begin{vmatrix} 1 - p\alpha_0 G_0(pR,\epsilon) & -p\alpha_\pi G_\pi(pR,\epsilon) \\ -p\alpha_0 G_\pi(pR,\epsilon) & 1 - p\alpha_\pi G_0(pR,\epsilon) \end{vmatrix}$$

with

$$G_0(pR,\epsilon) = \frac{1}{2\pi R} \sum_{n=-\infty}^{+\infty} \frac{e^{i0n}}{p + i\, m_n} = \frac{1}{2} \coth \pi(pR + i\epsilon)$$

$$G_\pi(pR,\epsilon) = \frac{1}{2\pi R} \sum_{n=-\infty}^{+\infty} \frac{e^{i\pi n}}{p + i\, m_n} = \frac{1}{2} \operatorname{csch} \pi(pR + i\epsilon)$$

Putting these two pieces together and simplifying one finds:

$$\Delta W_\psi(\alpha_{0,\pi}, R, \epsilon)$$

$$= \text{Div.} - \frac{1}{2\pi^6 R^4} \text{Re} \int_0^\infty dl\, l^3 \ln \left| 1 - \frac{1+\alpha_0 l}{1-\alpha_0 l} \frac{1+\alpha_\pi l}{1-\alpha_\pi l} e^{-2(l+i\pi\epsilon)} \right|$$

The $\mathcal{O}(\epsilon^0)$ part cancels the contributions of other bulk fields. The $\mathcal{O}(\epsilon^2)$ part yields the relevant potential ΔV that we need. The $\mathcal{O}(\epsilon^{2n})$ terms map to D-terms with superderivatives.

Expanding $\Delta W_\psi|_{\epsilon^2}$ in powers of $\alpha_{0,\pi}$ and comparing with the general expression for ΔV, one extracts the coefficients $c_{m,n}$. The first few ones are:

$$c_{0,0} = \frac{\zeta(3)}{4\pi^2}\,, \quad c_{1,0} = c_{0,1} = \frac{\zeta(3)}{6\pi^2}\,, \quad c_{1,1} = \frac{\zeta(3)}{6\pi^2}\,, \quad \ldots$$

An independent and direct computation exploiting supergraph techniques leads to the same results.

Buchbinder et al.

Since we know the exact expression $\Delta W_\psi|_{\epsilon^2}$ for ΔV, we can do better and find the exact expression for $\Delta\Omega$ by solving the differential equation $\Delta V = -\epsilon^2\, \partial_R^2\, \Delta\Omega$. The result is:

$$\Delta\Omega(\Omega_{0,\pi}, T+T^\dagger)$$

$$= -\frac{9}{\pi^2} M_5^2 \int_0^\infty dx\, x \ln \left[1 - \frac{1+\frac{\Omega_0}{M_5^2}x}{1-\frac{\Omega_0}{M_5^2}x} \frac{1+\frac{\Omega_\pi}{M_5^2}x}{1-\frac{\Omega_\pi}{M_5^2}x} e^{-6(T+T^\dagger)M_5 x} \right]$$

This shows in particular that all the $c_{m,n}$s are positive.

Results

The results for the vacuum energy and soft masses are:

$$\delta\mathcal{E}^4 = -\frac{\zeta(3)}{16\pi^2}\left[\frac{1}{3}f_\pi\frac{|F_\pi|^2}{M_4^2}M_C^2+\frac{3}{2}f_T|F_T|^2M_C^4\right]$$

$$\delta m_0^2 = -\frac{\zeta(3)M_C^2}{16\pi^2M_4^2}\left[\frac{1}{6}g_\pi\frac{|F_\pi|^2}{M_4^2}+g_T|F_T|^2M_C^2\right]$$

These depend on the parameters $r_{0,\pi}$ through

$$M_4^2=\frac{1}{1+r_0+r_\pi}M_P^2$$

and the normalized functions

$$f_\pi=\frac{4}{3\zeta(3)}\int_0^\infty dl\, l^2e^{-2l}\frac{(1-r_0l)/(1+r_\pi l)}{[(1+r_0l)(1+r_\pi l)-(1-r_0l)(1-r_\pi l)e^{-2l}]}$$

$$f_T=\frac{2}{3\zeta(3)}\int_0^\infty dl\, l^3e^{-2l}\frac{(1-r_0^2l^2)(1-r_\pi^2l^2)}{[(1+r_0l)(1+r_\pi l)-(1-r_0l)(1-r_\pi l)e^{-2l}]^2}$$

$$g_\pi=\frac{8}{3\zeta(3)}\int_0^\infty dl\, l^3e^{-2l}\frac{1}{[(1+r_0l)(1+r_\pi l)-(1-r_0l)(1-r_\pi l)e^{-2l}]^2}$$

$$g_T=\frac{4}{3\zeta(3)}\int_0^\infty dl\, l^4e^{-2l}\frac{(1-r_\pi^2l^2)[(1+r_0l)(1+r_\pi l)+(1-r_0l)(1-r_\pi l)e^{-2l}]}{[(1+r_0l)(1+r_\pi l)-(1-r_0l)(1-r_\pi l)e^{-2l}]^3}$$

For $r_{0,\pi}=0$, $\delta\mathcal{E}^4$ and δm_0^2 are negative $\Rightarrow$ not interesting.

For $r_{0,\pi}\neq 0$, $\delta\mathcal{E}^4$ and δm_0^2 can have any sign $\Rightarrow$ interesting.

Three main cases for the dependence on R at fixed $L_{0,\pi}$:

- $L_0=0$, $L_\pi=0$: $\delta\mathcal{E}^4$ unstable, $\delta m_0^2\sim-(\delta\mathcal{E}^4)'$.
- $L_0=0$, $L_\pi\neq 0$: $\delta\mathcal{E}^4$ stable, $\delta m_0^2\sim-(\delta\mathcal{E}^4)'$.
- $L_0\neq 0$, $L_\pi\neq 0$: $\delta\mathcal{E}^4$ metastable, $\delta m_0^2\not\sim-(\delta\mathcal{E}^4)'$.

PROTOTYPE MODEL

The goal is to achieve values of T, F_T, F_π, F_S such that:

- $\mathcal{E}^4 \sim 0 \Rightarrow$ tuning of P.
- $\delta^{\rm grav} m_0^2 > 0 \Rightarrow$ needs $r_\pi \neq 0$.
- $\delta^{\rm grav} m_0^2 \sim \delta^{\rm gau} m_0^2 \Rightarrow$ indep. stab. mech.

One can try to combine localized kinetic terms with gaugino condensation, with:

$$\Omega = -\frac{3}{2}(T+T^\dagger)M_5^3 + \Phi_0\Phi_0^\dagger - 3L_\pi M_5^3 + \Phi_\pi\Phi_\pi^\dagger$$
$$P = \Lambda_\pi^3 + M_\pi^2\Phi_\pi + \Lambda^3 e^{-\alpha\Lambda T}$$

To have $\mathcal{E}^4 \sim 0$ we need $\Lambda_\pi^3 \sim M_\pi^2 M_{\rm P}$. We then get:

$$M_{\rm C} \sim \alpha\Lambda \,, \quad F_T \sim \frac{M_\pi^2}{\Lambda M_{\rm P}} \,, \quad F_S \sim \frac{M_\pi^2}{M_{\rm P}} \,, \quad F_\pi \sim M_\pi^2$$

To have $r_\pi \gg 1$ we need $L_\pi \gg (\alpha\Lambda)^{-1}$. In this limit:

$$f_\pi, g_\pi \to \frac{2\ln(2)}{3\zeta(3)}\frac{1}{r_\pi^2} \,, \quad f_T, g_T \to -\frac{3}{4}$$

$\delta^{\rm grav} m_0^2$ becomes positive for $r_\pi \sim \alpha^{-1}$; OK with $\alpha \ll 1$.
$\delta^{\rm grav} m_0^2$ is of the same order of magnitude as $\delta^{\rm gau} m_0^2$ if:

$$\alpha^2 \frac{M_{\rm C}^2}{16\pi^2 M_4^2} \sim \left(\frac{g^2}{16\pi^2}\right)^2 \Rightarrow \frac{M_{\rm C}}{M_{\rm P}} \sim \frac{g^2}{4\pi\sqrt{\alpha}}$$

CONCLUSIONS

- Bulk-to-boundary couplings now well understood and loop corrections under controll.
- Radius-dependent quantum corrections to sfermion squared masses generally negative, but can become positive with sizable localized kinetic terms.
- Sequestered models can work, but radion dynamics plays a crucial rôle.