

FIELD AND STRING THEORIES WITH SCHERK-SCHWARZ SYMMETRY BREAKING

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Masiero, S., Serone, Silvestrini : [hep-ph/0107201](#)

S., Serone, Silvestrini, Zwirner : [hep-th/0110073](#)

S., Serone : [hep-th/0107159](#)

- Scherk-Schwarz mechanism of supersymmetry and gauge symmetry breaking.
- Protected operators with finite radiative corrections. Softness versus non-locality.
- Chiral models from freely acting orbifolds.
- Diseases of field theory orbifolds. Anomalies and Fayet-Iliopoulos terms.
- String models and new structures.
- Outlook.

SCHERK-SCHWARZ BREAKING

Consider a field theory in D dimensions, with a symmetry G . One can break this symmetry by compactifying 1 dimension and imposing boundary conditions twisted through $g \in G$:

$$\phi(y + 2\pi R) = g \phi(y) g^{-1}$$

In the $D - 1$ dimensional effective theory below $M_c = 1/R$, the symmetry group is broken down to $G' = \{g' \in G | [g', g] = 0\}$.

The breaking mechanism is non-local.

Scherk, Schwarz

Consider a continuous twist of the form $g(\mathbf{a}) = e^{2\pi i \mathbf{a} T}$. A D -dimensional massless field with charge q under T can be decomposed in $D - 1$ -dimensional KK modes as:

$$\phi(x, y) = \sum_n \phi_n(x) e^{i \frac{(n+q\mathbf{a})y}{R}}$$

The mass of these modes has a q -dependent shift:

$$m_n = \frac{n + q\mathbf{a}}{R}$$

The symmetry breaking is then continuous and very similar to a spontaneous breaking. This suggests that there should exist an alternative description where this is manifest.

Supersymmetry

If G is the global R -symmetry group of a supersymmetric theory, the supersymmetries that do not commute with G/G' get broken, and therefore supersymmetry is partially broken at M_c .

Scherk-Schwarz

For certain particular supergravity examples, it has been argued that this twisting is equivalent to a spontaneous supersymmetry breaking through the VEV of certain auxiliary fields.

Marti, Pomarol; Gersdorff, Quiros

Gauge symmetry

If G is a local gauge symmetry, the gauge bosons in G/G' get a mass proportional to M_c , and G is broken to G' at M_c .

Hosotani; Fayet

Performing a non-periodic gauge transformation, one can in this case demonstrate that the twist in the b.c. is equivalent to a VEV of A_{D-1} . The breaking is therefore spontaneous and the corresponding order parameter is the VEV of the Wilson loop:

$$W = \text{Tr} \exp i \oint dy A_{D-1}(y)$$

Hosotani

RADIATIVE CORRECTIONS

Field theories in $D > 4$ are not renormalizable, and must be thought as effective theories valid below a physical cut-off Λ as approximations to a microscopic theory (*e.g.* a string theory).

The UV behaviour is very bad, but operators that are forbidden by the broken symmetry are expected to be finite and controlled by IR physics, since the SS breaking is non-local.

Supersymmetry breaking example

Consider a $N = 1$ theory in $D = 5$ compactified to $D = 4$, and twist it using a $U(1)_R \subset SU(2)_R$ symmetry:

$$\phi(y + 2\pi R) = e^{2\pi i \mathbf{a} T_R} \phi(y)$$

Since bosons and fermions have different R -charges, supersymmetry is completely broken, and the tree-level spectrum is:

$$m_n^B = \frac{n + q_R^B \mathbf{a}}{R}, \quad m_n^F = \frac{n + q_R^F \mathbf{a}}{R}$$

The simplest protected quantities are the scalar mass corrections. Supersymmetry forces $\Delta m^2 = 0$, but when it is broken:

$$\Delta m^2 \sim \begin{cases} \Lambda^2, & \text{hard breaking} \\ M_{\text{susy}}^2 \ln \frac{\Lambda}{M_{\text{susy}}}, & \text{soft breaking} \\ M_{\text{susy}}^2, & \text{SS breaking} \end{cases}$$

The one-loop correction to the scalar mass from the gauge couplings is given by:

$$\Delta m^2 = \frac{C g_4^2}{4\pi^4} \text{Re} \left[\text{Li}_3(e^{2\pi i \mathbf{a} q_R^B}) - \text{Li}_3(e^{2\pi i \mathbf{a} q_R^F}) \right] M_c^2$$

Antoniadis, Dimopoulos, Pomarol, Quiros

Gauge symmetry breaking example

Consider a $SU(M+N)$ gauge theory compactified from $D = 5$ to $D = 4$ with matter scalars in the fundamental representation, and impose the boundary conditions:

$$A(y + 2\pi R) = e^{2\pi i \mathbf{a} T} A(y) e^{-2\pi i \mathbf{a} T}$$

$$\phi(y + 2\pi R) = e^{2\pi i \mathbf{a} T} \phi(y)$$

where:

$$T = \begin{pmatrix} N \mathbf{1}_M & 0 \\ 0 & -M \mathbf{1}_N \end{pmatrix}$$

The gauge symmetry is broken to $SU(M) \times SU(N) \times U(1)$ and the tree-level mass spectrum is:

$$A^{\text{diag}} : m_n = \frac{n}{R}, \quad A^{\text{offdiag}} : m_n = \frac{n \pm (M + N)\mathbf{a}}{R}$$

$$\phi^{\text{up}} : m_n = \frac{n + N\mathbf{a}}{R}, \quad \phi^{\text{down}} : m_n = \frac{n - M\mathbf{a}}{R}$$

Consider now the corrections to the masses of ϕ^{up} and ϕ^{down} . Their difference $\Delta m^2 = \Delta m_{\phi^{\text{up}}}^2 - \Delta m_{\phi^{\text{down}}}^2$ is protected, since it preserves $SU(M) \times SU(N) \times U(1)$ but breaks $SU(M+N)$.

At one-loop, one finds:

$$\Delta m^2 = \frac{g_4^2}{8\pi^4} \text{Re} \left\{ \frac{1}{M+N} \left[\text{Li}_3(e^{2\pi i M a}) - \text{Li}_3(e^{2\pi i N a}) \right] + 8(M-N) \left[\text{Li}_3(1) - \text{Li}_3(e^{2\pi i (M+N) a}) \right] \right\} M_c^2$$

The twisted theory with $\langle A_4 \rangle = 0$ is equivalent to the untwisted theory with $\langle A_4 \rangle = -a T/R$. The tree-level shifts in the masses arise from the gauge couplings, whereas loop correction must come from the effective interactions involving **Wilson lines**.

The one-loop corrections come from:

$$\Delta \mathcal{L} = \frac{g_4^2 M_c^2}{16\pi^4} \sum_{n=1}^{\infty} \frac{1}{n^3} \left[\left(8 \text{Tr} W_n^\dagger - \frac{1}{M+N} \right) \phi^\dagger(y) W_n \phi(y + 2\pi n R) + \text{Tr} W_n \Phi^\dagger(y) \Phi(y) \right] + \text{h.c.}$$

where:

$$W_n = \exp i \int_y^{y+2\pi n R} A_4$$

Corrections to symmetry breaking operators are therefore finite because they are non-local.

Masiero, Scrucra, Serone, Silvestrini

General considerations

To get a finite result for protected quantities, it is crucial to include the full tower of **KK** modes in loops to respect all the symmetries. In the gauge symmetry example, the crucial symmetries are the y -dependent gauge transformation.

Finiteness is perhaps guaranteed just by the existence of a local extension of the symmetry. In the gauge symmetry example, this can be verified by computing corrections from **Yukawa** couplings; one finds that they are non-local and finite $\forall g$.

The spontaneous nature of **SS** symmetry breaking has an important implication: the twist should be dynamically determined through the effective potential for the order parameter.

CHIRAL MODELS FROM FREE ORBIFOLDS

Compactifications on a S^1 are non-chiral (with and without SS). Chirality can be achieved on an orbifold S^1/\mathbf{Z}_2 .

The SS mechanism itself can be reinterpreted as an orbifolding through a translation combined with a twist:

$$SS(g) \text{ on } S^1 = S^1/\mathbf{Z}_2(gt)$$

The most general situation combines these two kind of operations in a **freely acting orbifold**.

With a single compact dimension, one can use only reflections and translations. The most general situation is essentially given by the orbifold $S^1/\mathbf{Z}_2(\alpha) \times \mathbf{Z}_2(\beta)$, where:

$$\alpha : \phi(y) \rightarrow P_\alpha \phi(-y)$$

$$\beta : \phi(y) \rightarrow P_\beta \phi(y + \pi R)$$

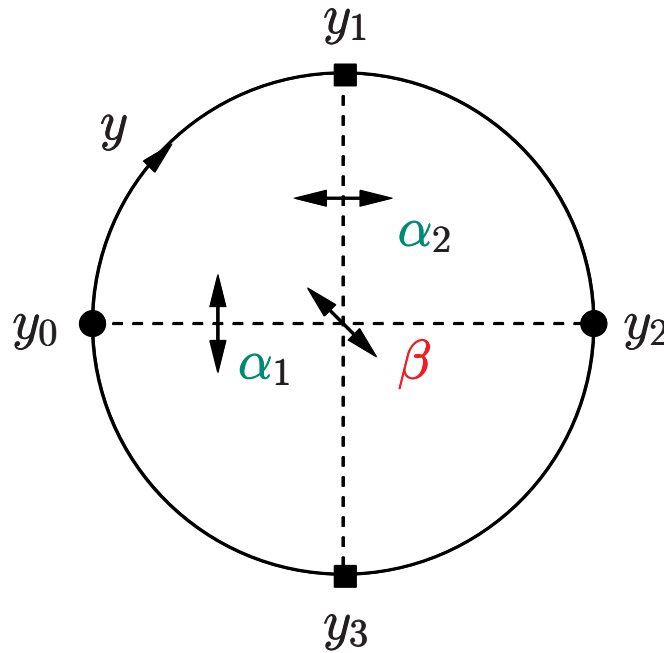
The third non-trivial element is:

$$\alpha\beta : \phi(y) \rightarrow P_{\alpha\beta} \phi(-y + \pi R)$$

The same orbifold group is generated by any pair of elements. Defining $\alpha_1 = \alpha$ and $\alpha_2 = \alpha\beta$, we have therefore:

$$S^1/\mathbf{Z}_2(\alpha_i) \times \mathbf{Z}_2(\beta) = SS(\beta) \text{ on } S^1/\mathbf{Z}_2(\alpha_j) , \quad \forall i, j$$

The α_i elements have fixed points at $y_a = a\pi R/2$:



The wave functions for the 4 possible (α_1, α_2) parities are:

$$\xi_n^{++}(y) = \frac{\eta_n}{\sqrt{\pi R}} \cos m_n^{++} y, \quad m_n^{++} = \frac{2n}{R}$$

$$\xi_n^{+-}(y) = \frac{1}{\sqrt{\pi R}} \cos m_n^{+-} y, \quad m_n^{+-} = \frac{2n+1}{R}$$

$$\xi_n^{-+}(y) = \frac{1}{\sqrt{\pi R}} \sin m_n^{-+} y, \quad m_n^{-+} = \frac{2n+1}{R}$$

$$\xi_n^{--}(y) = \frac{1}{\sqrt{\pi R}} \sin m_n^{--} y, \quad m_n^{--} = \frac{2n+2}{R}$$

Supersymmetry breaking example

Consider a $D = 5$ $N = 1$ theory and choose P_{α_1} and P_{α_2} to commute with a different half of the supersymmetry. $P_{\beta} = P_{\alpha_1} P_{\alpha_2}$ will then break all the supersymmetry.

A generic supermultiplet Φ takes the form $(\Phi^{++}, \Phi^{+-}, \Phi^{-+}, \Phi^{--})$, and at a generic point y there are 2 non-rigid supersymmetries: $Q_1^{+-}: \Phi^{\pm+} \leftrightarrow \Phi^{\pm-}$ and $Q_2^{-+}: \Phi^{+\pm} \leftrightarrow \Phi^{-\pm}$.

From the form of $\xi^{\pm\mp}(y)$ we see that:

$$\begin{aligned} \alpha_1\text{-fixed points} & : N = 1 (Q_1) \\ \alpha_2\text{-fixed points} & : N = 1' (Q_2) \\ \text{Bulk} & : N = 0 \end{aligned}$$

The breaking is non-local. The reflections α_1 and α_2 preserve different supersymmetries at distinct fixed points; the translation β breaks all supersymmetries but decouples for $R \rightarrow \infty$.

To build interesting models, introduce:

- Gauge vector-multiplet : $V(A^{++}, \lambda^{+-}, \psi_{\Sigma}^{-+}, \Sigma^{--})$
- Matter hyper-multiplet : $M(\psi_M^{++}, \phi_M^{+-}, \phi_M^{c-+}, \psi_M^{c--})$
- Higgs hyper-multiplet : $H(\phi_H^{++}, \psi_H^{+-}, \psi_H^{c-+}, \phi_H^{c--})$

The tree-level Higgs scalar potential is fixed by supersymmetry. ESB is radiative and triggered by supersymmetry breaking.

Using two conjugate Higgs hyper-multiplets, one can build models with the massless spectrum of the MSSM with broken supersymmetry at $M_c \sim 1$ TeV.

Pomarol, Quiros; Delgado, Pomarol, Quiros

Using a single Higgs hyper-multiplet, one can build models with the massless spectrum of the SM and one less parameter. Fitting the one-loop effective potential, one finds $M_c \sim 350$ GeV and predict that $m_{\phi_H} \sim 130$ GeV.

Barbieri, Hall, Nomura

Gauge symmetry breaking example

Consider a $D = 5$ $N = 1$ gauge theory, and choose P_{α_1} and P_{α_2} to preserve half of the supersymmetry, as well as G and $G' \subset G$ respectively. $P_{\beta} = P_{\alpha_1} P_{\alpha_2}$ breaks then G to G' .

One finds a non-local gauge symmetry breaking:

$$\begin{aligned} \alpha_1\text{-fixed points} & : G, \quad N = 1 (Q_1) \\ \alpha_2\text{-fixed points} & : G', \quad N = 1 (Q_1) \\ \text{Bulk} & : G', \quad N = 1 (Q_1) \end{aligned}$$

Taking $G = SU(5)$ and $G' = SU(3) \times SU(2) \times U(1)$, one can build interesting SGUT models with $M_c \sim 10^{16}$ GeV and the massless spectrum of the MSSM.

The two Higgs in the $\mathbf{5}$ and $\bar{\mathbf{5}}$ live in the bulk, and doublet-triplet splitting is automatic. The matter fields in 3 copies of $\mathbf{10}$ and $\bar{\mathbf{5}}$ live at the α_1 -fixed points.

Kawamura

DISEASES OF FIELD THEORY ORBIFOLDS

Orbifolds intended as manifolds have conical singularities at their fixed points, where the theory must be carefully defined.

In **string theory**, the twisted sectors provide blowing-up modes. The orbifold is obtained from a smooth manifold in the singular limit of vanishing **VEV** for these modes. This guarantees that orbifold projections preserve consistency.

In **field theory**, no twisted sectors are specified and the orbifold remains in general singular. The orbifold projections do not necessarily preserve consistency, and one must checked. It is not even enough to have an anomaly free zero-mode spectrum.

$S^1/\mathbf{Z}_2 \times \mathbf{Z}_2$ models

Consider a $D = 5$ $N = 1$ theory on $S^1/\mathbf{Z}_2 \times \mathbf{Z}_2$ with a charged hyper-multiplet with parities $(\phi^{++}, \psi^{+-}, \psi^{c-+}, \phi^{c--})$.

The theory is chiral, although free of fermionic zero-modes, because conjugate spinors have different wave functions.

The divergence of the left and right currents J_{\pm}^M induced by triangles are:

$$\partial_M J_{\pm}^M(x, y) = \pm \frac{1}{2} \sum_{n=0}^{\infty} [\xi_n^{\pm\mp}(y)]^2 \frac{g_4^2}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}(x, y)$$

The divergence of $J = J_+ + J_-$ is therefore controlled by:

$$\sum_{n=0}^{\infty} \left([\xi_n^{+-}(y)]^2 - [\xi_n^{-+}(y)]^2 \right) = \frac{1}{4} e^{-i\frac{2y}{R}} \sum_{a=-\infty}^{\infty} \delta(y - a\pi R/2)$$

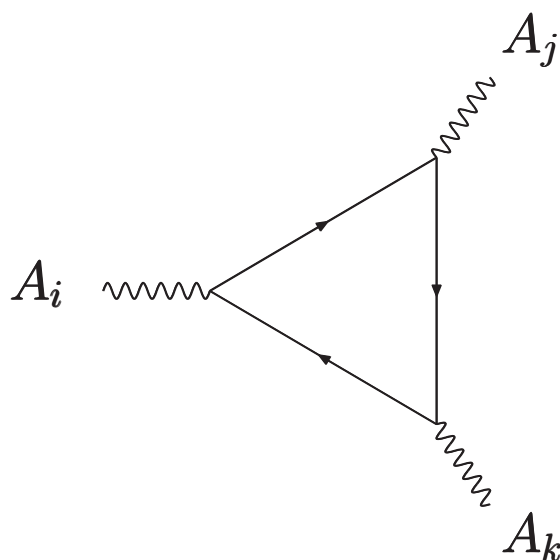
Restricting to $y \in [0, 2\pi R[$ one finds:

$$\begin{aligned} \partial_M J^M(x, y) = & \frac{1}{4} \left[\delta(y - y_0) - \delta(y - y_1) \right. \\ & \left. + \delta(y - y_2) - \delta(y - y_3) \right] \frac{g_4^2}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}(x, y) \end{aligned}$$

Therefore, there is $1/4$ of a chiral spinor anomaly at the α_1 -fixed points $y_{0,2}$ and $-1/4$ at the α_2 -fixed points $y_{1,3}$. The total anomaly in $D = 4$ vanishes, but the $D = 5$ theory is inconsistent.

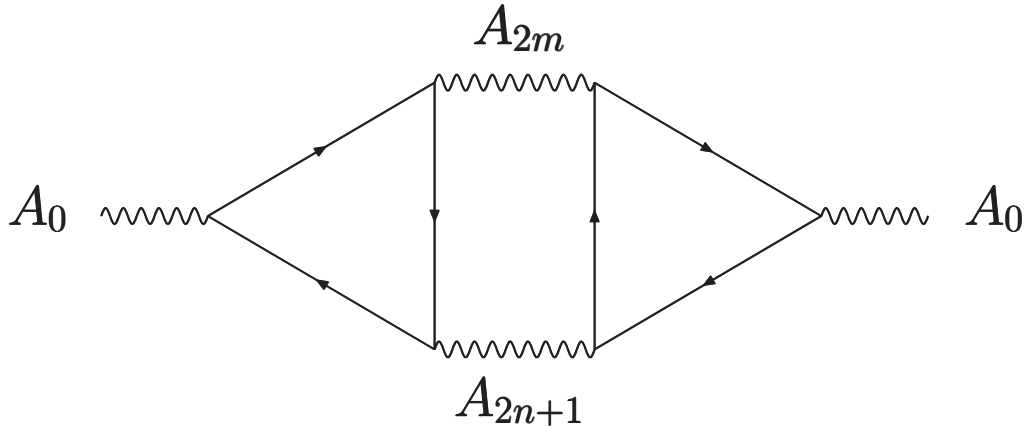
The same result can be derived using equivariant index theorems. The orbifold action is taken into account through the projector $P = 1/4 (1 + \alpha_1 + \alpha_2 + \beta)$ (only α_1 and α_2 contribute).

From the $D = 4$ point of view, triangle diagrams are subject to a selection rule:



anomalous if $i + j + k = \text{odd}$

Such diagrams lead to non-invariant operators in the effective $D = 4$ theory obtained by integrating out all massive KK modes. For example:



Scrucca, Serone, Silvestrini, Zwirner

There is also a FI term $\mathcal{L}_{FI} = \xi(y)D(x, y)$ induced at one-loop, since conjugate scalars have different wave functions. One gets:

$$\xi(y) = g_4 \sum_{n=0}^{\infty} \int \frac{d^4 p}{(2\pi)^4} \frac{[\xi_n^{++}(y)]^2 - [\xi_n^{--}(y)]^2}{p^2 + m_{2n}^2}$$

One can easily compute ($\tilde{y} = y \bmod \pi R/2$):

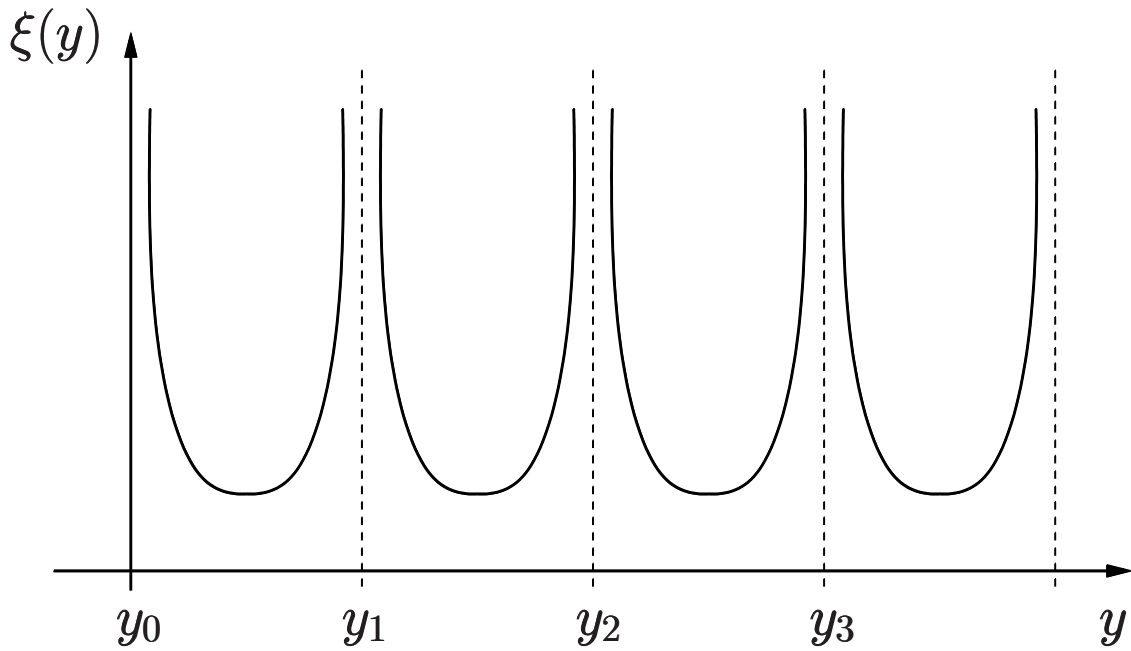
$$\sum_{n=0}^{\infty} \frac{[\xi_n^{++}(y)]^2 - [\xi_n^{--}(y)]^2}{p^2 + m_{2n}^2} = \frac{\cosh \pi R p/2 (1 - 4 \tilde{y}/\pi R)}{4 p \sinh \pi R p/2}$$

$$\xrightarrow{\text{UV}} \frac{1}{2 p^2} \left[\delta(\tilde{y}) + \delta(\tilde{y} - \pi R/2) \right]$$

The momentum integral converges everywhere in the bulk and:

$$\xi(y) = \frac{g_4}{8\pi^5 R^3} \left[\zeta \left(3, \frac{2\tilde{y}}{\pi R} \right) + \zeta \left(3, 1 - \frac{2\tilde{y}}{\pi R} \right) \right]$$

This corresponds to a symmetric profile:



To get a $D = 4$ interpretation of the divergence at each fixed point, one can introduce a momentum cut-off Λ . Then:

$$\xi(y) = \frac{g_4 \Lambda^2}{64\pi^2} \sum_{a=0}^3 \delta(y - y_a) + \xi_{\text{finite}}(y)$$

The first pieces are divergent $D = 4$ FI terms localized at the fixed points, whereas the second piece is a finite non-local term.

Ghencea, Groot Nibbelink, Nilles;
Scrucca, Serone, Silvestrini, Zwirner

The induced anomaly and FI terms might be related in a supergravity version of the model. It is not impossible that one can cure this kind of model by adding some physics at the fixed points.

STRING MODELS

String models with SS supersymmetry breaking can be constructed by deforming the partition functions of supersymmetric orbifolds.

Kounnas, Porrati; Ferrara, Kounnas, Porrati, Zwirner;
Antoniadis, Dudas, Sagnotti

A more convenient and general method is to look for freely acting orbifolds, constructed such that any element of the group either preserves some supersymmetry or acts freely and trivializes therefore in the large volume limit.

Kiritsis, Kounnas

The general set up is a $D = 10$ $N = 1$ string model compactified to $D = 4$. The Lorentz group decomposes as

$$SO(9, 1) \rightarrow SO(3, 1) \times SO(6)_R$$

The $D = 10$ supercharge gives rise to a $\mathbf{4}_R$ -plet of $D = 4$ supercharges Q_a , $a = 1, 2, 3, 4$, with different $SO(6)_R$ weights w_a :

$$w_1 = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right), \quad w_2 = \left(\frac{1}{2}, \frac{-1}{2}, \frac{-1}{2}\right)$$
$$w_3 = \left(\frac{-1}{2}, \frac{1}{2}, \frac{-1}{2}\right), \quad w_4 = \left(\frac{-1}{2}, \frac{-1}{2}, \frac{1}{2}\right)$$

The orbifold group G is a discrete subgroup of $SO(6)_R$.

A generic $g(v, \delta; v') \in G$ acts as a $2\pi(v_1, v_2, v_3)$ rotation and a $2\pi(\delta_1, \delta_2, \delta_3)$ translation in $T^6 = T_1^2 \times T_2^2 \times T_3^2$, and in addition as a $2\pi(v'_1, \dots, v'_K)$ twist in the gauge group.

On the Q_a 's:

$$g(v, \delta; v') : Q_a \rightarrow e^{2\pi i v \cdot w_a} Q_a$$

A given Q_a is thus preserved by $g(v, \delta; v') \in G$ if $v \cdot w_a = \text{integer}$. This means $v \in SU(3)_R^a$ and $SO(6)_R \rightarrow U(1)_R^a$.

Freely acting $\mathbf{Z}_N \times \mathbf{Z}_N$ models

Consider a $\mathbf{Z}_N(\alpha) \times \mathbf{Z}_N(\beta)$ orbifold model with $[\alpha, \beta] = 0$ and

α : preserves some Q_a and has fixed points

β : preserves no Q_a but acts freely in some T^2

To have a non-local breaking, we further require for all the other independent non-freely acting elements that:

$\alpha\beta^i$: preserve different Q_a but have distinct fixed points

The same orbifold group is generated from $\alpha\beta^i$ and β for any i .

Defining $\alpha_i = \alpha\beta^{i-1}$ for $i = 1, \dots, N$, one has therefore:

$$T^6/\mathbf{Z}_N(\alpha_i) \times \mathbf{Z}_N(\beta) = SS(\beta) \text{ on } T^6/\mathbf{Z}_N(\alpha_j), \quad \forall i, j$$

Supersymmetry and part of the gauge symmetry are broken below $M_c = 1/\sqrt{V_{T^2}}$.

The partition function of the model is given by:

$$Z = \frac{1}{N^2} \sum_{k,l=0}^{N-1} \sum_{p,q=0}^{N-1} Z \left[\begin{matrix} \alpha^l \beta^q \\ \alpha^k \beta^p \end{matrix} \right]$$

where $Z \left[\begin{matrix} h \\ g \end{matrix} \right] = \text{Tr}^{(h)} [g q^{L_0} \bar{q}^{\bar{L}_0}]$ is the g -twisted partition function in the h -twisted sector.

To get consistent models, one must impose modular invariance and tadpole cancellation. This will constrain the possible actions of the twists in the gauge group.

Using the fact that $Z \left[\begin{matrix} h \\ g \end{matrix} \right] = 0$ if g and h have no common fixed points, one can rewrite for any i :

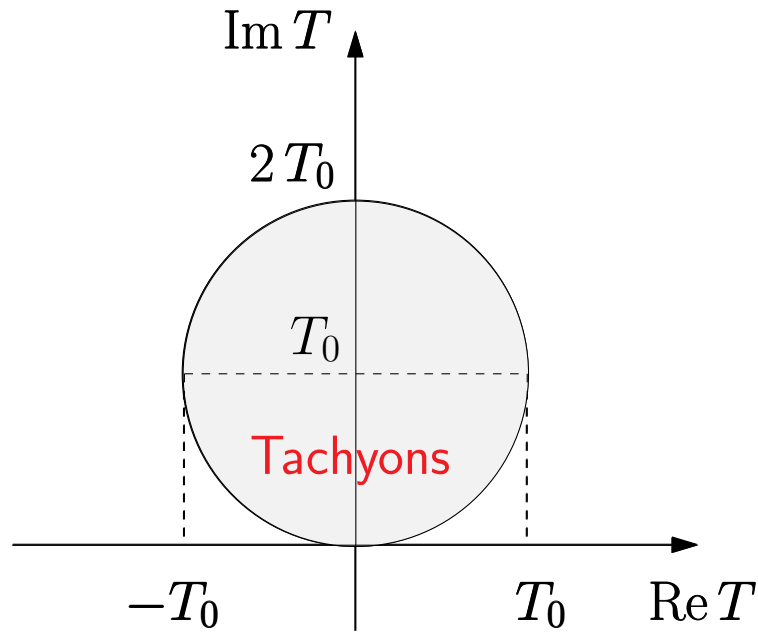
$$Z = Z_{\mathbf{Z}_N(\alpha_i) \times \mathbf{Z}_N(\beta)}^U + \sum_{j=1}^N Z_{\mathbf{Z}_N(\alpha_j)}^T + Z_{\mathbf{Z}_N(\beta)}^T$$

The only possible source of tachyons is the β -twisted sector. But since β acts as a pure translation along some T^2 , the states in this sector have a positive definite contribution to their m^2 depending on the moduli:

$$m^2 \geq m_0^2 + \frac{\delta^2 |T(1+U)|^2}{\alpha' \text{Im} T \text{Im} U}$$

The complex structure U is in general fixed, and the absence of tachyons restricts the Kähler modulus $T = (B + iR^2)/\alpha'$ to:

$$|T - iT_0| \geq T_0, \quad T_0 = \frac{\alpha'(-m_0^2)}{2\delta^2} \frac{\text{Im} U}{|1+U|^2}$$



One must then verify that the effective potential drives T into the stable region.

Scrucca, Serone

Explicit examples

Consider a $\mathbf{Z}_3(\alpha) \times \mathbf{Z}_3(\beta)$ example. This model is essentially unique; there is only one possible choice for β and three for α , but all lead to equivalent models. Take:

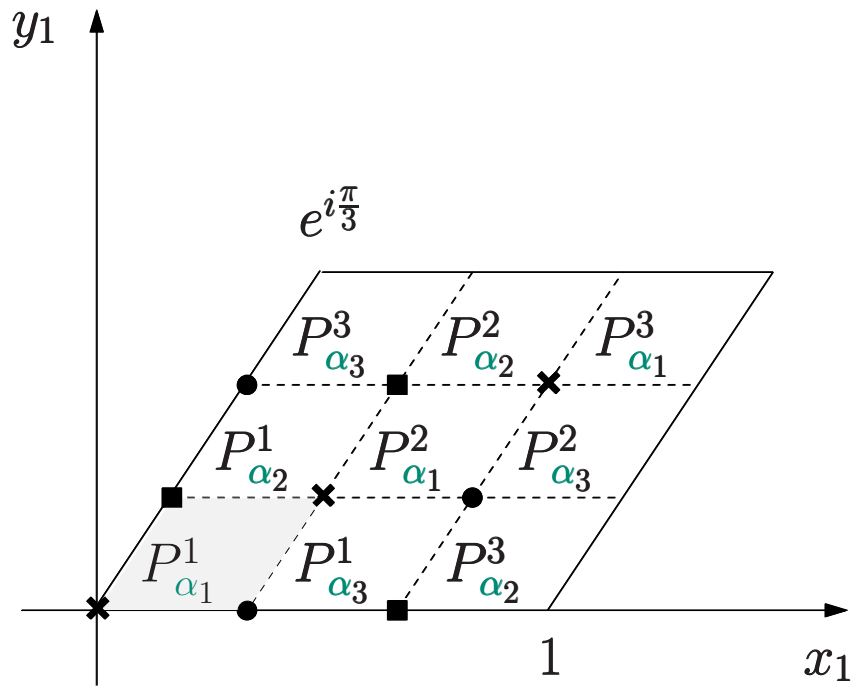
$$\alpha : v_\alpha = \left(\frac{1}{3}, \frac{1}{3}, 0\right), \quad \delta_\alpha = (0, 0, 0)$$

$$\beta : v_\beta = \left(0, 0, \frac{2}{3}\right), \quad \delta_\beta = \left(\frac{1}{3}, 0, 0\right)$$

As before, define $\alpha_i = \alpha\beta^{i-1}$ for $i = 1, 2, 3$. One can verify that α_1 preserves $Q_{2,3}$, α_2 preserves Q_4 and α_3 preserves Q_1 , whereas β preserves no Q_a but acts freely in the first T^2 .

The complex structure is $U = e^{i\pi/3}$, and the shift sends fixed points of each element into each other.

In the first T^2 :



Therefore:

$$\beta : P_{\alpha_j}^i \rightarrow P_{\alpha_j}^{i+1}$$

One can easily construct an explicit heterotic model of this type.

Choosing the standard embedding for both α and β , one finds:

$$\alpha_1\text{-fixed points} : E_7 \times U(1), \quad N = 2 (Q_{2,3})$$

$$\alpha_2\text{-fixed points} : E_6 \times SU(3), \quad N = 1 (Q_4)$$

$$\alpha_3\text{-fixed points} : E'_6 \times SU(3)', \quad N = 1' (Q_1)$$

$$\text{Bulk} : SO(10) \times SU(2) \times U(1)^2, \quad N = 0$$

Scrucca, Serone

One can also construct similar **unoriented** models. The simplest solution to the tadpole cancellation conditions gives:

$$\begin{aligned}
 \alpha_1\text{-fixed points} & : SO(16) \times U(8) , & N = 2 (Q_{2,3}) \\
 \alpha_2\text{-fixed points} & : SO(8) \times U(12) , & N = 1 (Q_4) \\
 \alpha_3\text{-fixed points} & : SO(8) \times U(12) , & N = 1' (Q_1) \\
 \text{Bulk} & : SO(8) \times U(4) \times U(8) , & N = 0
 \end{aligned}$$

Scrucca, Serone, Trapletti (to appear)

Scales

In order to solve the hierarchy problem, one needs $M_c \sim 1$ TeV. In string theory, however, the natural value is $M_c \sim M_s$.

In **heterotic** models, $M_s \sim 10^{18}$ GeV. One must then look for models free of dangerous threshold corrections (the $\mathbf{Z}_3 \times \mathbf{Z}_3$ model is the first example of this kind).

Antoniadis

In **unoriented** models, $1 \text{ TeV} \leq M_s \leq 10^{18}$ GeV. One can thus arrange that $M_c \sim M_s \sim 1$ TeV.

Antoniadis, Arkani-Hamed, Dimopoulos, Dvali

M-theory perspective

The microscopic quantum theory is not yet under control, but one can use efficiently consistency arguments to investigate orbifold compactifications of M-theory.

The basic example is M-theory compactified from $D = 11$ to $D = 10$ on S^1/\mathbf{Z}_2 . This represents the exact non-perturbative description of the supersymmetric $E_8 \times E_8$ heterotic string.

Horava, Witten

One can construct a similar model with SS supersymmetry breaking by compactifying on $S^1/\mathbf{Z}_2 \times \mathbf{Z}_2$. One gets then a non-perturbative non-supersymmetric $E_8 \times \bar{E}_8$ theory.

Antoniadis, Quiros; Dudas, Grojean; Fabinger, Horava

One can obtain interesting $D = 4$ models by further compactifying these theories on a CY. From an effective theory point of view:

- $M_{S^1} \gg M_{CY} : D = 11 \rightarrow D = 10 \rightarrow D = 4$
- $M_{S^1} \ll M_{CY} : D = 11 \rightarrow D = 5 \rightarrow D = 4$

In this case, supersymmetry breaking is mediated only by gravity, and $M_{\text{susy}} \sim M_c^2/M_P$. We need then $M_c \sim 10^{11}$ GeV.

OUTLOOK

- Freely acting orbifolds with **Scherk-Schwarz** symmetry breaking are very attractive for model building.
- The quantities protected by the broken symmetry are finite thanks to the non-local nature of the breaking.
- Field theory realizations have in general diseases at the orbifold singularities.
- String realizations can be constructed systematically. The only potential problem are tachyons.