STABILITY OF NON-SUSY MINKOWSKI VACUA IN SUPERGRAVITY

Claudio Scrucca

University of Neuchâtel

- SUSY breaking in SUGRA scenarios.
- SUGRA models with chiral multiplets.
- Flatness and stability constraints.
- Factorizable scalar manifolds.
- Homogeneous scalar manifolds.
- Moduli in string models.

Gomez-Reino, Scrucca (hep-th/0602246)
Gomez-Reino, Scrucca (work in progress)

SUSY BREAKING AND SUGRA

The standard paradigm is that SUSY is spontaneously broken at M in a hidden sector with fields Φ_i and transmitted only indirectly to the visible sector with fields Q_a, through gravity interactions suppressed by $M_{\rm P}$.

Assuming $M \ll M_{\rm P}$, one can use a SUGRA description. The soft terms parametrizing the effect of SUSY breaking in the visible sector originate from operators mixing the Φ_i to the Q_a and suppressed by powers of $M_{\rm P}$, so that:

$$m_{\rm soft} \sim \frac{M^2}{M_{\rm P}}$$

Chamseddine, Arnowitt, Nath
Barbieri, Ferrara, Savoy
Hall, Lykken, Weinberg

The main delicate features that are needed in order to get a satisfactory situation are:

- Soft terms with $m_{\rm soft} \sim M_{\rm EW}$ and peculiarities.
- Cosmological constant with $M_{\rm CC} \ll M_{\rm EW}$.
- Hidden scalars with $m > M_{\rm EW}$ and stable.

CHIRAL SUGRA MODELS

A SUGRA theory with n chiral multiplets Φ_i is specified by a real function G. Setting $M_{\rm P} = 1$, this can be written as

$$G(\Phi_i, \Phi_i^\dagger) = K(\Phi_i, \Phi_i^\dagger) + \log W(\Phi_i) + \log \bar{W}(\Phi_i^\dagger)$$

This decomposition is however ambiguous, due to the Kähler symmetry changing $K \to K + F + \bar{F}$ and $W \to e^{-F} W$.

Mixed holomorphic/antiholomorphic derivatives of G depend only on K and define a Kähler geometry for the manifold parametrized by the scalars ϕ^i. The metric, the Chirstoffel connection and the Riemann tensor are given by:

$$g_{i\bar{j}} = G_{i\bar{j}}$$
$$\Gamma^k_{ij} = G^k_{ij} \,, \quad \Gamma^{\bar{k}}_{\bar{i}\bar{j}} = G^{\bar{k}}_{\bar{i}\bar{j}}$$
$$R_{i\bar{j}p\bar{q}} = G_{i\bar{j}p\bar{q}} - G^r_{ip} G_{\bar{j}\bar{q}r}$$

Pure holomorphic or antiholomorphic derivatives of G depend instead also on W, and determine the way SUSY is broken. In particular, the auxiliary fields F^i are given simply by:

$$F^i = e^{G/2} G^i$$

Cremmer, Julia, Scherk, Ferrara, Girardello, Van Nieuwenhuizen
Bagger, Witten

The scalars ϕ^i have a wave function factor given by $Z_{i\bar{j}} = g_{i\bar{j}}$ and a potential, which determines their vev and mass and controls spontaneous SUSY breaking, of the form:

$$V = e^G \left(G^k G_k - 3\right)$$

The flatness condition of vanishing cosmological constant is that $V = 0$ on the vacuum and implies that at that point:

$$g_{i\bar{j}}\, G^i G^{\bar{j}} = 3$$

The first derivatives of the potential controlling its variations can be computed as $\delta_i = \nabla_i V$ and are given by:

$$\delta_i = e^G \Big(G_i + G^k \nabla_i G_k\Big)$$

The stationarity conditions defining extrema of the potential are $\delta_i = 0$ and imply:

$$G_i + G^k \nabla_i G_k = 0$$

The two types of second derivatives of the potential controlling the squared masses can be computed as $m^2_{i\bar{j}} = \nabla_i \nabla_{\bar{j}} V$ and $m^2_{ij} = \nabla_i \nabla_j V$, and one easily finds:

$$m^2_{i\bar{j}} = e^G \Big(g_{i\bar{j}} + \nabla_i G^k\, \nabla_{\bar{j}} G_k - R_{i\bar{j}p\bar{q}}\, G^p G^{\bar{q}}\Big)$$
$$m^2_{ij} = e^G \Big(\nabla_i G_j + \nabla_j G_i + \frac{1}{2} G^k \big\{\nabla_i, \nabla_j\big\} G_k\Big)$$

The stability condition is that the $2n$-dimensional squared-mass matrix is positive definite:

$$m^2 = \begin{pmatrix} m^2_{i\bar{j}} & m^2_{ij} \\ m^2_{\bar{i}\bar{j}} & m^2_{\bar{i}j} \end{pmatrix} > 0$$

The only systematic way to determine the constraints that this implies is to study the mass eigenvalues.

The fermions ψ^i split into 1 Goldstino combination $\psi = G_i \psi^i$ and $n-1$ physical combinations $\tilde{\psi}^i$. They have wave-function factor $\tilde{Z}_{i\bar{j}} = g_{i\bar{j}}$, and their mass is encoded in:

$$\tilde{m}_{ij} = e^G \Big(\nabla_i G_j + \frac{1}{3} G_i G_j\Big)$$

More precisely, the $2n$-dimensional mass matrix is given by

$$\tilde{m} = \begin{pmatrix} 0 & \tilde{m}_{ij} \\ \tilde{m}_{\bar{i}\bar{j}} & 0 \end{pmatrix}$$

The graviton and gravitino $h^{\mu\nu}$ and ψ^μ have wave-function factors $Z_{\rm gra} = 1$ and $Z_{3/2} = 1$, and masses given by:

$$m^2_{\rm gra} = 0 \,, \quad m_{3/2} = e^{G/2}$$

The supertrace of the squared mass matrix for the whole theory is found to be:

$$\mathrm{STr} M^2 \;=\; 2\, e^G \Big(n - 1 - R_{i\bar{j}}\, G^i G^{\bar{j}}\Big)$$

Cremmer, Ferrara, Girardello, Van Proeyen

FLATNESS AND STABILITY CONSTRAINTS

It would be interesting to understand better what flatness and stability imply on G. More precisely, it would be very useful to derive a condition concerning only K and the geometry, independently of W and the mechanism of SUSY breaking.

Our strategy is to look for some simpler condition that is only necessary and in general not sufficient for having $m^2 > 0$ for the scalars.

The crucial point is that all the upper-left submatrices of m^2 must also be positive definite. In particular, the n-dimensional submatrix $m^2_{i\bar{j}}$ should be positive definite:

$$m^2_{i\bar{j}} > 0$$

This condition means that $\forall z^i$ one must have $m^2_{i\bar{j}}\, z^i \bar{z}^{\bar{j}} > 0$. One can then look for a specific z^i that leads to a particularly simple condition. The right choice is $z^i = G^i$, for which:

$$m^2_{i\bar{j}}\, G^i G^{\bar{j}} = e^G \Big(6 - R_{i\bar{j}p\bar{q}}\, G^i G^{\bar{j}} G^p G^{\bar{q}}\Big)$$

The corresponding necessary condition $m^2_{i\bar{j}}\, G^i G^{\bar{j}} > 0$ reduces then to the extremely simple curvature constraint:

$$R_{i\bar{j}p\bar{q}}\, G^i G^{\bar{j}} G^p G^{\bar{q}} < 6$$

Note that the special direction $z^i = G^i$ considered to derive $m^2_{i\bar{j}}\, G^i G^{\bar{j}} > 0$ for the scalars corresponds to the direction of the Goldstino for the fermions, and $\tilde{m}_{ij}\, G^i G^j = 0$.

Summarizing, a stationary point can lead to a satisfactory situation only if the following two conditions are satisfied:

Flatness: $g_{i\bar{j}}\, G^i G^{\bar{j}} = 3$ (necessary & sufficient)

Stability: $R_{i\bar{j}p\bar{q}}\, G^i G^{\bar{j}} G^p G^{\bar{q}} < 6$ (necessary)

The tensors $g_{i\bar{j}}$ and $R_{i\bar{j}p\bar{q}}$ depend only on K and characterize the geometry. The vectors G^i depend also on W and control the SUSY breaking direction, since $G^i = F^i/m_{3/2}$.

For a given geometry, the flatness condition determines the overall amount of SUSY breaking, and the stability condition constrains its direction to lie with a certain cone.

To solve the conditions, one must first determine the direction that minimizes $R_{i\bar{j}p\bar{q}}\, G^i G^{\bar{j}} G^p G^{\bar{q}}$ for fixed $g_{i\bar{j}}\, G^i G^{\bar{j}}$, and then check how far apart from it the former stays small enough.

This variational problem is hard to solve in full generality. However, it is possible to obtain very simple and strong results for the subclass of models based on spaces that are factorizable or homogeneous.

FACTORIZABLE SPACES

Suppose that $\mathcal{M}$ is a product of n 1-dimensional manifolds. The function K splits then into a sum of terms depending on a single field, while W can instead still be arbitrary:

$$K = \sum_k K^{(k)}(\Phi_k, \Phi_k^\dagger)$$

$$W = W(\Phi_1, \ldots, \Phi_n)$$

This assumption represents a Kähler-invariant constraint on G, implying that all its mixed holomorphic/antiholomorphic off-diagonal derivatives vanish.

In this situation, $g_{i\bar{j}}$ and $R_{i\bar{j}p\bar{q}}$ become both diagonal and have only n non-vanishing components. This simplifies the problem sufficiently much to solve it exactly.

The non-vanishing components of the metric are $g_{i\bar{i}} = G_{i\bar{i}}$, and those of the curvature tensor are related to these by:

$$R_{i\bar{i}i\bar{i}} = R_i \, g_{i\bar{i}}^2$$

where the crucial parameters are the n curvature scalars R_i associated to each complex scalar field:

$$R_i = \frac{G_{ii\bar{i}\bar{i}}}{G_{i\bar{i}}^2} - \frac{G_{ii\bar{i}} G_{i\bar{i}\bar{i}}}{G_{i\bar{i}}^3}$$

The two flatness and stability conditions derived before then simplify to the following expressions:

$$\text{Flatness: } \sum_k \Theta_k^2 = 1$$

$$\text{Stability: } \sum_k R_k\, \Theta_k^4 < \frac{2}{3}$$

where

$$\Theta_i = \sqrt{\frac{G^i G_i}{3}} = \sqrt{\frac{g_{i\bar{i}} G^i G^{\bar{i}}}{3}} \quad \text{(no sum)}$$

It is now straightforward to show that when $R_i > 0$ these two constraints can admit solutions only if the following curvature bound is satisfied:

$$\sum_k R_k^{-1} > \frac{3}{2}$$

The SUSY breaking direction must lie in a certain Goldstino cone specified by the curvature scalars. Its axis is the preferred direction minimizing the quartic curvature form:

$$\Theta_i^0 = \sqrt{\frac{R_i^{-1}}{\sum_k R_k^{-1}}}$$

Its solid angle grows with the excess of the effective inverse curvature $\sum_k R_k^{-1}$ with respect to the threshold $3/2$.

More precisely, the allowed configurations correspond to a bounded domain in the space of variables:

$$\Theta_i \in \left[\Theta_i^-, \Theta_i^+\right]$$

One easily finds:

$$\Theta_i^+ = \begin{cases} \sqrt{\dfrac{R_i^{-1} + \sqrt{\frac{2}{3} R_i^{-1} \Big(\sum_{k\neq i} R_k^{-1}\Big)\Big(\sum_k R_k^{-1} - \frac{3}{2}\Big)}}{\sum_k R_k^{-1}}}\,, & R_i^{-1} < \dfrac{3}{2} \\ 1\,, & R_i^{-1} > \dfrac{3}{2} \end{cases}$$

$$\Theta_i^- = \begin{cases} \sqrt{\dfrac{R_i^{-1} - \sqrt{\frac{2}{3} R_i^{-1} \Big(\sum_{k\neq i} R_k^{-1}\Big)\Big(\sum_k R_k^{-1} - \frac{3}{2}\Big)}}{\sum_k R_k^{-1}}}\,, & \sum_{k\neq i} R_k^{-1} < \dfrac{3}{2} \\ 0\,, & \sum_{k\neq i} R_k^{-1} > \dfrac{3}{2} \end{cases}$$

The most important qualitative result is that the direction of SUSY breaking must align more along the directions of low curvature than those of high curvature.

A given Θ_i can become as large as 1 only if its curvature satisfies $R_i^{-1} > 3/2$, and as low as 0 only if the curvatures of the remaining fields satisfy $\sum_{k\neq i} R_k^{-1} > 3/2$.

The relevance of a particular chiral multiplet Φ_i for SUSY breaking depends thus on the size of the corresponding inverse curvature R_i^{-1} with respect to the threshold value $3/2$.

HOMOGENEOUS SPACES

Suppose that $\mathcal{M}$ is a coset space G/H, where G is a group of global isometries and H a local stability group. The function K has then some special form, but W can be arbitrary:

$$K = K^{(G/H)}(\Phi_1, \Phi_1^\dagger, \ldots, \Phi_n, \Phi_n^\dagger)$$

$$W = W(\Phi_1, \ldots, \Phi_n)$$

The metric and curvature tensors are G-invariant and there are relations among their components. The problem simplifies then again sufficiently much to be able to solve it exactly.

For all the possible coset Kähler manifolds, the components of the metric and the Rieman tensor are somehow related:

$$R_{i\bar{j}p\bar{q}} \text{ related to } g_{r\bar{s}}$$

Calabi, Vesentini

The crucial ingredients are in this case the overall scale $R_{\rm all}$ of the curvature and the group structure of the space.

Maximally symmetric spaces

Suppose that the model has n fields Φ_i and

$$R_{i\bar{j}p\bar{q}} = \frac{R_{\rm all}}{2}\Big(g_{i\bar{j}}g_{p\bar{q}} + g_{i\bar{q}}g_{p\bar{j}}\Big)$$

Then, the two conditions become simply

$$\text{Flatness:}\ \ \Theta^2 = 1$$

$$\text{Stability:}\ \ R_{\rm all}\,\Theta^4 < \frac{2}{3}$$

where

$$\Theta = \sqrt{\frac{G^k G_k}{3}} = \sqrt{\frac{g_{k\bar l} G^k G^{\bar l}}{3}}$$

The situation is then as for 1 field with $R = R_{\rm all}$:

$$R_{\rm all}^{-1} > \frac{3}{2}$$

Next-to-maximally symmetric spaces

Suppose that the model has $n = p\,(p+q)$ fields $\Phi_{i\alpha}$ with

$$g_{i\alpha\,\bar j\bar\beta} = h_{i\bar j}\,\tilde h_{\alpha\bar\beta}$$

$$R_{i\alpha\,\bar j\bar\beta\,p\gamma\,\bar q\bar\delta} = \frac{R_{\rm all}}{2}\Big(h_{i\bar j}\,h_{p\bar q}\,\tilde h_{\alpha\bar\delta}\,\tilde h_{\gamma\bar\beta} + h_{i\bar q}\,h_{p\bar j}\,\tilde h_{\alpha\bar\beta}\,\tilde h_{\gamma\bar\delta}\Big)$$

The two conditions reduce then simply to

$$\text{Flatness:}\ \ \sum_k \Theta_k^2 = 1$$

$$\text{Stability:}\ \ \sum_k R_{\rm all}\,\Theta_k^4 < \frac{2}{3}$$

where

$$\Theta_i = {\rm Eig}_i^{1/2}\Big(\frac{G_I{}^{\alpha} G_{\bar J\alpha}}{3}\Big) = {\rm Eig}_i^{1/2}\Big(\frac{e_{Ik} e_{\bar J\bar l}\tilde h_{\alpha\bar\beta} G^{k\alpha} G^{\bar l\bar\beta}}{3}\Big)$$

The situation is then as for p fields with $R_i = R_{\rm all}$:

$$R_{\rm all}^{-1} > \frac{3}{2p}$$

MODULI IN STRING MODELS

In string models, a natural candidate for the hidden sector is the universal sector containing the neutral moduli controlling to the coupling strength and the compactification geometry.

Kaplunovky, Louis

The simplest geometry that can occur for n moduli fields Φ_i is described by

$$K = -\sum_k n_k \ln\Big(\Phi_k + \Phi_k^\dagger\Big)$$

Witten

This describes a factorized and also homogeneous space:

$$\mathcal{M} = \bigotimes_k \frac{SU(1,1)}{U(1)}$$

The curvature scalars are:

$$R_i = \frac{2}{n_i} \quad \Rightarrow \quad R_i^{-1} = \frac{n_i}{2}$$

The curvature condition $\sum_k R_k^{-1} > 3/2$ implies then:

$$\sum_k n_k > 3$$

The Goldstino cone is also fixed by the n_i, and its axis is:

$$\Theta_i^0 = \sqrt{\frac{n_i}{\sum_k n_k}}$$

There exist several relevant generalizations of this geometry involving extra moduli and matter fields. They are no longer factorizable but still homogeneous.

For instance, there are situations with $p(p+q)$ fields in total, p^2 moduli fields ϕ_{ij} and $p\,q$ matter fields X_{ia}, described by:

$$K = -\,n_{\rm all} \ln \det\Big(\Phi_{ij} + \Phi^\dagger_{ij} - \sum_a X^\dagger_{ia} X_{ja}\Big)$$

Ellis, Kounnas, Nanopoulos

Ferrara, Kounnas, Porrati

This corresponds to an $p(p+q)$-dimensional homogeneous space of the type:

$$\mathcal{M} = \frac{SU(p,p+q)}{U(1)\times SU(p)\times SU(p+q)}$$

One can show that this is of the next-to-maximally symmetric type, with overall curvature scale given by:

$$R_{\rm all} = \frac{2}{n_{\rm all}}$$

The curvature condition $R_{\rm all}^{-1} > 3/(2p)$ implies then:

$$n_{\rm all} > \frac{3}{p}$$

This shows that the addition of extra off-diagonal moduli or matter fields does not change the situation found for standard factorized moduli.

Dilaton and volume moduli

The most relevant moduli are the dilaton S, controlling the coupling, and the global Kähler modulus T, controlling the volume of the compact dimensions. One finds:

$$n_S = 1 \ , \ \ n_T = 3$$

Taking each field separately, the curvature bound is always violated. To fulfill the bound one would need corrections. These should be large for S, but could be small for T.

Keeping both fields, the curvature bound is instead fulfilled. But T must dominate over S, and the Goldstino angle θ is constrained to the quadrant $[0, \pi/4]$. This implies that:

$$\frac{|F_S|}{\mathrm{Re}S} < \frac{|F_T|}{\sqrt{3}\,\mathrm{Re}T}$$

This demonstrates in a very robust way that the scenario where S dominates over T is impossible to realize, at least in the controllable limit where both are large.

CONCLUSIONS AND OUTLOOK

- In SUGRA models with only chiral multiplets, there exist a necessary condition for stability that strongly constrains the curvature of the geometry and the SUSY breaking direction.
- The form of these constraints can be worked out in full detail for factorizable and homogeneous geometries, as those occurring for instance in the moduli sector of string models.
- It would be of great interest to generalize this study to models involving also vector multiplets gauging isometries of the scalar manifold.