ASPECTS OF SUPERSYMMETRIC D-BRANE DYNAMICS

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- VARIOUS SPACE-TIME 0-BRANES IN 4D ORBIFOLD COMPACTIFICATIONS.
  TWO INTERESTING CASES: 0-BRANE OF TYPE IIA AND 3-BRANE OF TYPE IIB.
- DYNAMICS AND BOUNDARY STATE; FORCE.
  LARGE DISTANCE INTERACTIONS AND RELATION WITH SOLUTIONS OF 4D EFFECTIVE SUGRA.
- EMISSION OF MASSLESS NSNS STATES FROM TWO INTERACTING SPACE-TIME 0-BRANES.
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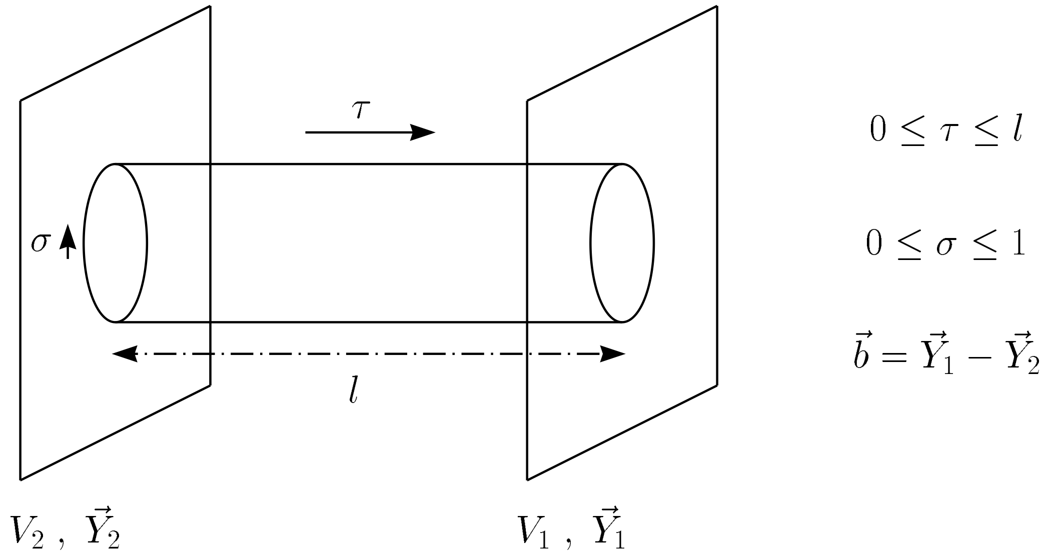
INTERACTIONS ON ORBIFOLDS

Consider two 0-branes moving with velocities $V_1 = \tanh v_1$, $V_2 = \tanh v_2$ (along 1) and transverse positions $\vec{Y}_1$, $\vec{Y}_2$ (along 2-3).



The amplitude in the closed string channel is

$$\mathcal{A} = \int_0^\infty dl \sum_s < B, V_1, \vec{Y}_1 | e^{-lH} | B, V_2, \vec{Y}_2 >_s$$

There are two sectors, RR and NSNS, and after the GSO projection four spin structures contribute, R$\pm$ and NS$\pm$.

In the static case, one has Neumann b.c. in time and Dirichlet b.c. in space. The velocity twist the 0-1 directions.

The moving boundary state is obtained by boosting the static one with $v = v_1 - v_2$ (Billó, Di Vecchia, Cangemi)

$$|B, V, \vec{Y} >= e^{-ivJ^{01}} |B, \vec{Y} >$$

In the large distance limit $b \to \infty$ only world-sheets with $l \to \infty$ will contribute.

Momentum or winding in the compact directions can be neglected since they correspond to massive components.

The moving boundary states

$$
\begin{aligned}
|B, V_1, \vec{Y}_1 > &= \int \frac{d^3\vec{k}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{Y}_1} |B, V_1 > \otimes |k_B > \\
|B, V_2, \vec{Y}_2 > &= \int \frac{d^3\vec{q}}{(2\pi)^3} e^{i\vec{q}\cdot\vec{Y}_2} |B, V_2 > \otimes |q_B >
\end{aligned}
$$

can only carry the boosted space-time momenta

$$
\begin{aligned}
k_B^\mu &= (V_1\gamma_1 k^1, \gamma_1 k^1, \vec{k}_T) = (\sinh v_1 k^1, \cosh v_1 k^1, \vec{k}_T) \\
q_B^\mu &= (V_2\gamma_2 q^1, \gamma_2 q^1, \vec{q}_T) = (\sinh v_2 q^1, \cosh v_2 q^1, \vec{q}_T)
\end{aligned}
$$

Taking into account momentum conservation ($k_B^\mu = q_B^\mu$), the amplitude factorizes

$$
\begin{aligned}
\mathcal{A} &= \frac{1}{\sinh v} \int_0^\infty dl \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} e^{-\frac{q_B^2}{2}} \sum_s Z_B Z_F^s \\
&= \frac{1}{\sinh v} \int_0^\infty \frac{dl}{2\pi l} e^{-\frac{b^2}{2l}} \sum_s Z_B Z_F^s
\end{aligned}
$$

with (from now on $X^\mu \equiv X^\mu_{osc}$)

$$
Z_{B,F} =< B, V_1 | e^{-lH} | B, V_2 >^s_{B,F}
$$

Group the fields into pairs

$$
\begin{aligned}
X^\pm = X^0 \pm X^1 &\rightarrow \alpha_n, \beta_n = a_n^0 \pm a_n^1 \\
X^i, X^{i*} = X^i \pm iX^{i+1} &\rightarrow \beta_n^i, \beta_n^{i*} = a_n^i \pm i a_n^{i+1} \quad , \quad i = 2,4,6,8 \\
\chi^{A,B} = \psi^0 \pm \psi^1 &\rightarrow \chi_n^{A,B} = \psi_n^0 \pm \psi_n^1 \\
\chi^i, \chi^{i*} = \psi^i \pm i\psi^{i+1} &\rightarrow \chi_n^i, \chi_n^{i*} = \psi_n^i \pm i\psi_n^{i+1} \quad , \quad i = 2,4,6,8
\end{aligned}
$$

with

$$
\begin{aligned}
[\alpha_m, \beta_{-n}] = -2\delta_{mn} \quad &, \quad [\beta_m^i, \beta_{-n}^{i*}] = 2\delta_{mn} \\
\{\chi_m^A, \chi_{-n}^B\} = -2\delta_{mn} \quad &, \quad \{\chi_m^i, \chi_n^{i*}\} = 2\delta_{mn}
\end{aligned}
$$

Orbifold construction

Identify points connected by discrete rotations $g = e^{2\pi i \sum_a z_a J_{aa+1}}$ on some of the compact pairs X^a,χ^a, a=4,6,8.

In order to preserve at least on SUSY: $\Sigma_a z_a = 0$.

- For T_6/Z_3 ($N = 2$ SUSY) take $z_4, z_6 = \frac{1}{3}, \frac{2}{3}$, $z_8 = -z_4 - z_6$
- For $T_2 \otimes T_4/Z_2$ ($N = 4$ SUSY) take $z_4 = -z_6 = \frac{1}{2}$, $z_8 = 0$
- For T_6 ($N = 8$ SUSY) take $z_4 = z_6 = z_8 = 0$

There can be additional twisted sectors. One can diagonalize the fields such that ($g_a = e^{2\pi i z_a}$)

$$X^a(\sigma + 1) = g_a X^a(\sigma) \quad , \quad X^{*a}(\sigma + 1) = g_a^* X^{*a}(\sigma)$$

and similarly for fermions. This leads to fractional moding.

The twisted states become massless only at fixed points of the orbifold.

In all sectors, one has to project onto invariant states to get the physical spectrum.

The physical boundary state is

$$|B_{phys}>= \frac{1}{N}(|B, 1> +|B, g> +... + |B, g^{N-1}>)$$

in terms of the twisted boundary states

$$|B, g^k>= g^k|B>$$

0-brane: untwisted sector

Consider first the static case. The b.c. are Neumann for time and Dirichlet for all other directions (i=2,4,6,8 and a=2,4,6).

For the bosons, the b.c. are

$$(\alpha_n + \tilde{\beta}_{-n})|B>_B = 0 \quad , \quad (\beta_n + \tilde{\alpha}_{-n})|B>_B = 0$$

$$(\beta^i_n - \tilde{\beta}^i_{-n})|B>_B = 0 \quad , \quad (\beta^{i*}_n - \tilde{\beta}^{i*}_{-n})|B>_B = 0$$

They are solved by the following boundary state

$$|B>_B = \exp\frac{1}{2}\sum_{n=1}^{\infty}(\alpha_{-n}\tilde{\alpha}_{-n} + \beta_{-n}\tilde{\beta}_{-n} + \beta^i_{-n}\tilde{\beta}^{i*}_{-n} + \beta^{i*}_{-n}\tilde{\beta}^i_{-n})|0>$$

For the fermions, one has integer or half-integer moding in the RR and NSNS sectors respectively. The b.c are

$$(\chi^A_n + i\eta\tilde{\chi}^B_{-n})|B,\eta>_F = 0 \quad , \quad (\chi^B_n + i\eta\tilde{\chi}^A_{-n})|B,\eta>_F = 0$$

$$(\chi^i_n - i\eta\tilde{\chi}^i_{-n})|B,\eta>_F = 0 \quad , \quad (\chi^{i*}_n - i\eta\tilde{\chi}^{i*}_{-n})|B,\eta>_F = 0$$

where $\eta = \pm 1$ to deal with the GSO projection.

The corresponding boundary state can be factorized into zero mode and oscillator parts:

$$|B,\eta>_F = |B_o>_F \otimes |B_{osc}>_F$$

The oscillator part is the same for both sectors, with appropriate moding

$$|B_{osc},\eta>_F = \exp\frac{i\eta}{2}\sum_{n>0}(\chi^A_{-n}\tilde{\chi}^A_{-n} + \chi^B_{-n}\tilde{\chi}^B_{-n} - \chi^i_{-n}\tilde{\chi}^{i*}_{-n} - \chi^{i*}_{-n}\tilde{\chi}^i_{-n})|0>$$

The zero mode part exist only in the RR sector.

The zero modes are proportional to Γ-matrices

$$\psi_o^\mu = \frac{i}{\sqrt{2}}\Gamma^\mu \quad , \quad \tilde{\psi}_o^\mu = \frac{i}{\sqrt{2}}\tilde{\Gamma}^\mu$$

One can construct

$$a, a^* = \frac{1}{2}(\Gamma^0 \pm \Gamma^1)$$

$$b^i, b^{i*} = \frac{1}{2}(-i\Gamma^i \pm \Gamma^{i+1})$$

and similar for tilded, satisfying

$$\{a, a^*\} = \{b^i, b^{i*}\} = 1$$

The b.c. for the zero modes can be rewritten as

$$(a + i\eta\tilde{a}^*)|B_o, \eta>_F = 0 \quad , \quad (a^* + i\eta\tilde{a})|B_o, \eta>_F = 0$$

$$(b^i - i\eta\tilde{b}^i)|B_o, \eta>_F = 0 \quad , \quad (b^{i*} - i\eta\tilde{b}^{i*})|B_o, \eta>_F = 0$$

Defining the spinor vacuum $|0> \otimes |\tilde{0}>$ such that

$$a|0>= 0 \quad , \quad \tilde{a}|\tilde{0}>= 0$$

$$b^i|0>= 0 \quad , \quad \tilde{b}^{i*}|\tilde{0}>= 0$$

the zero mode part of the boundary state can be written as

$$|B_o, \eta>_{RR} = \exp -i\eta(a^*\tilde{a}^* - b^{i*}\tilde{b}^i)|0> \otimes |\tilde{0}>$$

The complete boundary state is already invariant under orbifold rotations, for which

$$\beta_n^a \to g_a\beta_n^a \quad , \quad \beta_n^{a*} \to g_a^*\beta_n^{a*}$$

$$\chi_n^a \to g_a\chi_n^a \quad , \quad \chi_n^{a*} \to g_a^*\chi_n^{a*}$$

$$b^a \to g_a b^a \quad , \quad b^{a*} \to g_a^* b^{a*}$$

For a boost of rapidity v:

$$\alpha_n \to e^{-v}\alpha_n \quad , \quad \beta_n \to e^{v}\beta_n$$

$$\chi^A_n \to e^{-v}\chi^A_n \quad , \quad \chi^B_n \to e^{v}\chi^B_n$$

$$a \to e^{-v}a \quad , \quad a^* \to e^{v}a^*$$

The spinor vacuum is not invariant, but transforms as

$$|0> \otimes|\tilde{0}> \to e^{-v}|0> \otimes|\tilde{0}>$$

Finally, the complete boosted boundary state is

$$|B, V>_B = \exp\frac{1}{2}\sum_{n>0}(e^{-2v}\alpha_{-n}\tilde{\alpha}_{-n} + e^{2v}\beta_{-n}\tilde{\beta}_{-n} + \beta^i_{-n}\tilde{\beta}^{i*}_{-n} + \beta^{i*}_{-n}\tilde{\beta}^i_{-n})|0>$$

$$|B_{osc}, V, \eta>_F = \exp\frac{i\eta}{2}\sum_{n>0}(e^{-2v}\chi^A_{-n}\tilde{\chi}^A_{-n} + e^{2v}\chi^B_{-n}\tilde{\chi}^B_{-n}$$
$$-\chi^i_{-n}\tilde{\chi}^{i*}_{-n} - \chi^{i*}_{-n}\tilde{\chi}^i_{-n})|0>$$

$$|B_o, V, \eta>_{RR} = e^{-v}\exp -i\eta(e^{2v}a^*\tilde{a}^* - b^{i*}\tilde{b}^i)|0> \otimes|\tilde{0}>$$

In both sectors one gets

$$(-1)^F|B, V, \eta> = -|B, V, -\eta>$$

and the GSO-projected boundary state is

$$|B, V> = \frac{1}{2}(|B, V, +> - |B, V, ->)$$

In the partition function, the ghosts cancel one untwisted pair, say 2-3, and the result is the product of the contributions of the 0-1 pair and the 3 compact pairs.

For the bosons, one finds ($q = e^{-2\pi l}$)

$$< B, V_1|e^{-lH}|B, V_2>^{(0,1)}_B = \prod_{n=1}^{\infty}\frac{1}{(1 - e^{-2v}q^{2n})(1 - e^{2v}q^{2n})}$$

$$< B, V_1|e^{-lH}|B, V_2>^{(a,a+1)}_B = \prod_{n=1}^{\infty}\frac{1}{(1 - q^{2n})^2}$$

The total bosonic partition function is (zero-point energy $q^{-\frac{2}{3}}$)

$$Z_B = 2i \sinh v \; \eta^{-8}(2il) \frac{\vartheta_1'(0|2il)}{\vartheta_1(i\frac{v}{\pi}|2il)}$$

For the fermions, the 0-1 pair gives

$$< B, V_1, \eta | e^{-lH} | B, V_2, \eta' >_F^{s(0,1)} = Z_o^s(\eta\eta') \prod_{n>0} (1 + \eta\eta' e^{-2v} q^{2n})(1 + \eta\eta' e^{2v} q^{2n})$$

with $\eta\eta' = \pm 1$ and

$$Z_o^R(+) = 2\cosh v \quad , \quad Z_o^R(-) = 2\sinh v$$

$$Z_o^{NS}(\pm) = 1$$

Each compact pair gives instead

$$< B, V_1, \eta | e^{-lH} | B, V_2, \eta' >_F^{s(a,a+1)} = Z_o^s(\eta\eta') \prod_{n>0} (1 + \eta\eta' q^{2n})^2$$

with

$$Z_o^R(+) = 2 \quad , \quad Z_o^R(-) = 0$$

$$Z_o^{NS}(\pm) = 1$$

After the GSO projection, only the three even spin structures R+ and NS$\pm$ contribute, and (zero-point energy $q^{-\frac{1}{3}}$ for NSNS and $q^{\frac{2}{3}}$ for RR)

$$\begin{aligned} Z_F = \eta^{-4}(2il) \Big\{ & \vartheta_2(i\frac{v}{\pi}|2il)\vartheta_2(0|2il)^3 \\ & -\vartheta_3(i\frac{v}{\pi}|2il)\vartheta_3(0|2il)^3 + \vartheta_4(i\frac{v}{\pi}|2il)\vartheta_4(0|2il)^3 \Big\} \\ \sim V^4 & \end{aligned}$$

corresponding to the usual SUSY cancellation of the force (Bachas).

0-brane: twisted sector

The boundary state is similar to the one of the untwisted sector, with fractional moding.

In the Z_3 case, each pair of compact bosons gives

$$< B, V_1|e^{-lH}|B, V_2 >_B^{(a,a+1)} = \prod_{n=1}^{\infty} \frac{1}{(1-q^{2(n-\frac{1}{3})})(1-q^{2(n-\frac{2}{3})})}$$

For a pair of compact fermions (no zero modes)

$$< B, V_1, \eta|e^{-lH}|B, V_2, \eta' >_R^{s(a,a+1)} = \prod_{n=1}^{\infty} (1+\eta\eta' q^{2(n-\frac{1}{3})})(1+\eta\eta' q^{2(n-\frac{2}{3})})$$

$$< B, V_1, \eta|e^{-lH}|B, V_2, \eta' >_{NS}^{s(a,a+1)} = \prod_{n=1}^{\infty} (1+\eta\eta' q^{2(n-\frac{1}{6})})(1+\eta\eta' q^{2(n-\frac{5}{6})})$$

The total partition functions after the GSO projection are (the zero-point energies add to zero)

$$\begin{aligned}
Z_B &= 2\sinh v\, \eta^4(2il)\vartheta\begin{bmatrix}1/2\\1/2\end{bmatrix}(i\frac{v}{\pi}|2il)^{-1}\vartheta\begin{bmatrix}1/6\\1/2\end{bmatrix}(0|2il)^{-3} \\
Z_F &= \eta^{-4}(2il)\left\{\vartheta\begin{bmatrix}0\\0\end{bmatrix}(i\frac{v}{\pi}|2il)\vartheta\begin{bmatrix}-1/3\\0\end{bmatrix}(0|2il)^3\right. \\
&\quad -\vartheta\begin{bmatrix}0\\1/2\end{bmatrix}(i\frac{v}{\pi}|2il)\vartheta\begin{bmatrix}-1/3\\1/2\end{bmatrix}(0|2il)^3 \\
&\quad \left.+\vartheta\begin{bmatrix}1/2\\0\end{bmatrix}(i\frac{v}{\pi}|2il)\vartheta\begin{bmatrix}1/6\\0\end{bmatrix}(0|2il)^3\right\} \\
&\sim V^2
\end{aligned}$$

In the Z_2 case, the analysis is similar and the results have the same qualitative behavior. In particular, $Z_F \sim V^2$ again.

3-brane

In the static case, take Neumann b.c. for time, Dirichlet b.c. for space and mixed b.c. for each pair of compact directions, say Neumann for the a directions and Dirichlet for the a+1 directions.

The new b.c. for the compact directions are

$$(\beta_n^a + \tilde{\beta}_{-n}^{a*})|B>_B = 0 \quad , \quad (\beta_n^{a*} + \tilde{\beta}_{-n}^{a})|B>_B = 0$$

$$(\chi_n^a + i\eta\tilde{\chi}_{-n}^{a*})|B_{osc},\eta>_F = 0 \quad , \quad (\chi_n^{a*} + i\eta\tilde{\chi}_{-n}^{a})|B_{osc},\eta>_F = 0$$

$$(b^a + i\eta\tilde{b}^{a*})|B_o,\eta>_F = 0 \quad , \quad (b^{a*} + i\eta\tilde{b}^{a})|B_o,\eta>_F = 0$$

Defining a new spinor vacuum $|0>\otimes|\tilde{0}>$ such that

$$b^a|0>=0 \ , \ \tilde{b}^a|\tilde{0}>=0$$

the compact part of the boundary state is

$$|B>_B = \exp -\frac{1}{2}\sum_{n>0}(\beta_{-n}^a\tilde{\beta}_{-n}^a + \beta_{-n}^{a*}\tilde{\beta}_{-n}^{a*})|0>$$

$$|B_{osc},\eta>_F = \exp\frac{i\eta}{2}\sum_{n>0}(\chi_{-n}^a\tilde{\chi}_{-n}^a + \chi_{-n}^{a*}\tilde{\chi}_{-n}^{a*})|0>$$

$$|B_o,\eta>_{RR} = \exp -i\eta b^{a*}\tilde{b}^{a*}|0>\otimes|\tilde{0}>$$

In this case, the boundary state is not invariant under orbifold rotations. Recall that ($g_a = e^{2\pi i z_a}$)

$$\beta_n^a \to g_a\beta_n^a \quad , \quad \beta_n^{a*} \to g_a^*\beta_n^{a*}$$

$$\chi_n^a \to g_a\chi_n^a \quad , \quad \chi_n^{a*} \to g_a^*\chi_n^{a*}$$

$$b^a \to g_a b^a \quad , \quad b^{a*} \to g_a^* b^{a*}$$

Moreover, the spinor vacuum now transform

$$|0>\otimes|\tilde{0}>\to g_a|0>\otimes|\tilde{0}>$$

The compact part of the twisted boundary state is

$$|B,V,g_a>_B = \exp -\frac{1}{2}\sum_{n>0}(g_a^2\beta^a_{-n}\tilde\beta^a_{-n} + g_a^{*2}\beta^{a*}_{-n}\tilde\beta^{a*}_{-n})|0>$$

$$|B_{osc},V,g_a,\eta>_F = \exp\frac{i\eta}{2}\sum_{n>0}(g_a^2\chi^a_{-n}\tilde\chi^a_{-n} + g_a^{*2}\chi^{a*}_{-n}\tilde\chi^{a*}_{-n})|0>$$

$$|B_o,V,g_a,\eta>_{RR} = g_a \exp -i\eta g_a^{*2} b^{a*}\tilde b^{a*}|0>\otimes|\tilde 0>$$

A pair of compact bosons gives ($(g_a^*g_a')^2 = e^{2\pi i w_a}$)

$$<B,V_1,g_a|e^{-lH}|B,V_2,g_a'>_B^{(a,a+1)} = \prod_{n=1}^{\infty}\left|\frac{1}{1+\eta\eta' e^{2\pi i w_a}q^{2n}}\right|^2$$

For fermions

$$<B,V_1,g_a,\eta|e^{-lH}|B,V_2,g_a',\eta'>_F^{s(a,a+1)} = Z_o^s(\eta\eta')\prod_{n>0}\left|1+\eta\eta' e^{2\pi i w_a}q^{2n}\right|^2$$

where

$$Z_o^R(+) = 2\cos\pi w_a \quad , \quad Z_o^R(-) = 2i\sin\pi w_a$$

$$Z_o^{NS}(\pm) = 1$$

After the GSO projection, the total partition functions for a given relative twist are

$$Z_B = 16i\sinh v\ \eta^4(2il)\frac{1}{\vartheta_1(i\frac{v}{\pi}|2il)}\prod_a\frac{\sin\pi w_a}{\vartheta_1(w_a|2il)}$$

$$Z_F = \eta^{-4}(2il)\Big\{\vartheta_2(i\frac{v}{\pi}|2il)\prod_a\vartheta_2(w_a|2il)$$
$$-\vartheta_3(i\frac{v}{\pi}|2il)\prod_a\vartheta_3(w_a|2il) + \vartheta_4(i\frac{v}{\pi}|2il)\prod_a\vartheta_4(w_a|2il)\Big\}$$

$$\sim\begin{cases} V^4 \ , & w_a = 0\\ V^2 \ , & w_a\neq 0\end{cases}$$

To obtain the invariant amplitude, one has to average over all possible orbifold twists. There is no twisted sector.

Large distance limit

Explicit expressions for $l \to \infty$

0-brane

a) Untwisted sector

$$\mathcal{A} \sim 4\cosh v - \cosh 2v - 3 \sim V^4$$

b) Twisted sector

$$\mathcal{A} \sim \cosh v - 1 \sim V^2$$

3-brane

$$\mathcal{A}(w_a) \sim 4\prod_a \cos \pi w_a \cosh v - \cosh 2v - \sum_a \cos 2\pi w_a$$

$$\mathcal{A} \sim \begin{cases} \cosh v - \cosh 2v \sim V^2 \quad , \quad T_6/Z_3 \\ 4\cosh v - \cosh 2v - 3 \sim V^4 \quad , \quad T_2 \otimes T_4/Z_2 \; , \; T_6 \end{cases}$$

Field theory interpretation

The possible contributions in the eikonal approximation are

Scalar: $-a^2$

Vector: $e^2 \cosh v$

Graviton: $-M^2 \cosh 2v$

Thus

$4\cosh v - \cosh 2v - 3 \quad \Leftrightarrow \quad$ N=4,8 Grav. multiplet

$\cosh v - \cosh 2v \quad \Leftrightarrow \quad$ N=2 Grav. multiplet

$\cosh v - 1 \quad \Leftrightarrow \quad$ Vec. multiplet

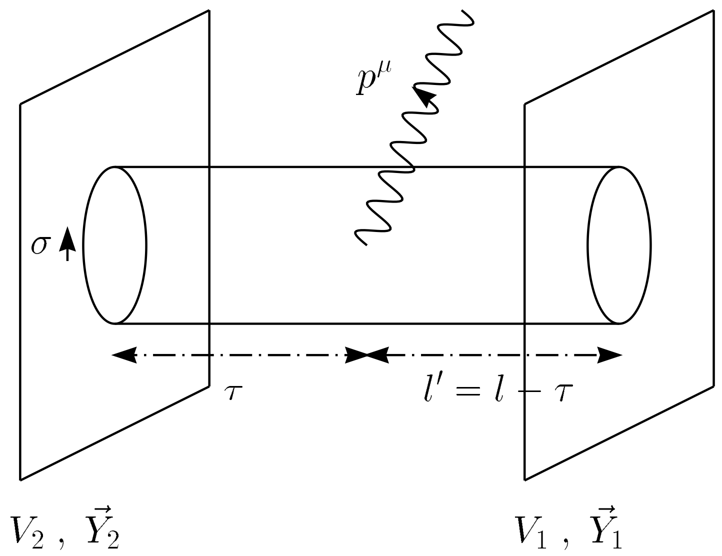
V^2 terms

In the dual open string channel, V corresponds to an electric field E, and V^2 terms correspond to a renormalization of the Maxwell term E^2. This can not happen for the maximally supersymmetric theory.

The V^2 behavior is thus forbidden for $N = 8$ and allowed for $N < 8$; our results are compatible with this.

EMISSION OF MASSLESS NSNS BOSONS

Consider two moving 0-branes in interaction emitting a massless NSNS boson.



$0 \leq \tau, l' \leq l$

$0 \leq \sigma \leq 1$

$\vec{b} = \vec{Y}_1 - \vec{Y}_2$

The amplitude is computed inserting the vertex operator $(z = \sigma + i\tau)$

$$V(z,\bar{z}) = G_{ij}(\partial X^i - \frac{1}{2}p\cdot\psi\psi^i)(\bar{\partial}X^j + \frac{1}{2}p\cdot\bar{\psi}\bar{\psi}^j)e^{ip\cdot X}$$

between the two boundary states

$$\begin{aligned}\mathcal{A} &= \int_0^\infty dl \int_0^l d\tau \sum_s < B, V_1, \vec{Y}_1| e^{-lH} V(z,\bar{z}) | B, V_2, \vec{Y}_2 >_s \\ &= \int_0^\infty d\tau \int_0^\infty dl' \sum_s < V(z,\bar{z}) >_s\end{aligned}$$

Split the bosons into zero mode and oscillators to be treated separately (again $X^\mu \equiv X^\mu_{osc}$).

The zero mode part gives the kinematics ($p^\mu = k^\mu_B - q^\mu_B$).

The energies and longitudinal momenta are completely fixed ($\cos\theta = \frac{p^1}{p}$, $p = p^0$)

$$k^0_B = V_1 k^1_B \quad , \quad k^1_B = \frac{p}{V_1 - V_2}(1 - V_2\cos\theta)$$

$$q^0_B = V_2 q^1_B \quad , \quad q^1_B = \frac{p}{V_1 - V_2}(1 - V_1\cos\theta)$$

The zero mode contribution is ($v = v_1 - v_2$)

$$< e^{ip\cdot X} >_o = \frac{1}{\sinh v} \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} e^{-\frac{q^2}{2}\tau} e^{-\frac{k^2}{2}l'}$$

Further zero mode insertions give

$$\begin{aligned}
\partial X_o^i \; &\Rightarrow \; -\frac{1}{2}k_B^i \\
\bar{\partial} X_o^j \; &\Rightarrow \; \frac{1}{2}k_B^j \\
\partial X_o^i \bar{\partial} X_o^j \; &\Rightarrow \; -\frac{1}{4}k_B^i k_B^j
\end{aligned}$$

Finally, the amplitude is (from now on $q^\mu \equiv q_B^\mu$ and $k^\mu \equiv k_B^\mu$)

$$\mathcal{A} = \frac{1}{\sinh v} \int_0^\infty d\tau \int_0^\infty dl' \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} e^{-\frac{q^2}{2}\tau} e^{-\frac{k^2}{2}l'} < e^{ip\cdot X} > \sum_s Z_B Z_F^s \mathcal{M}_s$$

with

$$\begin{aligned}
\mathcal{M}^s = G_{ij} \Big\{ & < \partial X^i \bar{\partial} X^j > - < \partial X^i p\cdot X >< \bar{\partial} X^j p \cdot X > \\
& + \frac{1}{4}\Big(< p\cdot\psi p\cdot\bar{\psi} >_s < \psi^i \bar{\psi}^j >_s - < p\cdot\psi\psi^i >_s < p\cdot\bar{\psi}\bar{\psi}^j >_s \\
& \qquad + < p\cdot\bar{\psi}\psi^i >_s < p\cdot\psi\bar{\psi}^j >_s \Big) \\
& + \frac{i}{2}\Big(< \partial X^i p\cdot X >< p\cdot\bar{\psi}\bar{\psi}^j >_s - < \bar{\partial} X^j p\cdot X >< p\cdot\psi\psi^i >_s \Big) \\
& - \frac{1}{2}k^i \left(i < \bar{\partial} X^j p\cdot X > + \frac{1}{2} < p\cdot\bar{\psi}\bar{\psi}^j >_s \right) \\
& + \frac{1}{2}k^j \left(i < \partial X^i p\cdot X > - \frac{1}{2} < p\cdot\psi\psi^i >_s \right) \\
& - \frac{1}{4}k^i k^j \Big\}
\end{aligned}$$

Obviously, the partition function factorizes, leaving connected correlators. In the odd spin structure, appropriate zero modes insertion is understood.

Correlators

The boundary state provides a systematic way of computing correlators with non trivial b.c.

$$< X^\mu X^\nu >= \frac{< B_1,V_1| e^{-lH} X^\mu X^\nu |B_2,V_2 >_B}{< B_1,V_1| e^{-lH} |B_2,V_2 >_B}$$

$$< \psi^\mu \psi^\nu >_s= \frac{< B_1,V_1,\eta| e^{-lH} \psi^\mu \psi^\nu |B_2,V_2,\eta' >_F^s}{< B_1,V_1,\eta| e^{-lH} |B_2,V_2,\eta' >_F^s}$$

For the bosons, one obtains ($q = e^{-2\pi\tau}$)

$$\begin{aligned} < X^0(z)\bar{X}^0(\bar{z}) >=&< X^1(z)\bar{X}^1(\bar{z}) >= \\ &= \frac{1}{4\pi}\sum_{n=0}^{\infty}\Big\{\cosh 2[(v_1-v_2)n - v_2]\ln(1-q^{2n}e^{-4\pi\tau}) \\ &\qquad - \cosh 2[(v_2-v_1)n - v_1]\ln(1-q^{2n}e^{-4\pi l'})\Big\} \end{aligned}$$

$$\begin{aligned} < X^0(z)\bar{X}^1(\bar{z}) >=&< X^1(z)\bar{X}^0(\bar{z}) >= \\ &= -\frac{1}{4\pi}\sum_{n=0}^{\infty}\Big\{\sinh 2[(v_1-v_2)n - v_2]\ln(1-q^{2n}e^{-4\pi\tau}) \\ &\qquad + \sinh 2[(v_2-v_1)n - v_1]\ln(1-q^{2n}e^{-4\pi l'})\Big\} \end{aligned}$$

For the fermions in the NS± sectors, the results are

$$\begin{aligned} < \psi^0(z)\bar{\psi}^0(\bar{z}) >_{NS\pm}=&< \psi^1(z)\bar{\psi}^1(\bar{z}) >_{NS\pm}= \\ &= -i\sum_{n=0}^{\infty}(\mp)^n\left\{\cosh 2[(v_1-v_2)n - v_2]\frac{q^n e^{-2\pi\tau}}{1-q^{2n}e^{-4\pi\tau}}\right. \\ &\qquad \left. \pm \cosh 2[(v_2-v_1)n - v_1]\frac{q^n e^{-2\pi l'}}{1-q^{2n}e^{-4\pi l'}}\right\} \end{aligned}$$

$$\begin{aligned} < \psi^0(z)\bar{\psi}^1(\bar{z}) >_{NS\pm}=&< \psi^1(z)\bar{\psi}^0(\bar{z}) >_{NS\pm}= \\ &= i\sum_{n=0}^{\infty}(\mp)^n\left\{\sinh 2[(v_1-v_2)n - v_2]\frac{q^n e^{-2\pi\tau}}{1-q^{2n}e^{-4\pi\tau}}\right. \\ &\qquad \left. \pm \sinh 2[(v_2-v_1)n - v_1]\frac{q^n e^{-2\pi l'}}{1-q^{2n}e^{-4\pi l'}}\right\} \end{aligned}$$

For the fermions in the R$\pm$ sectors, the results are similar,

$$
\begin{aligned}
&< \psi^0(z)\bar{\psi}^0(\bar{z}) >_{R\pm} = < \psi^1(z)\bar{\psi}^1(\bar{z}) >_{R\pm} = \\
&\quad = F_o^R(\pm) - i\sum_{n=0}^{\infty}(\mp)^n \left\{ \cosh 2[(v_1 - v_2)n - v_2] \frac{q^{2n}e^{-4\pi\tau}}{1 - q^{2n}e^{-4\pi\tau}} \right. \\
&\qquad\qquad \left. \pm \cosh 2[(v_2 - v_1)n - v_1] \frac{q^{2n}e^{-4\pi l'}}{1 - q^{2n}e^{-4\pi l'}} \right\} \\
&< \psi^0(z)\bar{\psi}^1(\bar{z}) >_{R\pm} = < \psi^1(z)\bar{\psi}^0(\bar{z}) >_{R\pm} = \\
&\quad = G_o^R(\pm) + i\sum_{n=0}^{\infty}(\mp)^n \left\{ \sinh 2[(v_1 - v_2)n - v_2] \frac{q^{2n}e^{-4\pi\tau}}{1 - q^{2n}e^{-4\pi\tau}} \right. \\
&\qquad\qquad \left. \pm \sinh 2[(v_2 - v_1)n - v_1] \frac{q^{2n}e^{-4\pi l'}}{1 - q^{2n}e^{-4\pi l'}} \right\}
\end{aligned}
$$

with zero mode contributions

$$
F_o^R(+) = -\frac{i}{2}\frac{\cosh(v_1 + v_2)}{\cosh(v_1 - v_2)} \quad , \quad F_o^R(-) = -\frac{i}{2}\frac{\sinh(v_1 + v_2)}{\sinh(v_1 - v_2)}
$$

$$
G_o^R(+) = -\frac{i}{2}\frac{\sinh(v_1 + v_2)}{\cosh(v_1 - v_2)} \quad , \quad G_o^R(-) = -\frac{i}{2}\frac{\cosh(v_1 + v_2)}{\sinh(v_1 - v_2)}
$$

World-sheet SUSY means (for osc.)

$$
< \partial X^\mu(z)\bar{X}^\nu(\bar{z}) > = \frac{1}{2} < \psi^\mu(z)\bar{\psi}^\nu(\bar{z}) >_{R-}
$$

There are also non vanishing equal-point correlators, which can be computed in the same way. They can also be deduced from the previous ones using the b.c.

The correlators can be expressed in terms of twisted ϑ-functions.

Form the combinations $\psi^{\pm} = e^{\mp v_2}(\psi^0 \pm \psi^1)$, satisfying the b.c.

$$\psi^{\pm}(z) = -i\bar{\psi}^{\mp}(\bar{z}) \quad , \quad \tau = 0 \ \Leftrightarrow \ z = \bar{z}$$

$$\psi^{\pm}(z) = -ie^{\pm 2v}\bar{\psi}^{\mp}(\bar{z}) \quad , \quad \tau = l \ \Leftrightarrow \ z = \bar{z} + 2il$$

The propagators

$$P^{s}_{(\pm)}(z - \bar{z}) =< \psi^{\pm}(z)\bar{\psi}^{\pm}(\bar{z}) >_s$$

should have appropriate periodicity conditions on the covering torus with modulus 2il from which the cylinder can be obtained by the involution $z \doteq \bar{z} + 2il$.

In fact, under

$$w \to w + m + 2iln$$

the propagators transform as

$$P^{R+}_{(\pm)}(w + m + 2iln) = e^{i\pi n}e^{\pm 2nv}P^{R+}_{(\pm)}(w)$$

$$P^{R-}_{(\pm)}(w + m + 2iln) = e^{\pm 2nv}P^{R-}_{(\pm)}(w)$$

$$P^{NS+}_{(\pm)}(w + m + 2iln) = e^{i\pi m}e^{i\pi n}e^{\pm 2nv}P^{NS+}_{(\pm)}(w)$$

$$P^{NS-}_{(\pm)}(w + m + 2iln) = e^{i\pi m}e^{\pm 2nv}P^{NS-}_{(\pm)}(w)$$

These properties, together with the universal local behavior

$$P^{s}_{(\pm)}(w) \to \frac{1}{4\pi w}$$

imply for the even spin structures:

$$P^{s}_{(\pm)}(w) = \frac{1}{4\pi}\frac{\vartheta_s(w \pm i\frac{v}{\pi}|2il)\vartheta_1'(0|2il)}{\vartheta_s(\pm i\frac{v}{\pi}|2il)\vartheta_1(w|2il)}$$

Results

Axion

$$G_{ij} = \frac{1}{2}\epsilon_{ijk}\frac{p^k}{p}$$

Only the odd spin structure can contribute because of the antisymmetry of G_{ij}. In the twisted sector of the Z_3 case, there are only two fermionic zero modes in the 2-3 pair, and the amplitude could be non vanishing.

After integrating by parts the two-derivative bosonic term, world-sheet SUSY leads to

$$\mathcal{M}_{ax}^{R-} = \frac{i}{8}\cos\theta\left[-\partial_\tau < p\cdot X(z)p\cdot\bar{X}(\bar{z}) > +\frac{1}{2}(k^2-q^2)\right]$$

Since $\partial_\tau|_l = \partial_\tau|_{l'} - \partial_{l'}|_\tau$ the final amplitude is a total derivative ($Z_B Z_F^{R-} = 2\sinh v$ for the twisted sector of Z_3)

$$\begin{aligned}\mathcal{A}_{ax} &= \frac{i}{4}\cos\theta\int_0^\infty d\tau\int_0^\infty dl'\int\frac{d^2\vec{k}_T}{(2\pi)^2}e^{i\vec{k}\cdot\vec{b}}(\partial_\tau-\partial_{l'})\left\{e^{-\frac{q^2}{2}\tau}e^{-\frac{k^2}{2}l'} < e^{ip\cdot X} >\right\}\\ &= 0\end{aligned}$$

Dilaton

$$G_{ij} = \delta_{ij} - \frac{p^i p^j}{p^2}$$

Only the even spin structures contribute, because of the symmetry of G_{ij}. Again, the two-derivative bosonic term is integrated by parts.

In the large distance limit, one keep only leading terms for $l\to\infty$ in the propagators, and

$$< e^{ip\cdot X} >= \left(1-e^{-4\pi\tau}\right)^{-\frac{p^{(2)2}}{2\pi}}\left(1-e^{-4\pi l'}\right)^{-\frac{p^{(1)2}}{2\pi}}$$

with the boosted energies

$$p^{(1,2)} = p\gamma_{1,2}(1-V_{1,2}\cos\theta) = p(\cosh v_{1,2} - \sinh v_{1,2}\cos\theta)$$

One finds for the contractions

$$\mathcal{M}_{dil}^{R+} = \frac{1}{4p^2}\left[(k^2-q^2) - 2p^2\cos\theta\tanh v\right] \times$$
$$\left\{\frac{1}{4}(k^2-q^2) - p^{(2)2}\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}} + p^{(1)2}\frac{e^{-4\pi l'}}{1-e^{-4\pi l'}}\right\}$$
$$-\frac{k^0}{p}\left(\frac{q^2}{4} + p^{(2)2}\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}}\right) + \frac{q^0}{p}\left(\frac{k^2}{4} + p^{(1)2}\frac{e^{-4\pi l'}}{1-e^{-4\pi l'}}\right)$$

$$\mathcal{M}_{dil}^{NS\pm} = \frac{1}{4p^2}\left[(k^2-q^2) \mp 8e^{-2\pi l}p^2\cos\theta\sinh v\right] \times$$
$$\left\{\frac{1}{4}(k^2-q^2) - p^{(2)2}\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}} + p^{(1)2}\frac{e^{-4\pi l'}}{1-e^{-4\pi l'}}\right\}$$
$$-\frac{k^0}{p}\left(\frac{q^2}{4} + p^{(2)2}\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}}\right) + \frac{q^0}{p}\left(\frac{k^2}{4} + p^{(1)2}\frac{e^{-4\pi l'}}{1-e^{-4\pi l'}}\right)$$

Taking into account $< e^{ip\cdot X} >$ and integrating by parts in the final amplitude, one gets the rules

$$\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}} \doteq -\frac{1}{4}\frac{q^2}{p^{(2)2}} \quad , \quad \frac{e^{-4\pi l'}}{1-e^{-4\pi l'}} \doteq -\frac{1}{4}\frac{k^2}{p^{(1)2}}$$

Using these equivalence relations, one finds

$$\mathcal{M}_{dil}^{R+} = \mathcal{M}_{dil}^{NS\pm} = 0$$

This means that the amplitude is a total derivative and

$$\mathcal{A}_{dil} = 0$$

Graviton

$$G_{ij} = h_{ij} = h_{ji} \quad , \quad p^i h_{ij} = h^i_i = 0$$

Proceeding as for the dilaton, one obtains for $l \to \infty$

$$\begin{aligned}
&\mathcal{M}^{R+}_{grav} = \\
&\quad -\frac{1}{4}\left[h_{ij}k^i k^j - p\tanh v h_{i1}k^i\right] \\
&\quad -V_2\gamma_2\left[p^{(2)}\left(h_{i1}k^i - \frac{p}{2}\tanh v h_{11}\right) + \frac{1}{4}(k^2-q^2)V_2\gamma_2 h_{11}\right]\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}} \\
&\quad +V_1\gamma_1\left[p^{(1)}\left(h_{i1}k^i - \frac{p}{2}\tanh v h_{11}\right) + \frac{1}{4}(k^2-q^2)V_1\gamma_1 h_{11}\right]\frac{e^{-4\pi l'}}{1-e^{-4\pi l'}}
\end{aligned}$$

$$\begin{aligned}
&\mathcal{M}^{NS\pm}_{grav} = \\
&\quad -\frac{1}{4}\left[h_{ij}k^i k^j \mp 4e^{-2\pi l}\left(p\sinh 2v h_{i1}k^i - p^2\sinh^2 v h_{11}\right)\right] \\
&\quad -V_2\gamma_2\left[p^{(2)}\left(h_{i1}k^i \mp 2e^{-2\pi l}p\sinh v h_{11}\right) + \frac{1}{4}(k^2-q^2)V_2\gamma_2 h_{11}\right]\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}} \\
&\quad +V_1\gamma_1\left[p^{(1)}\left(h_{i1}k^i \mp 2e^{-2\pi l}p\sinh v h_{11}\right) + \frac{1}{4}(k^2-q^2)V_1\gamma_1 h_{11}\right]\frac{e^{-4\pi l'}}{1-e^{-4\pi l'}}
\end{aligned}$$

One can use the same equivalence relations as before to write $\mathcal{M}^s_{grav}$ in a τ, l'-independent form. Anyway:

$$\mathcal{A}_{grav} \neq 0$$

The general structure of the amplitude is

$$\mathcal{A}_{grav} = \frac{1}{\sinh v} \int_0^\infty d\tau \int_0^\infty dl' \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} e^{-\frac{q^2}{2}\tau} e^{-\frac{k^2}{2}l'} \times$$

$$\left(1 - e^{-4\pi\tau}\right)^{-\frac{p^{(2)2}}{2\pi}} \left(1 - e^{-4\pi l'}\right)^{-\frac{p^{(1)2}}{2\pi}} \sum_s Z_B Z_F^s \mathcal{M}^s_{grav}$$

$$= \frac{1}{\sinh v} \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} I_1 I_2 \sum_s Z_B Z_F^s \mathcal{M}^s_{grav}$$

with

$$\mathcal{M}^s_{grav} = B^s(p,k,q) + q^2 C_1^s(p,k,q) + k^2 C_2^s(p,k,q)$$

The kinematical integrals over the two proper times τ, l' give

$$I_1 = \int_0^\infty d\tau e^{-\frac{q^2}{2}\tau} \left(1 - e^{-4\pi\tau}\right)^{-\frac{p^{(2)2}}{2\pi}} = -\frac{1}{4\pi} \frac{\Gamma[\frac{q^2}{8\pi}]\Gamma[-\frac{p^{(2)2}}{2\pi}+1]}{\Gamma[\frac{q^2}{8\pi} - \frac{p^{(2)2}}{2\pi}+1]}$$

$$I_2 = \int_0^\infty dl' e^{-\frac{k^2}{2}l'} \left(1 - e^{-4\pi l'}\right)^{-\frac{p^{(1)2}}{2\pi}} = -\frac{1}{4\pi} \frac{\Gamma[\frac{k^2}{8\pi}]\Gamma[-\frac{p^{(1)2}}{2\pi}+1]}{\Gamma[\frac{k^2}{8\pi} - \frac{p^{(1)2}}{2\pi}+1]}$$

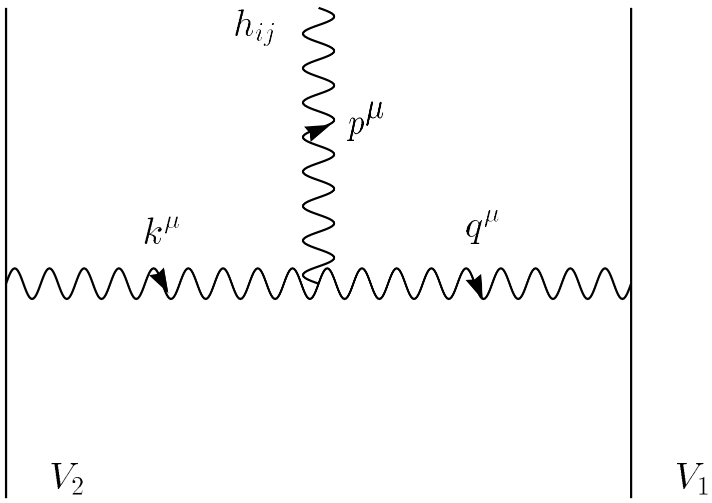
One finds the usual dual structure with a double serie of poles. However, in the eikonal approximation $p \ll M = 1$ and

$$I_1 \to -\frac{2}{q^2} \quad , \quad I_2 \to -\frac{2}{k^2}$$

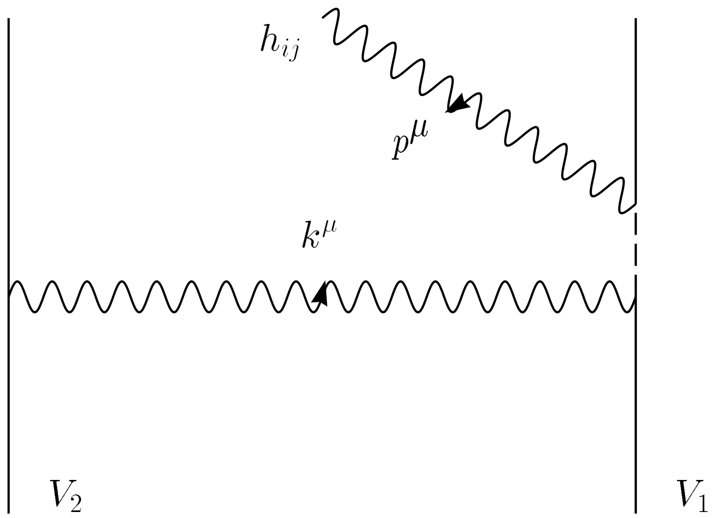
Finally, the amplitude becomes

$$\mathcal{A}_{grav} = \frac{4}{\sinh v} \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} \left\{ B^s \frac{1}{q^2 k^2} + C_1^s \frac{1}{k^2} + C_2^s \frac{1}{q^2} \right\}$$

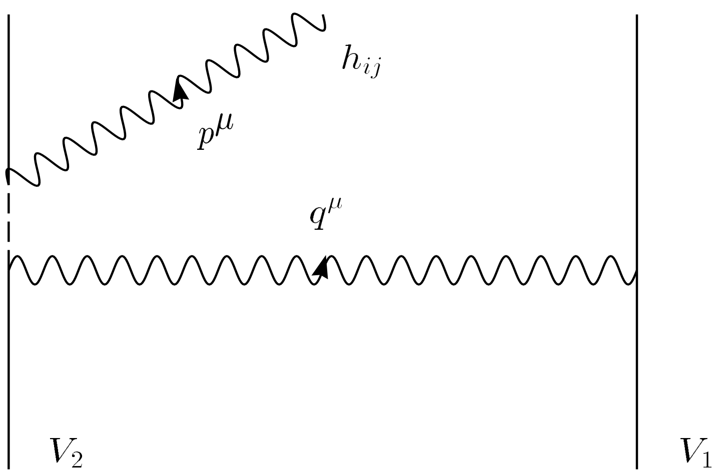
The graphical interpretation is the following



$$\Rightarrow \quad B^s \frac{1}{q^2 k^2} \quad (\tau, l' \neq 0)$$



$$\Rightarrow \quad C_1^s \frac{1}{k^2} \quad (\tau = l, l' = 0)$$



$$\Rightarrow \quad C_2^s \frac{1}{q^2} \quad (\tau = 0, l' = l)$$

Annihilation term

Field theory results

For the axion and the dilaton, there is no coupling in SUGRA allowing the emission process.

For the annihilation term of the graviton, there are three possible diagrams in SUGRA, involving the exchange of R vectors and NS scalars and gravitons. Their respective contributions in the eikonal approximation are

$$B^{R}_{V_\mu} = e^2 \left[\cosh v h_{ij} k^i k^j - p \sinh v h_{i1} k^i\right]$$
$$B^{NS}_{\phi} = -a^2 h_{ij} k^i k^j$$
$$B^{NS}_{g_{\mu\nu}} = -M^2 \left[\cosh 2v h_{ij} k^i k^j - 2p \sinh 2v h_{i1} k^i + 2p^2 \sinh^2 v h_{11}\right]$$

String results

The string results in the various compactification schemes are the following

0-brane: untwisted sector & **3-brane on** $T_2 \otimes T_4/Z_2, T_6$

$$Z^{R+} - Z^{NS+} + Z^{NS-} \to 16 \cosh v - 4 \cosh 2v - 12$$
$$Z^{NS+} + Z^{NS-} \to 2e^{2\pi l}$$

and

$$B^{R}_{grav} = 4 \left[\cosh v h_{ij} k^i k^j - p \sinh v h_{i1} k^i\right]$$
$$B^{NS}_{grav} = -\left[\cosh 2v h_{ij} k^i k^j - 2p \sinh 2v h_{i1} k^i + 2p^2 \sinh^2 v h_{11}\right]$$
$$-3 h_{ij} k^i k^j$$
$$\Rightarrow B_{grav} \sim V^4 h_{ij} k^i k^j + V^3 p h_{i1} k^i + V^2 p^2 h_{11}$$

0-brane: twisted sector

$$Z^{R+} - Z^{NS+} - Z^{NS-} \to 4\cosh v - 4$$

$$Z^{NS+} - Z^{NS-} \to 0$$

and

$$B^R_{grav} = \left[\cosh v h_{ij} k^i k^j - p \sinh v h_{i1} k^i\right]$$

$$B^{NS}_{grav} = -h_{ij} k^i k^j$$

$$\Rightarrow B_{grav} \sim V^2 h_{ij} k^i k^j + V p h_{i1} k^i + V^2 p^2 h_{11}$$

3-brane on T_6/Z_3

$$Z^{R+} - Z^{NS+} + Z^{NS-} \to 4\cosh v - 4\cosh 2v$$

$$Z^{NS+} + Z^{NS-} \to 2e^{2\pi l}$$

and

$$B^R_{grav} = \left[\cosh v h_{ij} k^i k^j - p \sinh v h_{i1} k^i\right]$$

$$B^{NS}_{grav} = -\left[\cosh 2v h_{ij} k^i k^j - 2p \sinh 2v h_{i1} k^i + 2p^2 \sinh^2 v h_{11}\right]$$

$$\Rightarrow B_{grav} \sim V^2 h_{ij} k^i k^j + V p h_{i1} k^i + V^2 p^2 h_{11}$$

Colinear emission

At $\theta = 0$ one finds

$$B_{grav} \sim V^n h_{ij} k^i k^j$$

$$C_{1grav} = C_{2grav} = 0$$

with $n = 2, 4$ depending on the amount of SUSY.

Radiated energy

The average energy radiated when the two branes pass each other at impact parameter $\vec{b}$ is

$$< p >\sim \int \frac{d^3\vec{p}}{p} p \left|\mathcal{A}\right|^2$$

For $\theta = 0$ and $V \ll 1$ one has

$$\mathcal{A} \sim V^{n-1} g_s l_s f(\frac{p \cdot b}{V}) e^{-\frac{p \cdot b}{V}}$$

where f is a slowly varying function and $n = 2, 4$. Notice that the emission is exponentially suppressed for $p \sim p_{max} = V/b$.

By dimensional analysis one finds

$$< p >\sim g_s^2 l_s^2 \frac{V^{1+2n}}{b^3}$$

SPIN EFFECTS

The 0-brane belongs to a BPS multiplet realizing half of the SUSY. Its $2^{N/2}$ components ($N = \#$ of SUSY generators) have different spins.

The annulus vacuum amplitude with Polchinski's prescription for the b.c gives only the universal spin-independent potential. The additional spin-dependent interactions are obtained by applying the broken SUSY.

In the closed string channel, one can construct higher spin boundary states in the G-S formalism by acting with the broken supercharges on the scalar one. In D=10, the complete potential behaves for $V \to 0$ ($V \simeq v$) generically like

$$V(r) \sim \sum_{k=0}^{4} \frac{v^{4-k}}{r^{7+k}}$$

This result is consistent with string duality (Harvey).

Consider type II theory in the light-cone gauge $X^+ = x^+ + p^+\tau$. X^- is completely determined in terms of the transverse X^I and the left and right spinors S^a and $\tilde{S}^a$, in the 8_v and 8_s of $SO(8)$ respectively.

Any boundary state (Green, Gutperle) automatically satisfies Dirichlet b.c. in the two light-cone directions $\pm = 0 \pm 9$, but can implement arbitrary b.c. in the transverse directions $I = 1, 2...8$.

We can define a p-brane-like configuration by taking Neumann b.c. for $I = \mu = 1, 2, ..., p+1$ and Dirichlet b.c. for $I = i = p+2, ..., 8-p$. The "time" is now the $p+1$ direction; the usual metric is recovered through a double analytic continuation $0 \to i(p+1)$, $(p+1) \to i0$ in the final result.

A boost of rapidity v is implemented through a rotation of angle v in the $p+1, p+2$ plane and taking $v \to iv$.

The 32 SUSY charges are

$$Q^a = \sqrt{2p^+}\int_0^1 d\sigma S^a \quad , \quad Q^{\dot a} = \frac{1}{\sqrt{2p^+}}\gamma^I_{\dot a a}\int_0^1 d\sigma \partial X^I S^a$$

$$\tilde Q^a = \sqrt{2p^+}\int_0^1 d\sigma \tilde S^a \quad , \quad \tilde Q^{\dot a} = \frac{1}{\sqrt{2p^+}}\gamma^I_{\dot a a}\int_0^1 d\sigma \bar\partial X^I \tilde S^a$$

and satisfy the algebra

$$\{Q^a, Q^b\} = 2p^+\delta^{ab} \;\; , \;\; \{Q^{\dot a}, Q^{\dot b}\} = P^-\delta^{\dot a\dot b}$$

$$\{Q^a, Q^{\dot a}\} = \tfrac{1}{\sqrt 2}\gamma^I_{a\dot a}p^I$$

A static boundary state can be defined to preserve a combination of left and right supersymmetries. Defining

$$Q^a_\pm = \frac{1}{\sqrt 2}\left(Q^a \pm iM_{ab}\tilde Q^b\right)$$

$$Q^{\dot a}_\pm = \frac{1}{\sqrt 2}\left(Q^{\dot a} \pm iM_{\dot a\dot b}\tilde Q^{\dot b}\right)$$

with the algebra

$$\{Q^a_+, Q^b_-\} = 2p^+\delta^{ab} \;\; , \;\; \{Q^{\dot a}_+, Q^{\dot b}_-\} = P^-\delta^{\dot a\dot b}$$

$$\{Q^a_+, Q^{\dot a}_-\} = \tfrac{1}{\sqrt 2}\left(\gamma^I_{a\dot a}p^I + M_{ab}M_{\dot a\dot b}\gamma^I_{b\dot b}\tilde p^I)\right)$$

we impose the BPS conditions

$$Q^a_+|B>= 0 \quad , \quad Q^{\dot a}_+|B>= 0 \;\Rightarrow\; Q^a_+, Q^{\dot a}_+ \text{ unbroken}$$

$$Q^a_-|B>\neq 0 \quad , \quad Q^{\dot a}_-|B>\neq 0 \;\Rightarrow\; Q^a_-, Q^{\dot a}_- \text{ broken}$$

The bosonic b.c. are

$$(\alpha^I_n + M_{IJ}\tilde\alpha^J_{-n})|B>= 0$$

with

$$M_{IJ} = \begin{pmatrix} -I_{p+1} & 0 \\ 0 & I_{7-p} \end{pmatrix}$$

For the fermionic b.c. we make the ansatz

$$(S_n^a + iM_{ab}\tilde{S}^b_{-n})|B>= 0$$

$$(S_n^{\dot a} + iM_{\dot a\dot b}\tilde{S}^{\dot b}_{-n})|B>= 0$$

Consistency with the BPS condition implies

$$M^T_{ac}M_{cb} = \delta_{ab}$$

$$\gamma^I_{\dot a a}M_{IJ} - M_{ab}M_{\dot a\dot b}\gamma^J_{\dot b b} = 0$$

which yield

$$M_{ab} = (\gamma^1\gamma^2...\gamma^{p+1})_{ab}\ ,\ M_{\dot a\dot b} = (\gamma^1\gamma^2...\gamma^{p+1})_{\dot a\dot b}$$

The solution for the boundary state is written as

$$|B>= \exp\sum_{n>0}\left(\frac{1}{n}M_{IJ}\alpha^I_{-n}\tilde{\alpha}^J_{-n} - iM_{ab}S^a_{-n}\tilde{S}^b_{-n}\right)|B_o>$$

the zero mode part being

$$|B_o>= M_{IJ}|I>|\tilde{J}> -iM_{\dot a\dot b}|\dot a>|\tilde{\dot b}>$$

The complete boundary state is

$$|B,y>= \int\frac{d^{9-p}q}{(2\pi)^{9-p}}\,e^{iq\cdot y}\,|B>\otimes|q>$$

and the cylinder amplitude reduces to

$$\begin{aligned}\mathcal{A} &= \int_0^\infty dl < B,y_2|e^{-2p^+(P^- - p^-)l}|B,y_1> \\ &= V_{p+1}\int_0^\infty\frac{dl}{(2\pi l)^{\frac{9-p}{2}}}e^{-\frac{b^2}{2l}}Z_BZ_F\end{aligned}$$

The static amplitude vanishes because of the fermionic zero modes:

$$Z^F_o =< B_o|B_o>= \mathrm{Tr}_V\mathbf{1} - \mathrm{Tr}_S\mathbf{1} = 8-8 = 0$$

The boost is implemented as a rotation on the boundary state

$$|B,v>= e^{-ivJ^{p+1p+2}}|B>$$

whose effect is

$$M \to M(v) = \Sigma(v)M\Sigma^T(v)$$

where $\Sigma(v)$ is the appropriate representation of the $SO(8)$ rotation:

$$\begin{aligned}\Sigma_V(v) &= \begin{pmatrix} I_p & 0 & 0 & 0 \\ 0 & \cos v & -\sin v & 0 \\ 0 & \sin v & \cos v & 0 \\ 0 & 0 & 0 & I_{6-p} \end{pmatrix} \\ \Sigma_S(v) &= \cos(v/2)\,\delta_{ab} - \sin(v/2)\,\gamma_{ab}^{p+1p+2}\end{aligned}$$

The fermionic zero modes part now give ($v = v_1 - v_2$)

$$\begin{aligned}Z_o^F =& < B_o, v_2|B_o, v_1 >= \mathrm{Tr}_V M^T(v_2)M(v_1) - \mathrm{Tr}_S M^T(v_2)M(v_1) \\ &= (6 + 2\cos 2v) - 8\cos v = 16\sin^4\frac{v}{2}\end{aligned}$$

Computing also the oscillator part one finds, after taking $v \to iv$,

$$\mathcal{A} = V_p \int_0^\infty \frac{dl}{(2\pi l)^{\frac{8-p}{2}}} e^{-\frac{b^2}{2l}} \eta^{-9}(2il) \frac{\vartheta_1(i\frac{v}{2\pi}|2il)^4}{\vartheta_1(i\frac{v}{\pi}|2il)}$$

which coincides with the well known result after using the Riemann identity.

In this framework, SUSY breaking for small velocity can be efficiently studied. In fact, the behavior for $v \to 0$ can be alternatively obtained simply by setting $v = 0$ in the oscillator part,

$$\mathcal{A} \underset{v\to 0}{\longrightarrow} \frac{V_p}{v} \int_0^\infty \frac{dl}{(2\pi l)^{\frac{8-p}{2}}} e^{-\frac{b^2}{2l}} Z_o^F(v \to 0)$$

and looking only at the fermionic zero modes:

$$Z_o^F(v \to 0) = \lim_{v\to 0} < B_o|e^{-ivJ^{p+1p+2}}|B_o >$$

Recall now that the fermionic zero mode part of J^{p+1p+2} is

$$R_o^{p+1p+2} = \frac{1}{4}\left(S_o\gamma^{p+1p+2}S_o + \tilde{S}_o\gamma^{p+1p+2}\tilde{S}_o\right)$$

Since the insertion of less than four of these factors gives a vanishing result, we find, expanding the boost operator

$$Z_o^F(v \to 0) = v^4 < B_o|(R_o^{p+1p+2})^4|B_o > \sim v^4$$

Doing the l-integral one finds

$$\mathcal{A} \sim \frac{v^3}{b^{6-p}} \Leftrightarrow V(r) \sim \frac{v^4}{r^{7-p}}$$

It is clear that this non-relativistic behavior is exact at all scales, both at large and short distances.

To compute spin effects, consider now the insertion of broken supercharges. Because of the q^+ integration, only pairs of dotted-undotted charges give a non-vanishing contribution.

To find the non-relativistic behavior, we look at the fermionic zero modes. Before integrating over the momentum, the amplitude with the insertion of $k < 4$ pairs of supercharges is

$$\mathcal{A} \underset{v\to 0}{\longrightarrow} \frac{V_p}{v}\int_0^\infty dl \int \frac{d^{9-p}q}{(2\pi)^{9-p}}\, e^{iq\cdot b}e^{-q^2 l} Z_o^F(v \to 0)$$

with

$$Z_o^F(v \to 0) = \lim_{v\to 0} < B_o|e^{-ivJ^{p+1p+2}} \prod_{i=1}^{k} Q_-^{a_i}Q_-^{\dot{a}_i}|B_o >$$

The leading contribution comes from taking the zero mode part in each pair of supercharges

$$Q_-^{a_i}Q_-^{\dot{a}_i} = p^{I_i}\gamma^{I_i}_{\dot{a}_i b_i}S_o^{-a_i}S_o^{-b_i} = \frac{1}{4}p^{I_i}\left(\gamma^{I_i}\gamma^{M_iN_i}\right)_{a_i\dot{a}_i} R_o^{-M_iN_i}$$

One finds, expanding the boost operator,

$$Z_o^F(v \to 0) = \frac{v^{4-k}}{4^k} \prod_{i=1}^{k} q^{i_i} \left(\gamma^{i_i}\gamma^{M_i N_i}\right)_{a_i \dot{a}_i} < B_o| \prod_{i=k}^{4} R_o^{M_i N_i} (R_o^{p+1 p+2})^{4-k} |B_o >$$
$$\sim q^k v^{4-k}$$

This leads, after integrating over the transverse momentum, to

$$\mathcal{A} \sim \frac{v^{3-k}}{b^{6+k-p}} \Leftrightarrow V(r) \sim \frac{v^{4-k}}{r^{7+k-p}}$$

Again this behavior is exact at all scales, both at large and short distances.

An equivalent argument can be given in the open string channel, in which one computes a vacuum amplitude with k pairs of supercharges inserted.

Each pair of charges provides 2 fermionic zero modes and a factor bt^2, whereas the velocity-dependent interaction contains 2 fermionic zero modes coming with a factor vt.

In order to soak up all the 8 fermionic zero modes in the path integral, we need $8 - 2k$ fermionic zero modes from the exponential of the interaction, giving a factor $(vt)^{4-k}$.

Finally, considering also the bosonic zero modes ($(vt)^{-1}$) and setting $v = 0$ for all the oscillators, one finds

$$\mathcal{A} \underset{v \to 0}{\longrightarrow} \int_0^\infty \frac{dt}{t^{1+\frac{p}{2}}} e^{-b^2 t} (vt)^{3-k} (bt^2)^k \sim \frac{v^{3-k}}{b^{6+k-p}}$$

For the 0-brane, the first spin-orbit interaction going like v^3/r^8 has been checked (Kraus) in the matrix model.