CONSTRAINTS FROM SUSY BREAKING IN SUPERGRAVITY THEORIES

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- SUSY breaking in SUGRA scenarios.
- SUGRA models with chiral multiplets.
- Flatness and stability constraints.
- Factorizable scalar manifolds.
- Symmetric scalar manifolds.
- Moduli in string models.

SUSY BREAKING AND SUGRA

In a renormalizable theory with rigid SUSY, spontaneous SUSY breaking implies a sum rule on the mass spectrum:

$$\mathbf{STr}\, M^2 = \sum_J (-1)^{2J}(2J+1)\, m_J^2 = 0$$

This predicts that some superparticle is lighter than its ordinary partner particle, in contradiction with experimental observation.

The standard paradigm to evade this difficulty is to assume that SUSY breaking occurs spontaneously in a hidden sector with fields Φ_i and is transmitted to the visible sector with fields Q_a only indirectly, through some suppressed interactions.

The effect of SUSY breaking on the visible sector can be parametrized through super-renormalizable soft breaking terms, which depend both on the details of the hidden sector theory and on the mediation mechanism.

The effective Lagrangian that is relevant for phenomenology has then the general form of a supersymmetric Lagrangian plus a set of soft breaking terms:

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{susy}} + \mathcal{L}_{\mathrm{soft}}$$

A natural mediation mechanism is provided by gravitational interactions, which have a scale M_P. The general setup then becomes that of SUGRA, with local SUSY.

SUSY breaking occurs spontaneously at some scale $M \ll M_{\mathrm{P}}$ in the hidden sector and is transmitted to the visible sector through gravitational interactions.

The microscopic theory might be some kind of superstring model. But below M_{P}, and in particular at M, this can be effectively described by a non-renormalizable SUGRA theory.

The soft terms originate from the higher-dimensional operators that mix visible fields Q_a to hidden fields Φ_i and are suppressed by powers of M_{P}, and their scale is

$$m_{\mathrm{soft}} \sim \frac{M^2}{M_{\mathrm{P}}}$$

Chamseddine, Arnowitt, Nath
Barbieri, Ferrara, Savoy
Hall, Lykken, Weinberg

The main delicate features that are needed in order to get a satisfactory situation are:

- Soft terms with $m_{\mathrm{soft}} \sim M_{\mathrm{EW}}$ and peculiarities.
- Cosmological constant with $M_{\mathrm{CC}} \ll M_{\mathrm{EW}}$.
- Hidden scalars with $m > M_{\mathrm{EW}}$ and stable.

CHIRAL SUGRA MODELS

A SUGRA theory with N chiral multiplets Φ_i is specified by a real function G. Setting $M_{\rm P} = 1$, this can be written as

$$G(\Phi_i, \Phi_i^\dagger) = K(\Phi_i, \Phi_i^\dagger) + \log W(\Phi_i) + \log \bar{W}(\Phi_i^\dagger)$$

This decomposition is however ambiguous, due to the Kähler symmetry changing $K \to K + F + \bar{F}$ and $W \to e^{-F}\, W$.

Mixed holomorphic/antiholomorphic derivatives of G depend only on K and define a Kähler geometry for the manifold parametrized by the scalars ϕ^i. The metric, Chirstoffel connection and Riemann tensor are:

$$g_{i\bar{j}} = G_{i\bar{j}}$$
$$\Gamma^k_{ij} = G^k_{ij} \;, \quad \Gamma^{\bar{k}}_{\bar{i}\bar{j}} = G^{\bar{k}}_{\bar{i}\bar{j}}$$
$$R_{i\bar{j}p\bar{q}} = G_{i\bar{j}p\bar{q}} - G^r_{ip}\, G_{\bar{j}\bar{q}r}$$

Pure holomorphic or antiholomorphic derivatives of G depend instead also on W, and determine the way SUSY is broken. In particular, the auxiliary fields F^i are given simply by:

$$F^i = e^{G/2}\, G^i$$

Cremmer, Julia, Scherk, Ferrara, Girardello, Van Nieuwenhuizen
Bagger, Witten

The scalars ϕ^i have a wave function factor given by $Z_{i\bar{\jmath}} = g_{i\bar{\jmath}}$ and a potential, which determines their vev and mass and controls spontaneous SUSY breaking, of the form:

$$V = e^G \Big(G^k G_k - 3 \Big)$$

The flatness condition of vanishing cosmological constant is that $V = 0$ on the vacuum and implies that at that point:

$$g_{i\bar{\jmath}}\, G^i G^{\bar{\jmath}} = 3$$

The first derivatives of the potential controlling its variations are computed as $\delta_i = \nabla_i V$ and are given by:

$$\delta_i = e^G \Big(G_i + G^k \nabla_i G_k \Big)$$

The stationarity conditions defining extrema of the potential are $\delta_i = 0$ and imply:

$$G_i + G^k \nabla_i G_k = 0$$

The second derivatives of the potential controlling the squared masses can be computed as $m^2_{i\bar{\jmath}} = \nabla_i \nabla_{\bar{\jmath}} V$ and $m^2_{ij} = \nabla_i \nabla_j V$, and one easily finds:

$$m^2_{i\bar{\jmath}} = e^G \Big(g_{i\bar{\jmath}} + \nabla_i G_k \nabla_{\bar{\jmath}} G^k - R_{i\bar{\jmath}p\bar{q}}\, G^p G^{\bar{q}} \Big)$$

$$m^2_{ij} = e^G \Big(\nabla_i G_j + \nabla_j G_i + G^k \nabla_i \nabla_j G_k \Big)$$

The stability condition is that the $2N$-dimensional squared-mass matrix is positive definite:

$$m_0^2 = \begin{pmatrix} m^2_{i\bar{j}} & m^2_{ij} \\ m^2_{\bar{i}\bar{j}} & m^2_{\bar{i}j} \end{pmatrix} > 0$$

The only systematic way to determine the constraints that this implies is to study the mass eigenvalues.

The fermions ψ^i split into 1 Goldstino combination $\psi = G_i \psi^i$ and $N-1$ physical combinations $\tilde{\psi}^i$. They have wave-function factor $\tilde{Z}_{i\bar{j}} = g_{i\bar{j}}$, and their mass is encoded in:

$$\tilde{m}_{ij} = e^G \Big(\nabla_i G_j + \frac{1}{3} G_i G_j \Big)$$

More precisely, the $2N$-dimensional mass matrix is given by

$$m_{1/2} = \begin{pmatrix} 0 & \tilde{m}_{ij} \\ \tilde{m}_{\bar{i}\bar{j}} & 0 \end{pmatrix}$$

The graviton and gravitino $h^{\mu\nu}$ and ψ^{μ} have wave-function factors given by $Z_2 = 1$ and $Z_{3/2} = 1$, and their masses are:

$$m_2^2 = 0 \ , \quad m_{3/2} = e^{G/2}$$

The supertrace of the squared mass matrix for the whole theory is found to be:

$$\mathrm{STr} M^2 = 2\, e^G \Big(N - 1 - R_{i\bar{\jmath}}\, G^i G^{\bar{\jmath}} \Big)$$

Cremmer, Ferrara, Girardello, Van Proeyen

FLATNESS AND STABILITY CONSTRAINTS

It would be interesting to understand better what flatness and stability imply on G. More precisely, it would be really very helpful to find some condition concerning only K and the geometry, independently of W and the mechanism of SUSY breaking.

Our strategy is to impose the flatness condition $V = 0$ and look for some simpler condition that is only necessary and in general not sufficient for having stability with $m_0^2 > 0$.

The crucial point is that all the upper-left submatrices of m_0^2 must also be positive definite. In particular, the N-dimensional submatrix $m_{i\bar{\jmath}}^2$ should be positive definite:

$$m_{i\bar{\jmath}}^2 > 0$$

This condition means that $\forall z^i$ one must have $m^2_{i\bar{\jmath}}\, z^i \bar{z}^{\bar{\jmath}} > 0$. One can then look for a specific z^i that leads to a particularly simple condition. The right choice is $z^i = G^i$, for which:

$$m^2_{i\bar{\jmath}}\, G^i G^{\bar{\jmath}} = e^G \Big(6 - R_{i\bar{\jmath}p\bar{q}}\, G^i G^{\bar{\jmath}} G^p G^{\bar{q}}\Big)$$

The corresponding necessary condition $m^2_{i\bar{\jmath}}\, G^i G^{\bar{\jmath}} > 0$ reduces then to the extremely simple curvature constraint:

$$R_{i\bar{\jmath}p\bar{q}}\, G^i G^{\bar{\jmath}} G^p G^{\bar{q}} < 6$$

Note that the special direction $z^i = G^i$ considered to get the condition $m^2_{i\bar{\jmath}}\, G^i G^{\bar{\jmath}} > 0$ for the scalars corresponds to the Goldstino direction for the fermions, and $\tilde{m}_{ij}\, G^i G^j = 0$.

Summarizing, a stationary point can lead to a satisfactory situation only if the following two conditions are satisfied:

Flatness: $g_{i\bar{\jmath}}\, G^i G^{\bar{\jmath}} = 3$ (necessary & sufficient)

Stability: $R_{i\bar{\jmath}p\bar{q}}\, G^i G^{\bar{\jmath}} G^p G^{\bar{q}} < 6$ (necessary)

The tensors $g_{i\bar{\jmath}}$ and $R_{i\bar{\jmath}p\bar{q}}$ depend exclusively on K and characterize the geometry. The vectors G^i depend also on W and control the SUSY breaking direction, since $G^i = F^i/m_{3/2}$.

For a given geometry, the flatness condition fixes the overall amount of SUSY breaking, and the stability condition constrains its direction to lie with a certain cone.

To solve these two conditions, one must first determine the direction that minimizes $R_{i\bar{\jmath}p\bar{q}}\, G^i G^{\bar{\jmath}} G^p G^{\bar{q}}$ for fixed $g_{i\bar{\jmath}}\, G^i G^{\bar{\jmath}}$, and then check how far apart from it the former stays small enough.

This variational problem is hard to solve in full generality. However, it is possible to obtain very simple and strong results for the subclass of models based on spaces that are factorizable or symmetric.

Notice finally that the conditions refer to a particular stationary point. It is then useful to switch to normal coordinates around that point, defined in the standard way with a holomorphic vielbein e_i^J and its inverse e_J^i.

In these special coordinates with flat indices, the metric at the stationary point is trivial, and the flatness and stability conditions defining the problem can then be rewritten simply as:

Flatness: $\delta_{I\bar{J}}\, G^I G^{\bar{J}} = 3$

Stability: $R_{I\bar{J}P\bar{Q}}\, G^I G^{\bar{J}} G^P G^{\bar{Q}} < 6$

Gomez-Reino, Scrucca

FACTORIZABLE SPACES

Suppose that $\mathcal{M}$ is a product of N 1-dimensional manifolds. The function K splits then into a sum of terms depending on a single field, while W can instead still be arbitrary:

$$K = \sum_k K^{(k)}(\Phi_k, \Phi_k^\dagger)$$
$$W = W(\Phi_1, \dots, \Phi_n)$$

This assumption represents a Kähler-invariant constraint on G: its mixed holomorphic/antiholomorphic off-diagonal derivatives vanish.

In this situation, $g_{i\bar{j}}$ and $R_{i\bar{j}p\bar{q}}$ are diagonal and have only N non-zero components, given by $g_{i\bar{\imath}} = G_{i\bar{\imath}}$ and:

$$R_{i\bar{\imath}i\bar{\imath}} = R_i \, g_{i\bar{\imath}}^2$$

This simplifies enough to problem to solve it exactly.

The crucial parameters are in this case the N curvature scalars:

$$R_i = \frac{G_{ii\bar{i}\bar{i}}}{G_{i\bar{i}}^2} - \frac{G_{ii\bar{i}} G_{\bar{i}\bar{i}i}}{G_{i\bar{i}}^3}$$

In flat coordinates, the Riemann tensor has the form:

$$R_{I\bar{J}P\bar{Q}} = \begin{cases} R_i \ , & \text{if } I = J = P = Q \\ 0 \ , & \text{otherwise} \end{cases}$$

The two flatness and stability conditions derived before then simplify to the following expressions:

Flatness: $\sum_k \Theta_k^2 = 1$

Stability: $\sum_k R_k \, \Theta_k^4 < \frac{2}{3}$

in terms of the N real and positive variables

$$\Theta_i = \frac{1}{\sqrt{3}} |G^I|$$

It is now easy to show that for $R_i > 0$ these constraints admit solutions only if the following curvature bound is satisfied:

$$\sum_k R_k^{-1} > \frac{3}{2}$$

The SUSY breaking direction must lie in a certain Goldstino cone fixed by the R_i's. Its axis is the direction minimizing the quartic curvature form:

$$\Theta_i^0 = \sqrt{\frac{R_i^{-1}}{\sum_k R_k^{-1}}}$$

Its solid angle grows with the excess of the effective inverse curvature $\sum_k R_k^{-1}$ with respect to the threshold $3/2$.

More precisely, the allowed configurations correspond to a certain limited domain in the space of variables:

$$\Theta_i \in \left[\Theta_i^-, \Theta_i^+\right]$$

One easily finds:

$$\Theta_i^+ = \begin{cases} \sqrt{\dfrac{R_i^{-1} + \sqrt{\frac{2}{3} R_i^{-1} \left(\sum_{k\neq i} R_k^{-1}\right)\left(\sum_k R_k^{-1} - \frac{3}{2}\right)}}{\sum_k R_k^{-1}}}, & R_i^{-1} < \dfrac{3}{2} \\ 1\,, & R_i^{-1} > \dfrac{3}{2} \end{cases}$$

$$\Theta_i^- = \begin{cases} \sqrt{\dfrac{R_i^{-1} - \sqrt{\frac{2}{3} R_i^{-1} \left(\sum_{k\neq i} R_k^{-1}\right)\left(\sum_k R_k^{-1} - \frac{3}{2}\right)}}{\sum_k R_k^{-1}}}, & \sum_{k\neq i} R_k^{-1} < \dfrac{3}{2} \\ 0\,, & \sum_{k\neq i} R_k^{-1} > \dfrac{3}{2} \end{cases}$$

A given Θ_i can become as large as 1 only if the related curvature satisfies $R_i^{-1} > 3/2$, and as low as 0 only if the curvatures of the remaining fields satisfy $\sum_{k \neq i} R_k^{-1} > 3/2$.

The relevance of each chiral multiplet Φ_i for SUSY breaking depends thus on the size of the corresponding inverse curvature R_i^{-1} with respect to the threshold value $3/2$.

Gomez-Reino, Scrucca

SYMMETRIC SPACES

Suppose that $\mathcal{M}$ is a coset space G/H, where G is a group of global isometries and H a local stability group. The function K has then some special form, but W can be arbitrary:

$$K = K^{(G/H)}(\Phi_1, \Phi_1^\dagger, \ldots, \Phi_n, \Phi_n^\dagger)$$

$$W = W(\Phi_1, \ldots, \Phi_n)$$

The metric and curvature tensors are G-invariant and there are relations among their components. The problem simplifies then again sufficiently much to be able to solve it exactly.

For all the possible coset Kähler manifolds, the components of the metric and the Rieman tensor are somehow related:

$$R_{i\bar{j}p\bar{q}} \text{ related to } g_{r\bar{s}}$$

Calabi, Vesentini

The crucial ingredients are in this case the overall curvature scale $R_{\rm all}$ and the group structure of the space.

In flat coordinates, the Riemann tensor has in these cases a particularly simple structure of the form:

$$R_{I\bar{J}P\bar{Q}} = R_{\rm all}\Big(G\text{-invariant combination of } H\text{-invariant } \delta\text{'s}\Big)$$

Generalized spheres

Suppose that there are $N = 1 + q$ fields Φ_i and

$$K = -\frac{2}{R_{\rm all}}\ln\Big(1 - \sum_i \Phi_i \Phi_i^\dagger\Big)$$

The corresponding scalar manifold is the Kählerian analogue of the usual Riemannian sphere:

$$\mathcal{M} = \frac{SU(1,1+q)}{U(1)\times SU(1+q)} \subset \frac{SU(1,1)}{U(1)}$$

The Riemann tensor in normal coordinates takes in this case the very simple form

$$R_{I\bar{J}P\bar{Q}} = \frac{R_{\rm all}}{2}\Big(\delta_{I\bar{J}}\,\delta_{P\bar{Q}} + \delta_{I\bar{Q}}\,\delta_{P\bar{J}}\Big)$$

The two flatness and stability conditions can then be rewritten in thye simple form:

Flatness: $\Theta^2 = 1$

Stability: $R_{\rm all}\,\Theta^4 < \frac{2}{3}$

in terms of just 1 real and positive variable

$$\Theta = \frac{1}{\sqrt{3}}\sqrt{\sum_k |G^K|^2}$$

The situation is then as for 1 field with $R = R_{\rm all}$:

$$R_{\rm all}^{-1} > \frac{3}{2}$$

Unitary Grassmannians

Suppose that there are $N = p\,(p+q)$ fields Φ_{ia} and

$$K = -\frac{2}{R_{\mathrm{all}}}\ln\det\Big(\delta_{i\bar{\jmath}} - \sum_a \Phi_{ia}\Phi^{\dagger}_{ja}\Big)$$

The corresponding scalar manifold is the following unitary Grassmannian manifold:

$$\mathcal{M} = \frac{SU(p,p+q)}{U(1)\times SU(p)\times SU(p+q)} \subset \left(\frac{SU(1,1)}{U(1)}\right)^p$$

The Riemann tensor in normal coordinates takes in this case the following form

$$R_{IA\,\bar{J}\bar{B}\,PC\,\bar{Q}\bar{D}} = \frac{R_{\mathrm{all}}}{2}\Big(\delta_{I\bar{J}}\,\delta_{P\bar{Q}}\,\delta_{A\bar{D}}\,\delta_{C\bar{B}} + \delta_{I\bar{Q}}\,\delta_{P\bar{J}}\,\delta_{A\bar{B}}\,\delta_{C\bar{D}}\Big)$$

The two conditions reduce then simply to

$$\text{Flatness: } \sum_k \Theta_k^2 = 1$$

$$\text{Stability: } \sum_k R_{\text{all}} \, \Theta_k^4 < \frac{2}{3}$$

in terms of the p real and positive variables

$$\Theta_i = \frac{1}{\sqrt{3}} \left| \text{Eigenvalue}_i \left(G^{IA} \right) \right|$$

The situation is then as for p fields with $R_i = R_{\text{all}}$:

$$R_{\text{all}}^{-1} > \frac{3}{2p}$$

Orthogonal Grassmannians

Suppose that there are $N = 2 + q$ fields Φ_i and

$$K = -\frac{2}{R_{\mathrm{all}}} \ln \left(1 - 2 \sum_i \Phi_i \Phi_i^\dagger + \sum_{i,j} (\Phi_i \Phi_j^\dagger)^2 \right)$$

The associated scalar manifold is the following orthogonal Grassmannian manifold

$$\mathcal{M} = \frac{SO(2, 2+q)}{SO(2) \times SO(2+q)} \subset \left(\frac{SU(1,1)}{U(1)} \right)^2$$

The Riemann tensor in normal coordinates takes in this case the simple form

$$R_{I\bar{J}P\bar{Q}} = \frac{R_{\mathrm{all}}}{2} \left(\delta_{I\bar{J}}\, \delta_{P\bar{Q}} + \delta_{I\bar{Q}}\, \delta_{P\bar{J}} - \delta_{IP}\, \delta_{\bar{J}\bar{Q}} \right)$$

The two conditions reduce then simply to

Flatness: $\boldsymbol{\Theta_+^2 + \Theta_-^2 = 1}$

Stability: $\boldsymbol{R_{\rm all}\left(\Theta_+^4 + \Theta_-^4\right) < \frac{2}{3}}$

in terms of the $\mathbf{2}$ real and positive variables

$$\boldsymbol{\Theta_\pm = \frac{1}{\sqrt{6}}\sqrt{\sum_k |G^K|^2 \pm \sqrt{\Big(\sum_k |G^K|^2\Big)^2 - \Big|\sum_k (G^K)^2\Big|^2}}}$$

The situation is then as for $\mathbf{2}$ fields with $\boldsymbol{R_i = R_{\rm all}}$:

$$\boldsymbol{R_{\rm all}^{-1} > \frac{3}{4}}$$

Gomez-Reino, Scrucca

MODULI IN STRING MODELS

In string models, a natural candidate for the hidden sector is that of the neutral moduli controlling the coupling strength and the compactification geometry, and the Wilson lines of the hidden gauge groups.

Kaplunovky, Louis

In the simplest models, the scalar manifold of the moduli sector turns out to be symmetric and sometimes also factorizable. This is because this sector emerges as a projection of a SUSY theory in $\mathbf{10}$ dimension.

The scalar manifold is a Kähler submanifold of the space that would occur by compactifying on a T^6. With a hidden gauge group of rank s, this has the form

$$\mathcal{M}_{\text{max}} = \frac{SU(1,1)}{U(1)} \times \frac{SO(6,6+s)}{SO(6) \times SO(6+s)}$$

Narain

The first factor is associated to the dilaton S, and is always present. The second factor is spanned by the Kähler moduli T_p, the complex structure moduli U_q, and the Wilson lines Z_a, and gets in general reduced.

Minimal moduli space

The simplest situation for each modulus Φ_i is that

$$K_i = -n_i \ln\left(\Phi_i + \Phi_i^\dagger\right)$$

Witten

This corresponds to the simplest symmetric space:

$$\mathcal{M}_i = \frac{SU(1,1)}{U(1)}$$

The curvature scalar is:

$$R_i = \frac{2}{n_i}$$

Unitary enhancement by Wilson lines

Certain moduli Φ_i can mix to some number q_i of related Wilson lines X_{a_i}, and these $1 + q_i$ fields have then

$$K_i = -\,n_i \ln\left(\Phi_i + \Phi_i^\dagger - \sum_{a_i} X_{a_i}^\dagger X_{a_i}\right)$$

Ellis, Kounnas, Nanopoulos
Ferrara, Kounnas, Porrati

The corresponding scalar manifold is given by:

$$\mathcal{M}_i = \frac{SU(1,1+q_i)}{U(1)\times SU(1+q_i)}$$

This is a generalized sphere, which behaves as 1 copy of the minimal geometry for the flatness and stability constraints, with curvature scale:

$$R_i = \frac{2}{n_i}$$

Unitary enhancement by extra moduli

A set of p_r moduli with the same n_r can get enhanced to a matrix of p_r^2 moduli $\Phi_{i_r j_r}$. These p_r^2 fields have then

$$K_r = -n_r \ln \det\left(\Phi_{i_r j_r} + \Phi^\dagger_{i_r j_r}\right)$$

Ferrara, Kounnas, Porrati

The corresponding scalar manifold is:

$$\mathcal{M}_r = \frac{SU(p_r, p_r)}{U(1) \times SU(p_r) \times SU(p_r)}$$

This is a unitary Grassmannian space, which behaves as p_r copies of the minimal geometry for the flatness and stability constraints, with overall curvature:

$$R_r = \frac{2}{n_r}$$

Unitary enhancement by Wilson lines and extra moduli

A set of p_r moduli with the same n_r can get enhanced to p_r^2 moduli $\Phi_{i_r j_r}$ and also couple to some number $p_r q_r$ of related Wilson lines $X_{i_r a_r}$. These $p_r(p_r + q_r)$ fields have then

$$K_r = -\,n_r \ln \det\Big(\Phi_{i_r j_r} + \Phi^\dagger_{i_r j_r} - \sum\nolimits_{a_r} X^\dagger_{i_r a_r} X_{j_r a_r}\Big)$$

Ferrara, Kounnas, Porrati

The corresponding scalar manifold is:

$$\mathcal{M}_r = \frac{SU(p_r, p_r + q_r)}{U(1) \times SU(p_r) \times SU(p_r + q_r)}$$

This is again a unitary Grassmannian space, which still behaves as p_r copies of the minimal geometry for the flatness and stability constraints, with overall curvature scale given by:

$$R_r = \frac{2}{n_r}$$

Orthogonal enhancement by matter

A pair of $\mathbf{2}$ moduli Φ_{1_r} and Φ_{2_r} with common n_r can also mix in more peculiar and synchronized way to a number q_r of related Wilson lines X_{a_r}. The $2 + q_r$ fields that are involved are then described by:

$$K_r = -\,n_r \ln\Big(\big(\Phi_{1_r} + \Phi^\dagger_{1_r}\big)\big(\Phi_{2_r} + \Phi^\dagger_{2_r}\big) - \sum\nolimits_{a_r}\big(X_{a_r} + X^\dagger_{a_r}\big)^2\Big)$$

Derendinger, Kounnas, Petropoulos, Zwirner

The corresponding scalar manifold has in this case a different structure and is given by:

$$\mathcal{M}_r = \frac{SO(2,2+q_r)}{SO(2)\times SO(2+q_r)}$$

This is an orthogonal Grassmannian space, which behaves as $\mathbf{2}$ copies of the minimal geometry for the flatness and stability constraints, with an overall curvature given by:

$$R_r = \frac{2}{n_r}$$

Flatness and stability constraints

The structure of the flatness and stability constraints for string moduli spaces is controlled by the minimal factorizable and symmetric geometry, involving at least $\mathbf{2}$ and at most $\mathbf{7}$ factors:

$$\mathcal{M}_{\min} = \frac{SU(1,1)}{U(1)} \times \frac{SU(1,1)}{U(1)} \times \cdots$$

All the enhancements that we have analyzed just reshuffle the relevant combinations of fields, and do not allow to alleviated the constraints for viable SUSY breaking.

The crucial parameters in the constraints are the numerical coefficients n_i characterizing the basic submanifolds and controlling the curvatures associated to the corresponding moduli Φ_i, with:

$$R_i = \frac{2}{n_i}$$

The necessary condition $\sum_k R_k^{-1} > 3/2$ on the curvatures does then imply the following restriction on these coefficients n_i:

$$\sum_k n_k > 3$$

The Goldstino cone is also entirely specified in terms of the n_i's, and puts severe restrictions on the sizes of the auxiliary fields F_i:

$$|F_i| : \begin{cases} \text{upper bound smaller than } \sqrt{3}\, m_{3/2} \text{ if } \; n_i < 3 \\ \text{lower bound larger than } 0 \text{ if } \; \sum_{k \neq i} n_k < 3 \end{cases}$$

Dilaton and volume moduli

The most relevant moduli are the dilaton S, controlling the coupling, and the global volume modulus T, controlling the size of the internal manifold. These universally occur in all models, with:

$$n_S = 1 \,, \quad n_T = 3$$

Taking each field separately, the curvature bound is always violated. To fulfill the bound one would need corrections. These should be large for S, but could be small for T.

Keeping both fields, the curvature bound is instead fulfilled. But T must dominate over S, and the Goldstino angle θ is constrained to the quadrant $[0, \pi/4]$. This implies that:

$$\frac{F_S}{\mathrm{Re}\, S} < \frac{\sqrt{3}\,|F_T|}{\mathrm{Re}\, T}$$

This demonstrates in an extremely robust way that the scenario where S dominates over T is impossible to realize, at least in the controllable limit where both are large.

CONCLUSIONS AND OUTLOOK

- In SUGRA models with only chiral multiplets, there exist necessary conditions for stability that strongly constrain the curvature of the geometry and the SUSY breaking direction.
- The form of these constraints can be worked out in full detail for factorizable and symmetric geometries, like those occurring in the moduli sector of string models.
- It would be of great interest to generalize this study to models involving also vector multiplets gauging isometries of the scalar manifold.