CONSTRAINTS ON METASTABLE SUSY BREAKING IN N=2 THEORIES

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- Metastability and sGoldstino masses.
- N = 1 models with only chiral multiplets.
- N = 1 models with chiral and vector multiplets.
- N = 2 models with only hyper multiplets.
- N = 2 models with only vector multiplets.

Based on works with M. Gomez-Reino, J.-C. Jacot, J. Louis

METASTABILITY AND SGOLDSTINO MASSES

Vacuum

Vacua are set by constant values of the fields minimizing the potential V. One has $V' = 0$, whereas $V = \Lambda^4$ defines the vacuum energy and $V'' = m^2$ the fluctuation mass matrix.

In SUSY theories, the form of V is constrained. As a result, vacua display special features.

SUSY breaking and metastability

Denef, Douglas 2005
Gomez-Reino, Scrucca 2006

When SUSY is broken, there is a Goldstino fermion which has zero mass. Its partners the sGoldstino bosons have masses controlled by breaking effects and difficult to adjust.

Requiring positive sGoldstino masses leads to metastability conditions. Gravity gives only quantitative modifications compared to the rigid case.

N = 1 MODELS WITH CHIRAL MULTIPLETS

Lagrangian and transformation laws

Zumino 1979
Freedman, Alvarez-Gaumé 1981

An $N = 1$ theory with n_C chirals $\Phi^i = (\phi^i, \psi^i, F^i)$ is defined by:

$$L = \int d^4\theta\, K(\Phi, \bar{\Phi}) + \int d^2\theta\, W(\Phi) + \mathrm{h.c.}$$

This gives a non-linear σ-model on a Kähler target space with metric $g_{i\bar{\jmath}} = K_{i\bar{\jmath}}$ and non-trivial potential V.

The SUSY transformations take the standard form

$$\delta\phi^i = \sqrt{2}\epsilon\psi^i$$
$$\delta\psi^i = \sqrt{2}\epsilon F^i + \sqrt{2}i\partial\!\!\!/\psi^i\bar{\epsilon}$$

where

$$F^i = -g^{i\bar{\jmath}}\bar{W}_{\bar{\jmath}} + \text{ferm.}$$

Vacuum

A vacuum breaks SUSY if $F^i \neq 0$:

$$V = F^i \bar{F}_i$$

The Goldstino fermion and the two sGoldstino real scalars are identified with the following combinations of fields:

$$\eta = \sqrt{2}\bar{F}_i \psi^i$$
$$\varphi_{\pm} = {}^{\mathrm{Re}}_{\mathrm{Im}}(\bar{F}_i \phi^i)$$

Average masses

Grisaru, Rocek, Karlhede 1983

There is a sum rule on the difference between the bosonic and fermionic average masses:

$$\mathrm{str}\, m^2 = 2\, R_{i\bar{\jmath}} F^i \bar{F}^{\bar{\jmath}}$$

Goldstino and sGoldstino masses

Using the stationarity condition one finds:

$$m_\eta = 0 \quad m^2_{\varphi_\pm} = R\, F^i \bar{F}_i \pm \Delta$$

where

$$R = -\frac{R_{i\bar{\jmath}k\bar{l}}\, F^i \bar{F}^{\bar{\jmath}} F^k \bar{F}^{\bar{l}}}{(F^n \bar{F}_n)^2}$$

Metastability

Gomez-Reino, Scrucca 2006

Taking the average of $m^2_{\varphi_\pm}$, one finds an upper bound for the lowest scalar mass eigenvalue, which should be positive for metastability:

$$m^2_{\mathrm{meta}} = R\, F^i \bar{F}_i$$

N = 1 MODELS WITH CHIRAL AND VECTOR MULTIPLETS

Lagrangian and transformation laws

Bagger, Witten 1982
Hull, Karlhede, Lindstrom, Rocek 1986

An $N = 1$ theory with n_C chirals $\Phi^i = (\phi^i, \psi^i, F^i)$ and n_V vectors $V^a = (\lambda^a, A^a_\mu, D^a)$ is defined by:

$$L = \int d^4\theta \Big[K(\Phi,\bar\Phi) + \big(K_a(\Phi,\bar\Phi) + \xi_a\big)V^a + 2X^i_a \bar X_{bi}(\Phi,\bar\Phi)V^a V^b\Big] + \int d^2\theta \Big[\frac{1}{4} f_{ab}(\Phi) W^a W^b + W(\Phi)\Big] + \mathrm{h.c.}$$

This gives a gauged non-linear σ-model on a Kähler target space with metric $g_{i\bar\jmath} = K_{i\bar\jmath}$, gauged isometries generated by Killing vectors X^i_a with Killing potentials K_a and non-trivial potential V.

The gauge couplings and angles are determined by $h_{ab} = \mathrm{Re}(f_{ab})$ and $\theta_{ab} = \mathrm{Im}(f_{ab})$, the matter charges by $Q_{ai}{}^j = i\nabla_i X^j_a$ and the vector masses by $M^2_{ab} = 2X^i_{(a}\bar X_{b)i}$.

The gauge symmetries imply several constraints and act as

$$\delta_{\mathrm{g}} \Phi^i = \Lambda^a X_a^i(\Phi)$$

$$\delta_{\mathrm{g}} V^a = -\frac{i}{2}\left(\Lambda^a - \bar{\Lambda}^a\right)$$

The SUSY transformations read

$$\delta \phi^i = \sqrt{2} \epsilon \psi^i$$

$$\delta \psi^i = \sqrt{2} \epsilon F^i + \sqrt{2} i \not{D} \phi^i \bar{\epsilon}$$

$$\delta A_\mu^a = i \epsilon \sigma_\mu \bar{\lambda}^a - i \lambda^a \sigma_\mu \bar{\epsilon}$$

$$\delta \lambda^a = i \epsilon D^a + \sigma^{\mu\nu} \epsilon F_{\mu\nu}^a$$

where

$$F^i = -g^{i\bar{\jmath}} \bar{W}_{\bar{\jmath}} + \text{ferm.}$$

$$D^a = -\frac{1}{2} h^{ab} (K_b + \xi_b) + \text{ferm.}$$

The constant FI terms are consistent with gravity only if there is also a $U(1)_R$ symmetry.

Vacuum

A vacuum breaks SUSY if $F^i, D^a \neq 0$:

$$V = F^i \bar{F}_i + \frac{1}{2} D^a D_a$$

Stationarity further implies the relation

$$M^2_{ab} D^b - f_{ab}{}^c \theta_{cd} D^b D^d = 2 Q_{ai\bar{\jmath}} F^i \bar{F}^{\bar{\jmath}}$$

The Goldstino fermion and the two sGoldstino real scalars are

$$\eta = \sqrt{2} \bar{F}_i \psi^i + i D_a \lambda^a$$

$$\varphi_\pm = {}^{\mathrm{Re}}_{\mathrm{Im}} (\bar{F}_i \phi^i)$$

Average masses

Grisaru, Rocek, Karlhede 1983

The bosonic and fermionic average masses satisfy a sum rule:

$$\begin{aligned} \mathrm{str}\, m^2 = {} & 2 \big(R_{i\bar{\jmath}} - h_{abi} h^{ac} h^{bd} h_{cd\bar{\jmath}} \big) F^i \bar{F}^{\bar{\jmath}} \\ & + 2 \big(Q_{ai}{}^i - 2 f_{ab}{}^c h^{bd} \theta_{cd} \big) D^a \end{aligned}$$

Goldstino and sGoldstino masses

Using the stationarity condition one finds

$$m_\eta = 0 \quad m^2_{\varphi_\pm} = R\, F^i \bar{F}_i + S\, D^a D_a + T\, \frac{(D^a D_a)^2}{4\, F^i \bar{F}_i} + M^2 \frac{D^a D_a}{F^i \bar{F}_i} \pm \Delta$$

where

$$R = -\frac{R_{i\bar{\jmath}k\bar{l}}\, F^i \bar{F}^{\bar{\jmath}} F^k \bar{F}^{\bar{l}}}{(F^n \bar{F}_n)^2} \quad S = \frac{h_{aci} h^{cd} h_{db\bar{\jmath}}\, F^i \bar{F}^{\bar{\jmath}} D^a D^b}{F^n \bar{F}_k\, D^c D_c}$$

$$T = \frac{h_{abi} h_{cb}{}^{i} D^a D^b D^c D^d}{(D^e D_e)^2} \quad M^2 = \frac{2 X^i_a \bar{X}_{bi} D^a D^b}{D^c D_c}$$

Metastability

Gomez-Reino, Scrucca 2007

Averaging over $m^2_{\varphi_\pm}$, one finds:

$$m^2_{\mathrm{meta}} = R\, F^i \bar{F}_i + \left(S\, F^i \bar{F}_i + \frac{1}{4} T\, D^a D_a + M^2 \right) \frac{D^b D_b}{F^j \bar{F}_j}$$

N = 2 MODELS WITH HYPER MULTIPLETS

Lagrangian and transformation laws

Alvarez-Gaumé, Freedman 1981
Hull, Karlhede, Lindstrom, Rocek 1986

An $N = 2$ theory with $n_{\mathcal{H}}$ hypers H^k is a subcase of $N = 1$ theory with $2n_{\mathcal{H}}$ chirals $Q^u = (q^u, \chi^u, F^u)$.

The second SUSY transformation has the general form

$$\hat{\delta} Q^u = \frac{1}{2}\bar{D}^2\Big(\bar{N}^u(Q, \bar{Q})(\hat{\epsilon}\theta + \bar{\hat{\epsilon}}\bar{\theta})\Big)$$

This is an $\hat{N} = 1$ symmetry if $\Omega_{uv} \equiv \nabla_u N_v$ and $X^u \equiv \bar{\Omega}^{uv} W_v$ satisfy

$$\Omega_{(uv)} = 0 \quad \nabla_w \Omega_{uv} = 0 \quad \nabla_{\bar{w}} \Omega_{uv} = 0 \quad \bar{\Omega}^u{}_w \Omega^w{}_v = -\delta^u_v$$

$$\nabla_{\bar{w}} X^u = 0 \quad \nabla_{(u} X_{\bar{v})} = 0 \quad \bar{\Omega}^u{}_{\bar{w}} \nabla_{\bar{v}} \bar{X}^{\bar{w}} - \bar{\Omega}^w{}_{\bar{v}} \nabla_w X^u = 0$$

This says that the geometry is Hyper-Kähler with three complex structures defined out of Ω_{uv} and that X^u is a triholomorphic Killing vector.

The SUSY algebra closes on-shell with a global central charge symmetry:

$$\delta_c Q^u = \lambda X^u(Q)$$

The two SUSY transformations take the form

$$\begin{aligned} \delta q^u &= \sqrt{2}\epsilon\chi^u & \hat{\delta} q^u &= -\sqrt{2}\bar{\Omega}^u{}_{\bar{v}}\hat{\bar{\epsilon}}\,\bar{\chi}^v \\ \delta\chi^u &= \sqrt{2}\epsilon F^u + \sqrt{2} i \partial\!\!\!/ q^u \bar{\epsilon} & \hat{\delta}\chi^u &= \sqrt{2}\hat{\epsilon}\hat{F}^u + \sqrt{2} i \bar{\Omega}^u{}_{\bar{v}} \partial\!\!\!/ \bar{q}^{\bar{v}} \hat{\bar{\epsilon}} \end{aligned}$$

where

$$F^u = \bar{\Omega}^u{}_{\bar{v}} \bar{X}^{\bar{v}} + \text{ferm.} \quad \hat{F}^u = -X^u + \text{ferm.}$$

There is a global $SU(2)_R$ symmetry rotating the complex structures:

$$\vec{J}^U{}_V = \left(\begin{pmatrix} 0 & \bar{\Omega}^u{}_{\bar{v}} \\ \Omega^{\bar{u}}{}_v & 0 \end{pmatrix}, \begin{pmatrix} 0 & i\bar{\Omega}^u{}_{\bar{v}} \\ -i\Omega^{\bar{u}}{}_v & 0 \end{pmatrix}, \begin{pmatrix} i\delta^u_v & 0 \\ 0 & -i\delta^{\bar{u}}_{\bar{v}} \end{pmatrix} \right)$$

The source of SUSY breaking is

$$X^U = \begin{pmatrix} X^u \\ \bar{X}^{\bar{u}} \end{pmatrix}$$

Vacuum

A vacuum breaks SUSY if F^u, $\hat{F}^u \neq 0$, i.e. $X^U \neq 0$:

$$V = F^u \bar{F}_u = \hat{F}^u \hat{\bar{F}}_u = \frac{1}{2} X^U X_U$$

The two Goldstino fermions and the four sGoldstino real scalars are given by the combinations

$$\eta = \sqrt{2} \bar{F}_u \chi^u \qquad \hat{\eta} = \sqrt{2} \hat{\bar{F}}_u \chi^u$$

$$\varphi_\pm = {}^{\mathrm{Re}}_{\mathrm{Im}} (\bar{F}_u q^u) \quad \hat{\varphi}_\pm = {}^{\mathrm{Re}}_{\mathrm{Im}} (\hat{\bar{F}}_u q^u)$$

Using real scalars and symplectic-Majorana fermions, these modes can be reorganized as doublet and singlet plus triplet of $SU(2)_R$:

$$\eta^\alpha = \left(g_{UV} \delta^\alpha_\beta + i J^x_{UV} \sigma^{x\alpha}{}_\beta \right) X^V \chi^{U\beta}$$

$$\varphi^0 = g_{UV} X^V q^U \quad \varphi^x = J^x_{UV} X^V q^U$$

Average masses

Grisaru, Rocek, Karlhede 1983

Due to the property $R_{u\bar{v}} = 0$ one finds:

$$\mathrm{str}\, m^2 = 0$$

Goldstino and sGoldstino masses

Using the stationarity condition one finds:

$$m_{\eta^\alpha} = 0 \quad m^2_{\varphi^0} = 0 \quad m^2_{\varphi^x} = -R_x\, X^U X_U$$

where

$$R_x = \frac{R_{UVMN} X^U (J^x X)^V X^M (J^x X)^N}{(X^K X_K)^2}$$

Metastability

Gomez-Reino, Louis, Scrucca 2009
Jacot, Scrucca 2010

Averaging of the $m^2_{\varphi^x}$ and using the property $\sum_x R_x = 0$ one finds:

$$m^2_{\mathrm{meta}} = 0$$

N = 2 MODELS WITH VECTOR MULTIPLETS

De Wit, Van Proeyen 1984
Hull, Karlhede, Lindstrom, Rocek 1986
Castellani, D'auria, Fre 1991

Lagrangian and transformation laws

An $N = 2$ theory with $n_{\mathcal{V}}$ vectors $\mathcal{V}^a$ is a subcase of $N = 1$ theory with $n_{\mathcal{V}}$ chirals $\Phi^i = (\phi^i, \psi^i, F^i)$ and $n_{\mathcal{V}}$ vectors $V^a = (A^a_\mu, \lambda^a, D^a)$.

The second SUSY transformation has the following general form:

$$\hat{\delta}\Phi^i = \sqrt{2} i f^i_a(\Phi) \hat{\epsilon} W^a$$

$$\hat{\delta} V^a = -\sqrt{2} i \big(\bar{L}^a(\bar{\Phi}) - i f_{bc}{}^a \bar{L}^b(\bar{\Phi}) V^c\big) \hat{\epsilon}\theta + \mathrm{h.c.}$$

This is a $\hat{N} = 1$ symmetry if $f^a_i \equiv \partial_i L^a$ is the inverse of f^i_a and:

$$K = \frac{i}{2}\big(\bar{M}_a L^a - \bar{L}^a M_a\big) \qquad X^i_a = f_{ac}{}^b f^i_b L^c$$

$$W = \sqrt{2} e_a L^a, f_{ab}{}^c e_c = 0 \quad f_{ab} = -i M_{ab}$$

This says that the geometry is Special-Kähler with prepotential M and that W is linear in the sections L^a.

The SUSY algebra closes off-shell and without any central charge.

The two SUSY transformations take the form

$$\delta\phi^i = \sqrt{2}\epsilon\psi^i \qquad \hat{\delta}\phi^i = \sqrt{2}\hat{\epsilon} f^i_a \lambda^a$$

$$\delta\psi^i = \sqrt{2}\epsilon F^i + \sqrt{2} i \not{D}\phi^i \bar{\epsilon} \qquad \hat{\delta}\psi^i = \sqrt{2}\hat{\epsilon}\hat{F}^i + \sigma^{\mu\nu}\hat{\epsilon} f^i_a F^a_{\mu\nu}$$

$$\delta A^a_\mu = i\epsilon\sigma_\mu\bar{\lambda}^a - i\lambda^a\sigma_\mu\bar{\epsilon} \qquad \hat{\delta}A^a_\mu = -i\hat{\epsilon}\sigma_\mu \bar{f}^a_{\bar{\imath}}\psi^{\bar{\imath}} + i f^a_i \psi^i \sigma_\mu \hat{\bar{\epsilon}}$$

$$\delta\lambda^a = i\epsilon D^a + \sigma^{\mu\nu}\epsilon F^a_{\mu\nu} \qquad \hat{\delta}\lambda^a = i\hat{\epsilon}\hat{D}^a + \sqrt{2} i f^a_i \not{D}\phi^i \hat{\bar{\epsilon}}$$

where

$$F^i = -\sqrt{2}\bar{f}^{ia}\bar{e}_a + \text{ferm.} \qquad \hat{F}^i = \frac{i}{\sqrt{8}}\bar{f}^{ia}(K_a - \xi_a) + \text{ferm.}$$

$$D^a = -\frac{1}{2}h^{ab}(K_b + \xi_b) + \text{ferm.} \qquad \hat{D}^a = 2ih^{ab}e_b + \text{ferm.}$$

The constant FI terms must all be aligned for consistency with gravity. This leaves a $U(1)_R \subset SU(2)_R$ global symmetry, defined by

$$\vec{P}_a = P_a\vec{v} = \left(2\mathrm{Re}(e_a), 2\mathrm{Im}(e_a), \frac{1}{2}\xi_a\right) \quad N_a = -\frac{1}{2}K_a$$

Vacuum

A vacuum breaks SUSY if $F^i, D^a, \hat{F}^i, \hat{D}^a \neq 0$, i.e. $P^a, N^a \neq 0$:

$$V = F^i \bar{F}_i + \frac{1}{2} D^a D_a = \hat{F}^i \hat{\bar{F}}_i + \frac{1}{2} \hat{D}^a \hat{D}_a = \frac{1}{2} P^a P_a + \frac{1}{2} N^a N_a$$

The two Goldstino fermions and the four sGoldstino real scalars are

$$\eta = \sqrt{2} \bar{F}_i \psi^i + i D_a \lambda^a \qquad \hat{\eta} = \sqrt{2} \hat{\bar{F}}_i \psi^i + i \hat{D}_a \lambda^a$$

$$\varphi_\pm = {}^{\mathrm{Re}}_{\mathrm{Im}} (\bar{F}_i \phi^i) \qquad \hat{\varphi}_\pm = {}^{\mathrm{Re}}_{\mathrm{Im}} (\hat{\bar{F}}_i \phi^i)$$

Using complex scalars and doublet fermions, these can be traded for

$$\eta^\alpha = \left(N_a \delta^\alpha_\beta + i P^x_a \sigma^{x\alpha}{}_\beta \right) f^a_i \lambda^{i\beta}$$

$$\varphi^0_\pm = {}^{\mathrm{Re}}_{\mathrm{Im}} (N_a f^a_i \phi^i) \quad \varphi_\pm = {}^{\mathrm{Re}}_{\mathrm{Im}} (P_a f^a_i \phi^i)$$

Average masses

Grisaru, Rocek, Karlhede 1983

Using the relation between $g_{i\bar{\jmath}}$ and h_{ab} and the form of X^i_a one gets

$$\mathrm{str}\, m^2 = 0$$

Goldstino and sGoldstino masses

Using the stationarity condition as well as the relations $X^i_a L^a = 0$ and $X^i_a \bar{L}^a = i\bar{f}^{ia} N_a$ one finds:

$$m_{\eta^\alpha} = 0 \quad m^2_{\varphi^0_\pm} = 0 \quad m^2_{\varphi_\pm} = S N^a N_a + T\frac{(N^a N_a)^2}{P^b P_b} + 3M^2 \frac{N^a N_a}{P^b P_b} \pm \Delta$$

where

$$S = -\frac{R_{i\bar{\jmath}p\bar{q}} f^i_a \bar{f}^{\bar{\jmath}}_b f^p_c \bar{f}^{\bar{q}}_d\, P^a P^b N^c N^d}{P^e P_e\, N^f N_f} \quad M^2 = \frac{2 X^i_a \bar{X}_{bi}\, N^a N^b}{N^c N_c}$$

$$T = -\frac{R_{i\bar{\jmath}p\bar{q}} f^i_a \bar{f}^{\bar{\jmath}}_b f^p_c \bar{f}^{\bar{q}}_d\, N^a N^b N^c N^d}{(N^e N_e)^2}$$

Metastability

Cremmer, Kounnas, Van Proeyen, Derendinger, Ferrara, de Wit, Girardello 1985
Jacot, Scrucca 2010

Averaging over $m^2_{\varphi_\pm}$, one finds:

$$m^2_{\rm meta} = \left(S P^a P_a + T N^a N_a + 3M^2\right)\frac{N^b N_b}{P^c P_c}$$

CONCLUSIONS AND OUTLOOK

- In $N = 1$ theories, there exists a sharp necessary condition for the existence of metastable SUSY-breaking vacua. The general case is understood.
- In $N = 2$ theories, there are similar but stronger constraints for metastable SUSY breaking. The general case is however not yet understood.
- In some cases, one finds no-go theorems. These seem to fit with the obstructions recently found in some cases by Antoniadis and Buican against defining a low-energy theory for just the Goldstini.