METASTABLE DE SITTER VACUA IN N = 2 GAUGED SUPERGRAVITY

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- SUSY breaking in SUGRA.
- N = 1 models with chiral multiplets.
- N = 2 models with hyper multiplets.

SUSY BREAKING IN SUGRA

Constraints on realistic models

In a SUGRA model, the scalar potential V should allow for spontaneous SUSY breaking with certain non-trivial features.

- Phenomenology: To get a viable particle vacuum, need a point where $V \gtrsim 0$, $V' = 0$ and $V'' > 0$.
- Cosmology: To get a viable period of slow-roll inflation, need a region where $V > 0$, $V' \simeq 0$ and $V'' \gtrsim 0$.

The condition on V' can be satisfied by adjusting the values of the fields. But the conditions on V and V'' need an adjustment of parameters.

The natural question is then whether these two conditions can be used to restrict the class of models of potential interest. The answer is yes.

Algebraic formulation of the problem

Consider the critical situation where the scalar fields ϕ take values such that $V' = 0$, leading to broken SUSY and a gravitino mass $m_{3/2}$.

The value of V is linked to SUSY breaking. This gives a first relevant parameter given by:

$$\gamma = \frac{V}{3\, m_{3/2}^2}$$

The value of V'' along a generic direction is not related to SUSY breaking and can be easily adjusted, whereas along the sGoldstino direction η it is related to SUSY breaking. This gives a second relevant parameter:

$$\lambda = \frac{V''(\eta)}{m_{3/2}^2}$$

The structure of SUGRA implies $\gamma \geq -1$ and most importantly that λ is constrained in terms of γ.

Necessary conditions

The requirements coming from phenomenology and cosmology imply that both at the final vacuum and in the rolling region one should have

$$\gamma \gtrsim 0$$

More quantitatively:

$$\gamma_{\mathrm{vac}} \ll 1\,, \quad \gamma_{\mathrm{rol}} \gg 1$$

Similarly, since λ sets un upper bound on the minimal eigenvalue of V'', one should also have, again both for vacuum metastablity and inflationary slow rolling:

$$\lambda \gtrsim 0$$

More quantitatively:

$$\lambda_{\mathrm{vac}} : \text{sizable}\,, \quad \lambda_{\mathrm{rol}} : \text{free}$$

N = 1 MODELS WITH CHIRAL MULTIPLETS

Geometric formulation

A model with n_c chiral multiplets $\Phi^i = (\phi^i_{1,2}, \psi^i, F^i_{1,2})$ is specified by a real Kähler potential K and a holomorphic superpotential W. It has a $U(1)$ Kähler symmetry.

The $2n_c$ scalars span a Hodge-Kähler manifold with metric $g_{i\bar{\jmath}} = K_{i\bar{\jmath}}$ and Kähler form $J_{i\bar{\jmath}} = g_{i\bar{\jmath}}$, with a $U(1)$ bundle on it with curvature $J_{i\bar{\jmath}}$. The holonomy is $U(n_c) \times U(1)$. The vielbein has the form e_i^I and $e_{\bar{\imath}}^{\bar{I}}$.

It is convenient to introduce covariant derivative ∇_i including both the Christoffel and $U(1)$ connections Γ^k_{ij} and $\omega_i = K_i$. On a quantity of weights $(p, \bar{p})$, one has:

$$\nabla_i = D_i(\Gamma) + p\,\omega_i\,, \quad \nabla_{\bar{\jmath}} = D_{\bar{\jmath}}(\Gamma) + \bar{p}\,\omega_{\bar{\jmath}}$$

The gravitino mass has weights $(\frac{1}{2}, -\frac{1}{2})$ and reads:

$$L = e^{K/2} W \qquad \left(m_{3/2} = |L|\right)$$

The auxiliary fields also have weights $(\frac{1}{2}, -\frac{1}{2})$ and read:

$$F_i = e^{K/2}(W_i + K_i W)$$

These quantities satisfy the following relations:

$$\nabla_i L = F_i\,, \quad \nabla_{\bar{\jmath}} L = 0\,, \quad \nabla_{\bar{\jmath}} F_i = g_{i\bar{\jmath}} L$$

Scalar potential

The scalar potential is given by:

$$V = \bar{F}^i F_i - 3|L|^2$$

Its first first derivatives read:

$$\nabla_i V = -2 F_i \bar{L} + \nabla_i F_j \bar{F}^j$$

The second derivatives are also easily calculated, and one finds:

$$\nabla_i \nabla_{\bar{\jmath}} V = -2 g_{i\bar{\jmath}} |L|^2 + \nabla_i F_k \nabla_{\bar{\jmath}} \bar{F}^k - R_{i\bar{\jmath}p\bar{q}} F^p \bar{F}^{\bar{q}} + g_{i\bar{\jmath}} \bar{F}^k F_k - F_i \bar{F}_{\bar{\jmath}}$$
$$\nabla_i \nabla_j V = -\nabla_i F_j \bar{L} + \nabla_i \nabla_j F_k \bar{F}^k$$

Fermions and susy breaking

The n_c chiral fermions ψ^I are naturally defined on the tangent bundle of the scalar manifold, locally defined by the vielbein e_i^I and $\bar{e}_{\bar{\jmath}}^{\bar{J}}$.

The SUSY transformations give $\delta \psi^I \supset -\sqrt{2}\, e_i^I F^i \xi$. At a stationary point where $\nabla_i V = 0$, SUSY is spontaneously broken whenever $F_i \neq 0$, and the Goldstino direction in the tangent space is thus:

$$\eta^I = e_i^I F^i$$

The corresponding sGoldstino direction on the scalar manifold is:

$$\eta^i = e_I^i \eta^I = F^i$$

This defines $\mathbf{2}$ orthogonal directions in the real scalar-field space:

$$\eta^u = (F^i, \bar{F}^{\bar{\imath}})\,, \quad \tilde{\eta}^u = J^u{}_v \eta^v = (iF^i, -i\bar{F}^{\bar{\imath}})$$

Metastability

The strongest constraint on metastability comes from averaging over the $\mathbf{2}$ real sGoldstino directions $\eta, \tilde{\eta}$, and considering:

$$\lambda = \frac{\nabla_i \nabla_{\bar{\jmath}} V\, \bar{F}^i F^{\bar{\jmath}}}{|L|^2 \bar{F}^k F_k}$$

A simple computation shows that at a stationary point this is given by:

$$\lambda = 2 + R\,\frac{\bar{F}^i F_i}{|L^2|}$$

The quantity R is the sectional curvature in the plane $\eta, \tilde{\eta}$:

$$R = -\frac{R_{i\bar{\jmath}p\bar{q}} \bar{F}^i F^{\bar{\jmath}} \bar{F}^p F^{\bar{q}}}{(\bar{F}^k F_k)^2}$$

In terms of the parameter $\gamma = V/(3|L|^2)$, this reads:

$$\lambda = 2 + 3(1+\gamma)R$$

For a given positive γ, one gets thus a positive λ only if:

$$R \geq -\frac{2}{3}\frac{1}{1+\gamma} = \begin{cases} -\frac{2}{3}\,, & \gamma \ll 1 \\ 0\,, & \gamma \gg 1 \end{cases}$$

This defines a necessary condition for metastability. One can show that if K is kept fixed and W is allowed to be tuned, it becomes also sufficient. This allows a discrimination of models based just on K and not W.

Notice that $\mathcal{M} = \times_x \mathcal{M}_x$ is Hodge-Kähler if each $\mathcal{M}_x$ is Hodge-Kähler. The total R gets then diluted compared to each individual R_x, and:

$$R_{\rm best} = \Big(\sum_x R_x^{-1}\Big)^{-1}$$

Gomez-Reino, Scrucca 2006

N = 2 MODELS WITH HYPER MULTIPLETS

Geometric formulation

A model with $n_{\mathcal{H}}$ hyper multiplets $\mathcal{H}^i = (\phi^i_{1,2,3,4}, \psi^i_{1,2}, N^i_{1,2,3,4})$ is set by a scalar metric h_{uv}, a triplet of Hyperkähler forms J^x_{uv}, and a real Killing vector k^u. It has an $SU(2)$ symmetry.

The $4n_{\mathcal{H}}$ scalars span a Quaternionic-Kähler manifold, with an $SU(2)$ bundle on it with curvatures J^x_{uv}. The holonomy is $SP(2n_{\mathcal{H}}) \times SU(2)$. The vielbein $U^{\alpha A}_u$ has the property $U^{\alpha A}_u U^B_{\alpha v} = \frac{1}{2}\epsilon^{AB} h_{uv} + \frac{i}{2}\sigma^{xAB} J^x_{uv}$. Moreover, there should be an isometry associated to k^u.

It is convenient to introduce a covariant derivative ∇_u involving both the Christoffel and the $SU(2)$ connections Γ^w_{uv} and ω^x_u. On doublets and triplets one has:

$$\nabla_u{}^A{}_B = D_u(\Gamma)\delta^A_B - i\sigma^{xA}{}_B\omega^x_u\,,\quad \nabla_u{}^{xy} = D_u(\Gamma)\delta^{xy} + i\epsilon^{xyz}\omega^z_u$$

The Hyperkähler forms satisfy $\nabla_u J^x_{vw} = 0$ and:

$$J^x_{uw} J^{yw}{}_v = -h_{uv}\delta^{xy} + \epsilon^{xyz} J^z_{uv}$$

The Riemann tensor is constrained to take the following form:

$$R_{uvrs} = -h_{u[r}h_{vs]} - J^x_{uv}J^x_{rs} - J^x_{u[r}J^x_{vs]} + \Sigma_{uvrs}$$

The tensor Σ_{uvrs} is constructed out of a symmetric $SP(2n_{\mathcal{H}})$ tensor $\Sigma_{\alpha\beta\gamma\delta}$ as $\Sigma_{uvrs} = \epsilon_{AB}\epsilon_{CD}U^{\alpha A}_u U^{\beta B}_v U^{\gamma C}_r U^{\delta D}_s \Sigma_{\alpha\beta\gamma\delta}$. It represents the Weyl part of the curvature, so that the Ricci part is universal:

$$R_{uv} = -2(n_{\mathcal{H}} + 2)h_{uv}\,, \quad R = -8n_{\mathcal{H}}(n_{\mathcal{H}} + 2)$$

The gravitino masses are described by a triplet of real quantities, which represent Killing potentials for the Killing vector k^u:

$$P^x = \frac{1}{2n_{\mathcal{H}}} J^x_{uv} \nabla^u k^v \qquad \left(m^{AB}_{3/2} = P^x \sigma^{xAB}\,, \;\; m_{3/2} = \sqrt{P^x P^x}\right)$$

The auxiliary fields are instead given by:

$$N_u = 2\, k_u$$

The above quantities satisfy the following relations:

$$\nabla_u P^x = J^x_{uv} N^v \,, \quad \nabla_{(u} N_{v)} = 0$$

Scalar potential

The scalar potential takes the following simple form:

$$V = N^r N_r - 3\, P^x P^x$$

Its first derivatives are given by

$$\nabla_u V = -6 P^x J^x_{ur} N^r + 2 \nabla_u N_r N^r$$

Its second derivatives read instead:

$$\nabla_u \nabla_v V = 2\nabla_u N^r \nabla_v N_r - 2(R_{urvs} + 3J^x_{ur} J^x_{vs}) N^r N^s$$
$$-6P^x J^x_{(ur} \nabla_{v)} N^r$$

Fermions and susy breaking

The $2n_{\mathcal{H}}$ chiral fermions ψ^α are naturally defined on the tangent bundle of the scalar manifold, which is locally defined by the vielbein $U^{\alpha A}_u$.

The 2 SUSY transformations give $\delta\psi^\alpha \supset U^{\alpha A}_u N^u \xi_A$. At a stationary point where $\nabla_u V = 0$, SUSY is spontaneously broken when $N_u \neq 0$. The 2 Goldstino directions are thus described in the tangent space by:

$$\eta^{\alpha A} = U^{\alpha A}_u N^u$$

The corresponding sGoldstino directions on the scalar manifold are:

$$\eta^u_{AB} = U^u{}_{\alpha A} \eta^\alpha_B = \frac{1}{2}\epsilon_{AB} N^u + \frac{i}{2}\sigma^x_{AB} J^{xu}{}_v N^v$$

This defines $\mathbf{4}$ orthogonal directions in scalar-field space:

$$\eta^u = N^u\,, \quad \tilde{\eta}^u_x = J^{xu}{}_v N^v$$

The first corresponds however to the Goldstone flat direction k^u.

Metastability

The crucial condition on metastability comes in this case from averaging over the $\mathbf{3}$ non-trivial sGoldstino directions $\tilde{\eta}_x$, and considering:

$$\lambda = \frac{1}{6}\,\frac{\nabla_u \nabla_v V\, J^{xu}{}_r N^r J^{xv}{}_s N^s}{P^y P^y\, N^w N_w}$$

A non-trivial computation shows that at a stationary point this is given by:

$$\lambda = \frac{8}{3} - (R+3)\,\frac{N^u N_u}{P^x P^x}$$

Here R is the averaged sectional curvature in the planes $\eta, \tilde{\eta}_x$:

$$R = \frac{1}{3}\,\frac{R_{urvs} N^u J^{xr}{}_p N^p N^v J^{xs}{}_q N^q}{(N^w N_w)^2}$$

But using the constrained form of R_{urvs}, one finds that the Weyl part Σ_{urvs} does not contribute and the Ricci gives a universal answer:

$$R = -2$$

In terms of $\gamma = V/(3P^x P^x)$, it follows then that:

$$\lambda = -\frac{1}{3}(1 + 9\gamma)$$

For any γ that is positive, λ is therefore always negative, and there is unavoidably an instability.

Gomez-Reino, Louis, Scrucca 2009

CONCLUSIONS

- In N = 1 SUGRA theories, there exist a strong necessary condition on the Kähler potential for the existence of metastable stationary points with broken SUSY, no matter what the superpotential is.
- In N = 2 SUGRA theories, there are similar constraints which are even stronger and completely exclude the existence of such points in some particular classes of models.