GENERAL CONSTRAINTS ON METASTABLE SUPERSYMMETRY BREAKING

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- Vacuum stability and sGoldstino masses.
- Metastability constraints in SUSY models.
- Metastability constraints in SUGRA models.
- Application to SUSY breaking in string models.
- Other possible issues about SUSY breaking.

Based on works with L. Brizi, L. Covi, M. Gomez-Reino,
C. Gross, J.-C. Jacot, J. Louis, G. Palma

METASTABILITY AND SGOLDSTINO MASSES

Vacuum

Vacua are set by constant values of the fields minimizing the potential V. One has $V' = 0$, whereas $V = \Lambda^4$ defines the vacuum energy and $V'' = m^2$ the fluctuation mass matrix.

In SUSY theories, the form of V is constrained. As a result, vacua display special features. The two main issues are to get $\Lambda^4 > 0$ and $m^2 > 0$.

SUSY breaking and metastability

Denef, Douglas 2005
Gomez-Reino, Scrucca 2006

When SUSY is broken, there is a Goldstino fermion which has zero mass. Its partners the sGoldstino bosons have masses controlled by breaking effects and difficult to adjust.

Requiring positive sGoldstino masses leads to metastability conditions. Gravity gives only quantitative modifications compared to the rigid case.

N = 1 SUSY WITH CHIRAL MULTIPLETS

Lagrangian and transformation laws

Zumino 1979
Freedman, Alvarez-Gaumé 1981

An $N = 1$ theory with n_C chirals $\Phi^i = (\phi^i, \psi^i, F^i)$ is defined by:

$$L = \int d^4\theta\, K(\Phi, \bar{\Phi}) + \int d^2\theta\, W(\Phi) + \mathrm{h.c.}$$

This gives a non-linear σ-model on a Kähler target space with metric $g_{i\bar{\jmath}} = K_{i\bar{\jmath}}$ and non-trivial potential V.

The SUSY transformations take the standard form

$$\delta\phi^i = \sqrt{2}\epsilon\psi^i$$
$$\delta\psi^i = \sqrt{2}\epsilon F^i + \sqrt{2}i\partial\!\!\!/\psi^i\bar{\epsilon}$$

where

$$F^i = -g^{i\bar{\jmath}}\bar{W}_{\bar{\jmath}} + \text{ferm.}$$

Vacuum

A vacuum breaks SUSY if $F^i \neq 0$:

$$V = F^i \bar{F}_i$$

The Goldstino fermion and the two sGoldstino real scalars are identified with the following combinations of fields:

$$\eta = \sqrt{2}\bar{F}_i \psi^i$$
$$\varphi_\pm = {}^{\mathrm{Re}}_{\mathrm{Im}}(\bar{F}_i \phi^i)$$

Average masses

Grisaru, Rocek, Karlhede 1983

There is a sum rule on the difference between the bosonic and fermionic average masses:

$$\mathrm{str}\, m^2 = 2\, R_{i\bar{\jmath}} F^i \bar{F}^{\bar{\jmath}}$$

Goldstino and sGoldstino masses

Using the stationarity condition one finds:

$$m_\eta = 0 \quad m^2_{\varphi_\pm} = R\, F^i \bar{F}_i \pm \Delta$$

where

$$R = -\frac{R_{i\bar{j}k\bar{l}}\, F^i \bar{F}^{\bar{j}} F^k \bar{F}^{\bar{l}}}{(F^n \bar{F}_n)^2}$$

Metastability

Gomez-Reino, Scrucca 2006

Taking the average of $m^2_{\varphi_\pm}$, one finds an upper bound for the lowest scalar mass eigenvalue, which should be positive for metastability:

$$m^2_{\mathrm{meta}} = R\, F^i \bar{F}_i$$

This gives a sharp and strong constraint.

N = 1 SUSY WITH CHIRAL AND VECTOR MULTIPLETS

Lagrangian and transformation laws

Bagger, Witten 1982
Hull, Karlhede, Lindstrom, Rocek 1986

An $N = 1$ theory with n_C chirals $\Phi^i = (\phi^i, \psi^i, F^i)$ and n_V vectors $V^a = (\lambda^a, A^a_\mu, D^a)$ is defined by:

$$L = \int d^4\theta \Big[K(\Phi,\bar\Phi) + \big(K_a(\Phi,\bar\Phi) + \xi_a\big)V^a + 2X^i_a \bar X_{bi}(\Phi,\bar\Phi)V^a V^b\Big]$$
$$+ \int d^2\theta \Big[\frac{1}{4}H_{ab}(\Phi)W^a W^b + W(\Phi)\Big] + \mathrm{h.c.}$$

This gives a gauged non-linear σ-model on a Kähler target space with metric $g_{i\bar\jmath} = K_{i\bar\jmath}$, gauged isometries generated by Killing vectors X^i_a with Killing potentials K_a and non-trivial potential V.

The gauge couplings and angles are determined by $h_{ab} = \mathrm{Re}(H_{ab})$ and $\theta_{ab} = \mathrm{Im}(H_{ab})$, the matter charges by $Q_{ai}{}^j = i\nabla_i X^j_a$ and the vector masses by $M^2_{ab} = 2X^i_{(a}\bar X_{b)i}$.

The gauge symmetries imply several constraints and act as

$$\delta_g \Phi^i = \Lambda^a X_a^i(\Phi)$$
$$\delta_g V^a = -\frac{i}{2}\left(\Lambda^a - \bar{\Lambda}^a\right)$$

The SUSY transformations read

$$\delta\phi^i = \sqrt{2}\epsilon\psi^i$$
$$\delta\psi^i = \sqrt{2}\epsilon F^i + \sqrt{2} i \not{D}\phi^i \bar{\epsilon}$$
$$\delta A_\mu^a = i\epsilon\sigma_\mu\bar{\lambda}^a - i\lambda^a\sigma_\mu\bar{\epsilon}$$
$$\delta\lambda^a = i\epsilon D^a + \sigma^{\mu\nu}\epsilon F_{\mu\nu}^a$$

where

$$F^i = -g^{i\bar{\jmath}}\bar{W}_{\bar{\jmath}} + \text{ferm.}$$
$$D^a = -\frac{1}{2}h^{ab}(K_b + \xi_b) + \text{ferm.}$$

The constant FI terms are consistent with gravity only if there is also a $U(1)_R$ symmetry.

Vacuum

A vacuum breaks SUSY if $F^i, D^a \neq 0$:

$$V = F^i \bar{F}_i + \frac{1}{2} D^a D_a$$

Stationarity further implies the relation

$$M^2_{ab} D^b - f_{ab}{}^c \theta_{cd} D^b D^d = 2 Q_{ai\bar{\jmath}} F^i \bar{F}^{\bar{\jmath}}$$

The Goldstino fermion and the two sGoldstino real scalars are

$$\eta = \sqrt{2} \bar{F}_i \psi^i + i D_a \lambda^a$$

$$\varphi_\pm = {}^{\mathrm{Re}}_{\mathrm{Im}} (\bar{F}_i \phi^i)$$

Average masses

Grisaru, Rocek, Karlhede 1983

The bosonic and fermionic average masses satisfy a sum rule:

$$\mathrm{str}\, m^2 = 2\big(R_{i\bar{\jmath}} - h_{abi} h^{ac} h^{bd} h_{cd\bar{\jmath}}\big) F^i \bar{F}^{\bar{\jmath}}$$

$$+ 2\big(Q_{ai}{}^i - 2 f_{ab}{}^c h^{bd} \theta_{cd}\big) D^a$$

Goldstino and sGoldstino masses

Using the stationarity condition one finds

$$m_\eta = 0 \quad m^2_{\varphi\pm} = R\, F^i \bar{F}_i + S\, D^a D_a + T\, \frac{(D^a D_a)^2}{4\, F^i \bar{F}_i} + M^2 \frac{D^a D_a}{F^i \bar{F}_i} \pm \Delta$$

where

$$R = -\frac{R_{i\bar{\jmath}k\bar{l}}\, F^i \bar{F}^{\bar{\jmath}} F^k \bar{F}^{\bar{l}}}{(F^n \bar{F}_n)^2} \quad S = \frac{h_{aci} h^{cd} h_{db\bar{\jmath}}\, F^i \bar{F}^{\bar{\jmath}} D^a D^b}{F^n \bar{F}_k\, D^c D_c}$$

$$T = \frac{h_{abi} h_{cb}{}^{i} D^a D^b D^c D^d}{(D^e D_e)^2} \quad M^2 = \frac{2 X^i_a \bar{X}_{bi} D^a D^b}{D^c D_c}$$

Metastability

Gomez-Reino, Scrucca 2007

Averaging over $m^2_{\varphi\pm}$, one finds a result with a new semi-positive term:

$$m^2_{\mathrm{meta}} = R\, F^i \bar{F}_i + \left(S\, F^i \bar{F}_i + \frac{1}{4} T\, D^a D_a + M^2 \right) \frac{D^b D_b}{F^j \bar{F}_j}$$

This gives a milder and more flexible constraint.

N = 2 SUSY WITH HYPER MULTIPLETS

Lagrangian and transformation laws

Alvarez-Gaumé, Freedman 1981
Hull, Karlhede, Lindstrom, Rocek 1986

An $N = 2$ theory with $n_{\mathcal{H}}$ hypers $\mathcal{H}^k$ is a subcase of $N = 1$ theory with $2n_{\mathcal{H}}$ chirals $Q^u = (q^u, \chi^u, F^u)$.

The second SUSY transformation has the general form

$$\hat{\delta} Q^u = \frac{1}{2} \bar{D}^2 \Big(\bar{N}^u(Q, \bar{Q})(\hat{\epsilon}\theta + \hat{\bar{\epsilon}}\bar{\theta}) \Big)$$

This is an $N = \hat{1}$ symmetry if $\Omega_{uv} \equiv \nabla_u N_v$ and $X^u \equiv \bar{\Omega}^{uv} W_v$ satisfy

$$\Omega_{(uv)} = 0 \quad \nabla_w \Omega_{uv} = 0 \quad \nabla_{\bar{w}} \Omega_{uv} = 0 \quad \bar{\Omega}^u{}_w \Omega^w{}_v = -\delta^u_v$$
$$\nabla_{\bar{w}} X^u = 0 \quad \nabla_{(u} X_{\bar{v})} = 0 \quad \bar{\Omega}^u{}_{\bar{w}} \nabla_{\bar{v}} \bar{X}^{\bar{w}} - \bar{\Omega}^w{}_{\bar{v}} \nabla_w X^u = 0$$

This says that the geometry is Hyper-Kähler with three complex structures defined out of Ω_{uv} and that X^u is a triholomorphic Killing vector.

The SUSY algebra closes on-shell with a global central charge symmetry:

$$\delta_c Q^u = \lambda X^u(Q)$$

The two SUSY transformations take the form

$$\delta q^u = \sqrt{2}\epsilon\chi^u \qquad \hat{\delta} q^u = -\sqrt{2}\bar{\Omega}^u{}_{\bar{v}}\hat{\bar{\epsilon}}\,\bar{\chi}^v$$

$$\delta\chi^u = \sqrt{2}\epsilon F^u + \sqrt{2} i \partial\!\!\!/ q^u \bar{\epsilon} \qquad \hat{\delta}\chi^u = \sqrt{2}\hat{\epsilon}\hat{F}^u + \sqrt{2} i \bar{\Omega}^u{}_{\bar{v}} \partial\!\!\!/ \bar{q}^{\bar{v}} \hat{\bar{\epsilon}}$$

where

$$F^u = \bar{\Omega}^u{}_{\bar{v}}\bar{X}^{\bar{v}} + \text{ferm.} \qquad \hat{F}^u = -X^u + \text{ferm.}$$

There is a global $SU(2)_R$ symmetry rotating the complex structures:

$$\vec{J}^U{}_V = \left(\begin{pmatrix} 0 & \bar{\Omega}^u{}_{\bar{v}} \\ \Omega^{\bar{u}}{}_v & 0 \end{pmatrix}, \begin{pmatrix} 0 & i\bar{\Omega}^u{}_{\bar{v}} \\ -i\Omega^{\bar{u}}{}_v & 0 \end{pmatrix}, \begin{pmatrix} i\delta^u_v & 0 \\ 0 & -i\delta^{\bar{u}}_{\bar{v}} \end{pmatrix} \right)$$

The source of SUSY breaking is

$$X^U = \begin{pmatrix} X^u \\ \bar{X}^{\bar{u}} \end{pmatrix}$$

Vacuum

A vacuum breaks SUSY if $F^u, \hat{F}^u \neq 0$, i.e. $X^U \neq 0$:

$$V = F^u \bar{F}_u = \hat{F}^u \bar{\hat{F}}_u = \frac{1}{2} X^U X_U$$

The two Goldstino fermions and the four sGoldstino real scalars are given by the combinations

$$\eta = \sqrt{2}\bar{F}_u \chi^u \qquad \hat{\eta} = \sqrt{2}\bar{\hat{F}}_u \chi^u$$
$$\varphi_\pm = {}^{\mathrm{Re}}_{\mathrm{Im}}(\bar{F}_u q^u) \quad \hat{\varphi}_\pm = {}^{\mathrm{Re}}_{\mathrm{Im}}(\bar{\hat{F}}_u q^u)$$

Using real scalars and symplectic-Majorana fermions, these modes can be reorganized as doublet and singlet plus triplet of $SU(2)_R$:

$$\eta^\alpha = \left(g_{UV}\delta^\alpha_\beta + i J^x_{UV}\sigma^{x\alpha}{}_\beta\right) X^V \chi^{U\beta}$$
$$\varphi^0 = g_{UV} X^V q^U \quad \varphi^x = J^x_{UV} X^V q^U$$

Average masses

Grisaru, Rocek, Karlhede 1983

Due to the property $R_{u\bar{v}} = 0$ one finds:

$$\mathrm{str}\, m^2 = 0$$

Goldstino and sGoldstino masses

Using the stationarity condition one finds:

$$m_{\eta^\alpha} = 0 \quad m^2_{\varphi^0} = 0 \quad m^2_{\varphi^x} = -R_x\, X^U X_U$$

where

$$R_x = \frac{R_{UVMN} X^U (J^x X)^V X^M (J^x X)^N}{(X^K X_K)^2}$$

Metastability

Gomez-Reino, Louis, Scrucca 2009
Jacot, Scrucca 2010

Averaging of the $m^2_{\varphi^x}$ and using the property $\sum_x R_x = 0$ one finds:

$$m^2_{\mathrm{meta}} = 0$$

This gives a no-go theorem.

N = 2 SUSY WITH VECTOR MULTIPLETS

De Wit, Van Proeyen 1984
Hull, Karlhede, Lindstrom, Rocek 1986
Castellani, D'auria, Frè 1991

Lagrangian and transformation laws

An $N = 2$ theory with $n_{\mathcal{V}}$ vectors $\mathcal{V}^a$ is a subcase of $N = 1$ theory with $n_{\mathcal{V}}$ chirals $\Phi^i = (\phi^i, \psi^i, F^i)$ and $n_{\mathcal{V}}$ vectors $V^a = (A^a_\mu, \lambda^a, D^a)$.

The second SUSY transformation has the following general form:

$$\hat{\delta}\Phi^i = \sqrt{2} i f^i_a(\Phi)\hat{\epsilon}\, W^a$$
$$\hat{\delta}V^a = \sqrt{2} i \big(L^a(\Phi) + i f_{bc}{}^a L^b(\Phi) V^c\big)\hat{\bar{\epsilon}}\bar{\theta} + \mathrm{h.c.}$$

This is a $N = \hat{1}$ symmetry if f^i_a is the inverse of $f^a_i \equiv \partial_i L^a$ and:

$$K = \frac{i}{2}\big(\bar{M}_a L^a - \bar{L}^a M_a\big) \qquad X^i_a = f_{ac}{}^b f^i_b L^c$$
$$W = \sqrt{2} e_a L^a, f_{ab}{}^c e_c = 0 \quad H_{ab} = -i M_{ab}$$

This says that the geometry is Special-Kähler with prepotential M and that W is linear in the sections L^a.

The SUSY algebra closes off-shell and without any central charge.

The two SUSY transformations take the form

$$\delta\phi^i = \sqrt{2}\epsilon\psi^i \qquad \hat{\delta}\phi^i = \sqrt{2}\hat{\epsilon} f^i_a \lambda^a$$

$$\delta\psi^i = \sqrt{2}\epsilon F^i + \sqrt{2} i \not{D}\phi^i \bar{\epsilon} \qquad \hat{\delta}\psi^i = \sqrt{2}\hat{\epsilon}\hat{F}^i + \sigma^{\mu\nu}\hat{\epsilon} f^i_a F^a_{\mu\nu}$$

$$\delta A^a_\mu = i\epsilon\sigma_\mu\bar{\lambda}^a - i\lambda^a\sigma_\mu\bar{\epsilon} \qquad \hat{\delta}A^a_\mu = -i\hat{\epsilon}\sigma_\mu \bar{f}^a_{\bar{\imath}}\psi^{\bar{\imath}} + i f^a_i \psi^i \sigma_\mu \hat{\bar{\epsilon}}$$

$$\delta\lambda^a = i\epsilon D^a + \sigma^{\mu\nu}\epsilon F^a_{\mu\nu} \qquad \hat{\delta}\lambda^a = i\hat{\epsilon}\hat{D}^a + \sqrt{2} i f^a_i \not{D}\phi^i \hat{\bar{\epsilon}}$$

where

$$F^i = -\sqrt{2}\bar{f}^{ia}\bar{e}_a + \text{ferm.} \qquad \hat{F}^i = \frac{i}{\sqrt{8}}\bar{f}^{ia}(K_a - \xi_a) + \text{ferm.}$$

$$D^a = -\frac{1}{2}h^{ab}(K_b + \xi_b) + \text{ferm.} \qquad \hat{D}^a = 2ih^{ab}e_b + \text{ferm.}$$

The constant FI terms must all be aligned for consistency with gravity. This leaves a $U(1)_R \subset SU(2)_R$ global symmetry, defined by

$$\vec{P}_a = P_a\vec{v} = \left(2\mathrm{Re}(e_a), 2\mathrm{Im}(e_a), \frac{1}{2}\xi_a\right) \quad N_a = -\frac{1}{2}K_a$$

Vacuum

A vacuum breaks SUSY if $F^i, D^a, \hat{F}^i, \hat{D}^a \neq 0$, i.e. $P^a, N^a \neq 0$:

$$V = F^i \bar{F}_i + \frac{1}{2} D^a D_a = \hat{F}^i \hat{\bar{F}}_i + \frac{1}{2} \hat{D}^a \hat{D}_a = \frac{1}{2} P^a P_a + \frac{1}{2} N^a N_a$$

The two Goldstino fermions and the four sGoldstino real scalars are

$$\eta = \sqrt{2} \bar{F}_i \psi^i + i D_a \lambda^a \quad \hat{\eta} = \sqrt{2} \hat{\bar{F}}_i \psi^i + i \hat{D}_a \lambda^a$$
$$\varphi_\pm = {}^{\mathrm{Re}}_{\mathrm{Im}} (\bar{F}_i \phi^i) \qquad \hat{\varphi}_\pm = {}^{\mathrm{Re}}_{\mathrm{Im}} (\hat{\bar{F}}_i \phi^i)$$

Using complex scalars and doublet fermions, these can be traded for

$$\eta^\alpha = \left(N_a \delta^\alpha_\beta + i P^x_a \sigma^{x\alpha}{}_\beta \right) f^a_i \lambda^{i\beta}$$
$$\varphi^0_\pm = {}^{\mathrm{Re}}_{\mathrm{Im}} (N_a f^a_i \phi^i) \quad \varphi_\pm = {}^{\mathrm{Re}}_{\mathrm{Im}} (P_a f^a_i \phi^i)$$

Average masses

Grisaru, Rocek, Karlhede 1983

Using the relation between $g_{i\bar{\jmath}}$ and h_{ab} and the form of X^i_a one gets

$$\mathrm{str}\, m^2 = 0$$

Goldstino and sGoldstino masses

Using the stationarity condition and the form of X_a^i one finds:

$$m_{\eta^\alpha} = 0 \quad m^2_{\varphi^0_\pm} = 0 \quad m^2_{\varphi_\pm} = S N^a N_a + T \frac{(N^a N_a)^2}{P^b P_b} + 3M^2 \frac{N^a N_a}{P^b P_b} \pm \Delta$$

where

$$S = -\frac{R_{i\bar{\jmath}p\bar{q}} f_a^i f_b^{\bar{\jmath}} f_c^p f_d^{\bar{q}} \, P^a P^b N^c N^d}{P^e P_e \, N^f N_f} \qquad M^2 = \frac{2 X_a^i \bar{X}_{bi} N^a N^b}{N^c N_c}$$

$$T = -\frac{R_{i\bar{\jmath}p\bar{q}} f_a^i f_b^{\bar{\jmath}} f_c^p f_d^{\bar{q}} \, N^a N^b N^c N^d}{(N^e N_e)^2}$$

Metastability

Cremmer, Kounnas, Van Proeyen, Derendinger, Ferrara, de Wit, Girardello 1985
Jacot, Scrucca 2010

Averaging over $m^2_{\varphi_\pm}$ one finds a semipositive result:

$$m^2_{\rm meta} = \Big(S P^a P_a + T N^a N_a + 3M^2 \Big) \frac{N^b N_b}{P^c P_c}$$

This gives a no-go theorem for Abelian or FI-free theories.

N = 2 SUSY WITH HYPER AND VECTOR MULTIPLETS

Hull, Karlhede, Lindstrom, Rocek 1986
Castellani, D'auria, Frè 1991
Jacot, Scrucca 2010

General structure of the theory

An $N = 2$ theory with $n_{\mathcal{H}}$ hypers $\mathcal{H}^k$ and $n_{\mathcal{V}}$ vectors $\mathcal{V}^a$ is a subcase of $N = 1$ theory with $n_{\mathcal{V}}$ chirals Φ^i and $n_{\mathcal{V}}$ vectors V^a.

Hull, Karlhede, Lindstrom, Rocek 1986
Jacot, Scrucca 2010

Average, Goldstino and sGoldstino masses

One finds:

$$\mathrm{str}\, m^2 = 0 \quad \text{and} \quad m_{\eta^{1,2}} = 0 \quad m^2_{\varphi^{1,2,3,4}} = \,?$$

Jacot, Scrucca 2010
Antoniadis, Buican 2010

Metastability

The form of sGoldstino masses does not seem to imply any sharp result. But it was argued that the form of the SUSY algebra forbids its non-linear realization and yields a no-go theorem for $SU(2)_R$-symmetric theories. Are we missing something?

N = 4 SUSY WITH VECTOR MULTIPLETS

General structure of the theory

An $N = 4$ theory with $n_{\mathcal{W}}$ vectors $\mathcal{W}^a$ is a subcase of $N = 1$ theory with $3n_{\mathcal{W}}$ chirals Φ^i and $n_{\mathcal{W}}$ vectors V^a.
This kind of theory is however uniquely fixed by the gauge group and the geometry is necessarily trivial:

$$\mathcal{M}_{\mathrm{N=4}} = \mathbb{R}^{n_{\mathcal{W}}}$$

Vacua

The potential has a fixed very special form, and it turns out that it does not admit any SUSY breaking stationary point. Even before talking about metastability, one can then conclude that it is impossible to break SUSY.

METASTABILITY IN N = 1 SUGRA

Metastability with only chiral multiplets

Gomes-Reino, Scrucca 2006

The case of $N = 1$ theories with only chiral multiplets is easy to study. The geometry becomes Hodge-Kähler and one finds:

$$m^2_{\rm meta} = \Big(R + \frac{2}{3}M_{\rm P}^{-2}\Big)F^i\bar{F}_i - \frac{2}{3}M_{\rm P}^{-2}V$$

For $V \simeq 0$ and $M_{\rm P} \to 1$, the metastability condition is thus:

$$\exists \text{ viable vacua only if } R \gtrsim -\frac{2}{3}$$

This is a necessary and sufficient condition on K and thus the curvature if W and thus the point and the direction of SUSY breaking are arbitrary.

The bound $R \gtrsim -2/3$ explains the difficulty to achieve a good vacuum. It allows to discriminate between viable and non-viable models whenever K is known even if W is unknown, as is often the case in string models.

The universal dilaton

The dilaton S which universally appears in string models is described by the approximate Kähler potential

$$K \simeq -\log(S + \bar{S})$$

The scalar manifold is:

$$\mathcal{M} \simeq \frac{SU(1,1)}{U(1)}$$

The curvature badly violates the bound:

$$R \simeq -2$$

Subleading corrections can modify this result and give a non-coset space. To overcome the bound, these need however to be large.

One then concludes that the domination of SUSY breaking by the dilaton is impossible at weak coupling but possible at strong coupling.

The universal volume modulus

The volume modulus T which also universally appears in string models is described by the approximate Kähler potential

$$K \simeq -3 \log(T + \bar{T})$$

The scalar manifold is:

$$\mathcal{M} \simeq \frac{SU(1,1)}{U(1)}$$

The curvature marginally violates the bound:

$$R \simeq -\frac{2}{3}$$

Subleading corrections can modify this result and give a non-coset space. To overcome the bound, these do not need to be large.

One then concludes that the domination of SUSY breaking by the volume modulus is possible both at large volume and at small volume.

The dilaton and the volume modulus together

Together, the dilaton S and volume modulus T are described by:

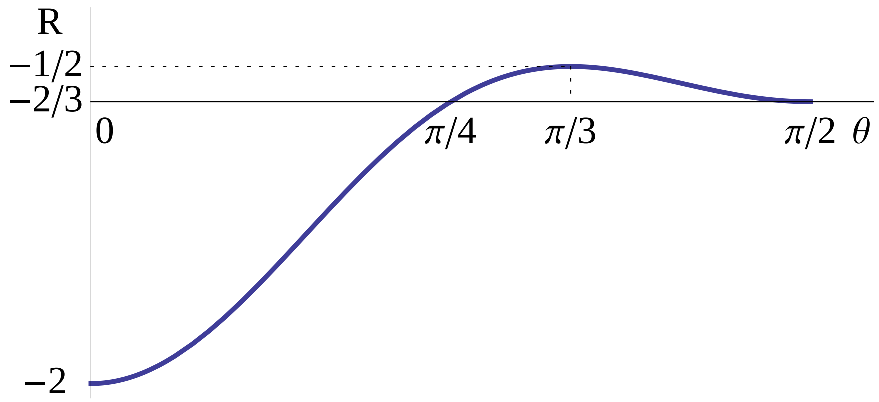
$$K \simeq -\log(S+\bar{S}) - 3\log(T+\bar{T})$$

The scalar manifold is:

$$\mathcal{M} \simeq \frac{SU(1,1)}{U(1)} \times \frac{SU(1,1)}{U(1)}$$

The curvature depends on a Goldstino angle θ and can satisfy the bound:

$$R \simeq -2\cos^4\theta - \frac{2}{3}\sin^4\theta$$



One concludes that the domination of SUSY breaking by the dilaton plus the volume modulus is always possible.

Effect of additional moduli

Extra non-minimal moduli modify the situation and bring more flexibility. In general there is a single S but several T^A, and the Kähler potential is controlled by a single real and degree-one homogeneous function Y:

$$K \simeq -\log(S + \bar{S}) - 3\log Y(T^A + \bar{T}^A)$$

The scalar manifold is:

$$\mathcal{M} \simeq \frac{SU(1,1)}{U(1)} \times \mathcal{M}_{\mathrm{ns}}$$

The curvature depends on some Goldstino angle θ and unit vector v^A:

$$R \simeq -2\cos^4\theta + r_{\mathrm{ns}}(v^A)\sin^4\theta$$

The quantity $r_{ns}(v^A)$ is not quite arbitrary. There always exist a direction for which it is $-2/3$, and the bound can thus be fulfilled. But there may also exist other directions giving larger and thus better values.

Case of singular orbifold models

Gomes-Reino, Scrucca 2006

In models based on singular orbifolds, $\mathcal{M}_{\rm ns}$ is still a coset, and it turns out that:

$$\forall v^a : \; r_{\rm ns}(v^A) \lesssim -\frac{2}{3}$$

The situation is then essentially unchanged.

Case of smooth manifold models

Covi, Gomes-Reino, Gross, Louis, Palma, Scrucca 2008

In models based on smooth manifolds, $\mathcal{M}_{\rm ns}$ is no-longer a coset space, and it turns out that generically:

$$\exists v^A : \; r_{\rm ns}(v^A) \gtrsim -\frac{2}{3}$$

The situation is then somewhat better.

Metastability with also vector multiplets

Gomes-Reino, Scrucca 2007

The case of $N = 1$ theories with also vector multiplets is somewhat more complicated but still rather straightforward to study.

The main qualitative result is again that the gauging by vector multiplets makes the metastability condition milder, and even unfavorably curved manifolds may lead to viable vacua.

Non-trivial examples

Villadoro, Zwirner 2005

A non-trivial example that illustrates how a gauging by vector multiplets can alleviate the metastability constraint is the following:

$$K = -\log(S + \bar{S}) \quad H = S \quad G = \text{shift symmetry}$$

This can admit a metastable de Sitter vacuum.

METASTABILITY IN N = 2 SUGRA

Metastability with only hyper multiplets

Gomes-Reino, Louis, Scrucca 2008

The case of $N = 2$ theories with only hyper multiplets is easy to study. The geometry becomes Quaternionic-Kähler and one finds:

$$m^2_{\mathrm{meta}} = -\frac{1}{9} M_{\mathrm{P}}^{-2} X^U X_U - \frac{16}{9} M_{\mathrm{P}}^{-2} V$$

For $V \simeq 0$ and $M_{\mathrm{P}} \to 1$, this means that there is at least one mode with $m^2 \lesssim -1/9\, X^2$ and metastability is thus impossible to achieve:

$\nexists$ viable vacua at all

This is a cathegorical no-go theorem applying to any theory with only hyper multiplets, where the potential can only originate from a graviphoton gauging.

The universal hyper

The universal hyper $\mathcal{H}$ which always appears in extended string models is described by the approximate Quaternionic-Kähler metric

$$ds^2 = \frac{1}{(\mathrm{Re}S)^2}\Big[\big(d\mathrm{Re}S\big)^2 + \big(d\mathrm{Im}S - \frac{i}{2}C^*\overleftrightarrow{d}C\big)^2\Big] + \frac{1}{\mathrm{Re}S}\big|dC\big|^2$$

The scalar manifold is:

$$\mathcal{M} \simeq \frac{SU(1,2)}{U(1)\times SU(2)}$$

Subleading corrections can modify this result and give a non-coset space. But no matter how large these are, there is no way to get a viable vacuum.

One concludes that the domination of SUSY breaking by the universal hyper is impossible in any regime.

Effect of addition hypers

Extra non-minimal hypers do not help at all.

Metastability with also vector multiplets

The case of $N = 2$ theories with also vector multiplets is much more complicated and no sharp condition has been found so far.

It is however expectable that the main qualitative result should again be that the gauging by vector multiplets makes the metastability condition milder, and that some models may lead to viable vacua.

Non-trivial examples

Frè, Trigiante, Van Proeyen 2003

A non-trivial and pretty unique example that proves that a gauging by vector multiplets can help is the following:

$$\mathcal{M} = \frac{SU(1,1)}{U(1)} \times \frac{SO(2,4)}{SO(2) \times SO(4)} \times \frac{SO(2,4)}{SO(2) \times SO(4)}$$
$$G = SO(2,1) \times SO(3)$$

This can admit a metastable de Sitter vacuum.

METASTABILITY IN N = 4 AND N = 8 SUGRA

Metastability with generic gaugings

Borghese, Roest 2010
Borghese, Linares, Roest 2011

The cases of $N = 4$ and $N = 8$ theories is different and more special. The geometry is fixed to coset spaces with $R \sim M_{\mathrm{P}}^{-2}$:

$$\mathcal{M}_{\mathrm{N=4}} = \frac{SU(1,1)}{U(1)} \times \frac{SO(6,6+n_{\mathcal{W}})}{SO(6) \times SO(6+n_{\mathcal{W}})} \quad \mathcal{M}_{\mathrm{N=8}} = \frac{E_{7(7)}}{SU(8)}$$

From a systematic study of the square masses of the sGoldstini, one finds that avoiding instabilities from them severely constrains the possible gaugings but still allows a small portion of the parameter space.

Non-trivial examples

Borghese, Linares, Roest 2011

No example of model admitting a metastable de Sitter vacuum is known. But some examples of models admitting unstable de Sitter vacua exist, in which the sGoldstini are massless. Are we missing something?

OTHER POSSIBLE ISSUES IN SUSY BREAKING

Instabilities from sGoldstones

Brizi, Scrucca 2011

In theories with vector fields the gauge symmetries are in general broken, and there are would-be Goldstone bosons of same mass as the vectors. Their bosonic partners the sGoldstone scalars have masses controlled by breaking effects and difficult to adjust.

Asking positive sGoldstone masses gives extra metastability conditions, and the sGoldstones can be tachyonic even when the sGoldstini are not. In general, the most dangerous mode is a combination of the two.

Absence of SUSY-breaking stationary points

In some classes of theories, there might be some obstructions against the existence of SUSY breaking vacua, independently of their stability.

CONCLUSIONS AND OUTLOOK

- In N = 1 theories, there exists a sharp necessary condition for the existence of metastable SUSY-breaking vacua, giving constraints. The general case is understood.
- In N = 2 theories, there are similar but stronger constraints for metastable SUSY breaking, giving in some cases no-go theorems. The general case is however not yet fully understood.
- In N = 4 and N = 8 theories, the situation is qualitatively different because the geometry is trivial in the rigid case and due to gravity. No fully conclusive result exists yet.