

MEDIATION OF SUPERSYMMETRY BREAKING IN EXTRA DIMENSIONS

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- Mechanisms for the transmission of **SUSY** breaking.
Generic problems and possible solutions.
- Geometrical sequestering through extra dimensions.
Soft masses and relevance of quantum corrections.
- Brane worlds from orbifold models.
Microscopic and low energy effective theories.
Loop corrections and **SUSY** cancellations.
- Radius-dependent loop effects in sequestered models.
Universal correction to soft masses.
- Prototype of viable model.

Rattazzi, Scrucca, Strumia (hep-th/0305184)

SUSY BREAKING

The standard scenario is that **SUSY** breaking occurs at a scale M in a **hidden** sector and is transmitted to the **visible** sector through some interactions, generating soft breaking terms.

There are two main delicate points for phenomenology:

- **Sflavour**: m_0^2 must be positive and nearly universal.
- **Shierarchy**: must have $\mu \sim m_0 \sim m_{1/2} \ll \Lambda_{UV}$.

Gauge mediation

SUSY breaking at $M < M_M$ mediated by **gauge** interactions:

- $m_{1/2} \sim \frac{g^2}{16\pi^2} \frac{M^2}{M_M}$, $m_0^2 \sim \left(\frac{g^2}{16\pi^2}\right)^2 \left(\frac{M^2}{M_M}\right)^2$.
- m_0^2 universal ($M_F \gg M_M$) and positive.
- $\mu \sim M_M$ from interactions.

Gravity mediation

SUSY breaking at $M \ll M_P$ mediated by **gravity** interactions:

- $m_{1/2}, m_{3/2} \sim \frac{M^2}{M_P}$, $m_0^2 \sim \left(\frac{M^2}{M_P}\right)^2$.
- m_0^2 generic ($M_F \sim M_P$).
- $\mu \sim \frac{M^2}{M_P}$ from interactions.

Symmetries

To naturally solve both problems, one can try to introduce new symmetries. Main options:

- Gauge mediation + complications for Shierarchy.

No simple and compelling model so far.

- Gravity mediation + constraints for Sflavour.

Difficult to forbid mixing of the two sectors at M_P .

Geometry

An interesting possibility, natural in string theory, is to separate the visible and the hidden sectors along an extra dimension.

This framework has very peculiar characteristics going beyond symmetries:

- Geometric distinction between visible sector, hidden sector, and mediating interactions.
- New physical scale M_C acting as cut-off for the mixing between the two sectors.

GEOMETRICAL SEQUESTERING IN SUGRA

Consider a general SUGRA theory with:

$$\text{Visible: } \Phi_0 = (\phi_0, \chi_0; F_0), V_0 = (A_0^\mu, \lambda_0; D_0).$$

$$\text{Hidden: } \Phi_\pi = (\phi_\pi, \chi_\pi; F_\pi), V_\pi = (A_\pi^\mu, \lambda_\pi; D_\pi).$$

$$\text{Interactions: } C = (e_\mu^a, \psi_\mu; a_\mu, b_\mu), S = (\phi_S, \psi_S; F_S).$$

After superconformal gauge-fixing, $b_\mu = 0$, $\phi_S = 1$, $\psi_S = 0$, and the structure of the matter action reads:

$$\begin{aligned} \mathcal{L}_{\text{mat}} = & \left[\Omega(\Phi, \Phi^\dagger) S S^\dagger \right]_D + \left[P(\Phi) S^3 \right]_F + \left[P(\Phi) S^3 \right]_F^\dagger \\ & + \left[\tau(\Phi) \mathcal{W}^2 \right]_F + \left[\tau(\Phi) \mathcal{W}^2 \right]_F^\dagger \end{aligned}$$

The functions Ω , $\tau \mathcal{W}^2$ and P have expansions of the type:

$$\Omega = -3M_{\text{P}}^2 + \Phi_0 \Phi_0^\dagger + \Phi_\pi \Phi_\pi^\dagger + \frac{h}{M_{\text{P}}^2} \Phi_0 \Phi_0^\dagger \Phi_\pi \Phi_\pi^\dagger + \dots$$

$$P = \Lambda^3 + M_\pi^2 \Phi_\pi + \dots$$

$$\tau \mathcal{W}^2 = \frac{1}{g_0^2} \mathcal{W}_0^2 + \frac{1}{g_\pi^2} \mathcal{W}_\pi^2 + \frac{k}{M_{\text{P}}} \Phi_\pi \mathcal{W}_0^2 + \dots$$

For a vanishing cosmological constant, we tune $\Lambda^3 \sim M_\pi^2 M_{\text{P}}$.

The SUSY breaking VEVs are then:

$$|F_\pi| \sim M_\pi^2, \quad |F_S| \sim \frac{\Lambda^3}{M_{\text{P}}^2} \sim \frac{M_\pi^2}{M_{\text{P}}}$$

Classical theory

Leading soft masses at classical level:

$$m_{3/2} \sim |F_S| \sim \frac{M_\pi^2}{M_P}$$
$$m_{1/2} \sim k \frac{|F_\pi|}{M_P} \sim k \frac{M_\pi^2}{M_P}$$
$$m_0^2 \sim h \frac{|F_\pi|^2}{M_P^2} \sim h \frac{M_\pi^4}{M_P^2}$$

Non-universal; separating **visible** and **hidden** sectors in an extra dimension, $h = k = 0 \Rightarrow$ quantum corrections important.

Quantum corrections

Corrections from gauge loops due to superconformal anomaly:

$$\delta m_{1/2} \sim \frac{g^2}{16\pi^2} |F_S| \sim \frac{g^2}{16\pi^2} \frac{M_\pi^2}{M_P}$$
$$\delta m_0^2 \sim \left(\frac{g^2}{16\pi^2}\right)^2 |F_S|^2 \sim \left(\frac{g^2}{16\pi^2}\right)^2 \frac{M_\pi^4}{M_P^2}$$

Universal; positive for squarks and negative for sleptons !

Randall, Sundrum;

Giudice, Luty, Murayama, Rattazzi

With an extra dimension, corrections from gravity loops are cut off at $M_C = (\pi R)^{-1}$ and computable:

$$\delta m_0^2 \sim \frac{M_C^2}{16\pi^2 M_P^2} \frac{|F_\pi|^2}{M_P^2} \sim \frac{M_C^2}{16\pi^2 M_P^2} \frac{M_\pi^4}{M_P^2}$$

Universal; positive or negative ?

Gauge and gravitational quantum corrections can compete if (gravity loop at M_C) \sim (gauge loop) 2 , that is:

$$\frac{M_C^2}{16\pi^2 M_P^2} \sim \left(\frac{g^2}{16\pi^2}\right)^2 \Rightarrow \frac{M_C}{M_P} \sim \frac{g^2}{4\pi}$$

This is reasonable \Rightarrow possible very interesting hybrid models of SUSY breaking.

Dynamics of extra dimensions

In the 4D effective theory for $E \ll M_C$, the dynamics of an extra dimension is described by a chiral multiplet:

$$\text{Radion: } T = (T, \psi_T; F_T).$$

The VEV of T controls the radius ($\text{Re} T = \pi R$), whereas a VEV for F_T gives additional SUSY-breaking effects.

There are various ways to get a satisfactory radion dynamics.

F-terms: e.g. strong coupling condensation of bulk gaugino.

D-terms: e.g. Casimir energy with localized kinetic terms.

Luty, Sundrum;

Ponton, Poppitz

To compute radiative effects involving the radion multiplet, one needs a full 5D supergravity description. All these effects are non-local and therefore finite and insensitive to UV physics.

S^1/\mathbf{Z}_2 ORBIFOLD MODELS

The extra dimension is a circle with coordinate $x^5 \in [0, 2\pi]$ and gauged parity $\mathbf{Z}_2 : \mathbf{x}^5 \rightarrow -\mathbf{x}^5$. The radius is $e_5^5 = R$.

The **visible** and **hidden** sectors are located at the fixed-points at 0 and π , and have $N = 1$ **SUSY** with $U(1)$ **R**-symmetry (bosons: q , fermions: $q - 1$, aux: $q - 2$):

$$\text{Visible: } \Phi_0 = (\phi_0, \chi_0; F_0), V_0 = (A_0^\mu, \lambda_0; D_0).$$

$$\text{Hidden: } \Phi_\pi = (\phi_\pi, \chi_\pi; F_\pi), V_\pi = (A_\pi^\mu, \lambda_\pi; D_\pi).$$

The interactions are in the bulk, and have $N = 2$ **SUSY** with $SU(2)$ **R**-symmetry (bosons: **1**, fermions: **2**, aux: **1** or **3**):

$$\text{Gauge: } \mathcal{V} = (A_M, \lambda, \Sigma; \vec{X}).$$

$$\begin{aligned} \text{Gravity: } \mathcal{M} &= (e_M^A, \psi_M, A_M; \vec{V}_M, \vec{t}, v_{AB}, \lambda, C), \\ \mathcal{T} &= (\vec{Y}, B_{MNP}, \rho; N). \end{aligned}$$

Bulk and boundary theories: fixed by $N = 2$ and $N = 1$ **SUSY**.

Bulk-boundary couplings: fixed by $N = 1$ **SUSY** with $N = 2$ bulk multiplets decomposed into $N = 1$ boundary multiplets.

The Lagrangian (with e factored out) has the form:

$$\mathcal{L} = \mathcal{L}_5 + e_5^5 \delta(x^5 - 0) \mathcal{L}_{4,0} + e_5^5 \delta(x^5 - \pi) \mathcal{L}_{4,\pi}$$

Singularities

Auxiliary fields have a dimensionless propagator and could give divergences in the sums over KK modes with $m_n = n/R$.

In the natural formulation, auxiliary and odd fields mix through $\partial_5 \Rightarrow$ propagators \square_4/\square_5 and $1/\square_5$. Matter couples to auxiliary fields \Rightarrow no singularities.

Making a shift, auxiliary and odd fields can be decoupled \Rightarrow propagators 1 and $1/\square_5$. Matter couples to odd fields through $\partial_5 \Rightarrow$ singularities cancelled by contact terms proportional to

$$\delta(0) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} 1 = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \frac{p^2 - m_n^2}{p^2 - m_n^2}$$

Gauge interactions

In this case, the off-shell formulation of the bulk theory is simple. The bulk-to-boundary couplings are well understood.

Mirabelli, Peskin

Gravity interactions

In this case, the off-shell formulation is rather involved and has been formulated only recently.

Zucker

The bulk-to-boundary couplings have been only partly studied.

Gherghetta, Riotto

GAUGE INTERACTIONS

The Lagrangian for the $N = 2$ bulk vector mult. \mathcal{V} is ($g_5 \rightarrow 1$):

$$\mathcal{L}_5 = -\frac{1}{4}F_{MN}^2 + \frac{i}{2}\bar{\lambda}\not{\partial}\lambda + \frac{1}{2}|\partial_M\Sigma|^2 + \frac{1}{2}\vec{X}^2$$

The \mathbf{Z}_2 parities of \mathcal{V} are:

\mathcal{V}	A_M	λ	Σ	\vec{X}
+	A_μ	λ^1		X^3
-	A_5	λ^2	Σ	$X^{1,2}$

At the fixed-points, the even components of \mathcal{V} form an $N = 1$ vector multiplet $V = (A_\mu, \lambda^1; D)$ with

$$D = X^3 - \partial_5\Sigma$$

The interaction with an $N = 1$ boundary chiral multiplet Φ is:

$$\mathcal{L}_4^\Phi = |D_\mu\phi|^2 + i\bar{\chi}\not{D}\chi + |F|^2 + |\phi|^2 D + \dots$$

with

$$D_\mu = \partial_\mu - iA_\mu$$

After integrating out $X^{1,2}$ and F , the total Lagrangian reads:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F_{MN}^2 + \frac{i}{2}\bar{\lambda}\not{\partial}\lambda + \frac{1}{2}|\partial_\mu\Sigma|^2 + \frac{1}{2}D^2 \\ & + e_5^5 \delta(x^5) \left[|D_\mu\phi|^2 + i\bar{\chi}\not{D}\chi \right] + \left(\partial_5\Sigma + \rho_5(x^5) \right) D + \dots \end{aligned}$$

The density which couples to D is given by:

$$\rho_{\dot{5}}(x^5) = e_{\dot{5}}^5 \delta(x^5) |\phi|^2$$

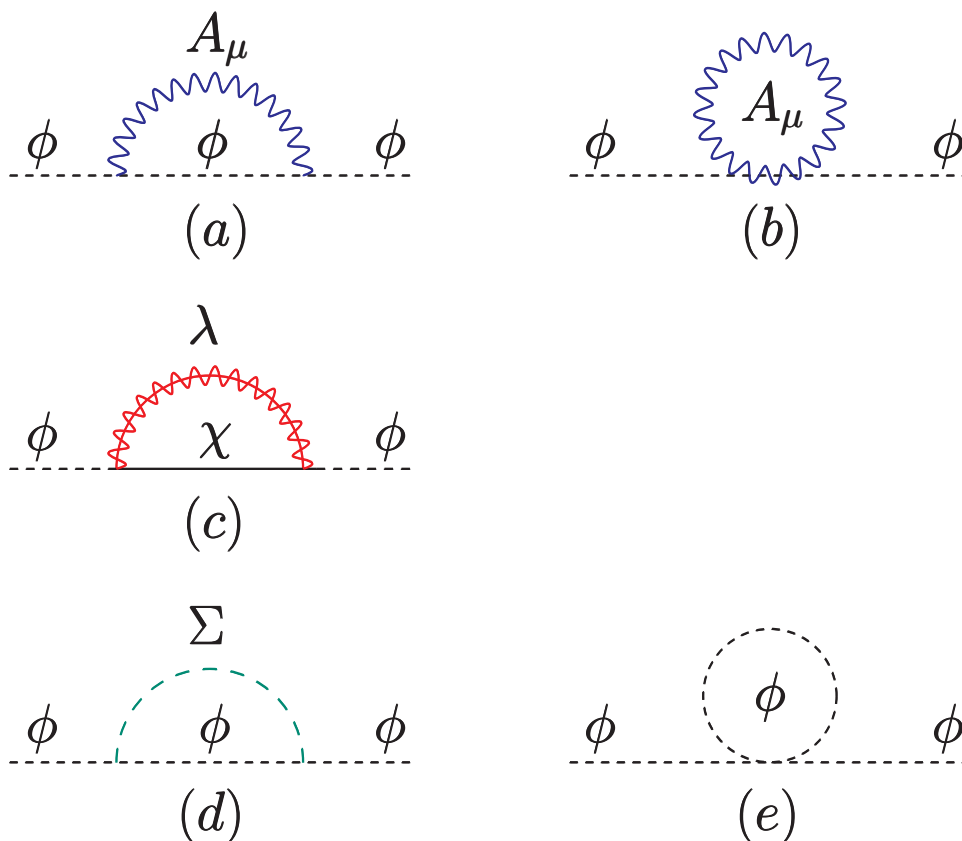
Redefining D through a shift to complete squares, one gets:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F_{MN}^2 + \frac{i}{2}\bar{\lambda}\not{D}\lambda + \frac{1}{2}|\partial_\mu\Sigma|^2 + \frac{1}{2}\tilde{D}^2 \\ & + e_{\dot{5}}^5 \delta(x^5) [|D_\mu\phi|^2 + i\bar{\chi}\not{D}\chi] \\ & - \frac{1}{2}(\partial_{\dot{5}}\Sigma + \rho_{\dot{5}}(x^5))^2 + \dots \end{aligned}$$

Loop corrections

Consider for example the 1-loop correction to the mass of ϕ .

This must vanish by **SUSY** non-renormalization theorem.



The result is:

$$\Delta m^2 = \frac{i}{2\pi R} \sum_{\alpha} \sum_{n=-\infty}^{\infty} \int \frac{d^4 p}{(2\pi)^4} \frac{N_{\alpha,n}}{p^2 - m_n^2}$$

with

$$\begin{aligned} N_{a,n} &= -p^2 & N_{b,n} &= 4p^2 \\ N_{c,n} &= -4p^2 & & \\ N_{d,n} &= m_n^2 & N_{e,n} &= p^2 - m_n^2 \end{aligned}$$

Low-energy theory

The low energy theory for $E \ll M_C$ is obtained by integrating out Σ . Neglecting $\partial_{\mu} \sim E$ with respect to $\partial_5 \sim M_C$, its equation of motion is:

$$\partial_{\dot{5}} \left(\partial_{\dot{5}} \Sigma + \rho_{\dot{5}}(x^5) \right) = 0$$

The solution is

$$\partial_{\dot{5}} \Sigma = - \left(\rho_{\dot{5}}(x^5) - \frac{1}{2\pi R} \rho \right)$$

with

$$\rho = \int_0^{2\pi} dx^5 e_{\dot{5}}^{\dot{5}} \rho_{\dot{5}}(x^5) = |\phi|^2$$

Substituting back in the Lagrangian and integrating over x^5 , one finds:

$$\begin{aligned} \mathcal{L}^{\text{eff}} &= -\frac{1}{4} F_{\mu\nu}^2 + \frac{i}{2} \bar{\lambda}^1 \not{\partial} \lambda^1 \\ &+ |D_{\mu} \phi|^2 + i \bar{\chi} \not{D} \chi - \frac{1}{22\pi R} \rho^2 + \dots \end{aligned}$$

GRAVITY INTERACTIONS

The Lagrangians for the $N = 2$ bulk minimal multiplet \mathcal{M} and tensor multiplet \mathcal{T} are ($M_5 \rightarrow 1$):

$$\begin{aligned}\mathcal{L}_5^{\mathcal{M}} = & -32\vec{t}^2 - \frac{1}{\sqrt{3}}F_{AB}v^{AB} + \bar{\psi}_M\vec{\tau}\gamma^{MN}\psi_N\vec{t} \\ & - \frac{1}{6\sqrt{3}}\varepsilon^{MNPQR}(A_M F_{NP} - \frac{3}{2}\bar{\psi}_M\gamma_N\psi_P)F_{QR} \\ & - 4C - 2i\bar{\lambda}\gamma^M\psi_M\end{aligned}$$

$$\begin{aligned}\mathcal{L}_5^{\mathcal{T}} = & Y^{-1}\left(-\frac{1}{4}|\mathcal{D}_M\vec{Y}|^2 + W_A^2 - \frac{i}{2}\bar{\rho}\mathcal{D}\rho - (N + 6\vec{t}\vec{Y})^2\right. \\ & - \frac{1}{24}\varepsilon^{MNPQR}\vec{Y}(\vec{H}_{MN} - Y^{-2}\mathcal{D}_M\vec{Y}\times\mathcal{D}_N\vec{Y})B_{PQR} \\ & \left. - \frac{1}{4}\bar{\psi}_M\vec{\tau}\gamma^{MNP}\psi_N(\vec{Y}\times\mathcal{D}_P\vec{Y}) + 4\bar{\rho}\vec{\tau}\lambda\vec{Y}Y\right) \\ & + Y\left(-\frac{1}{4}\mathcal{R}(\hat{\omega}) - \frac{i}{2}\bar{\psi}_M\gamma^{MNP}\mathcal{D}_N\psi_P - \frac{1}{6}\hat{F}_{MN}^2\right. \\ & + 20\vec{t}^2 + v_{AB}^2 - \frac{i}{2}\bar{\psi}_A\psi_Bv^{AB} - \bar{\psi}_M\vec{\tau}\gamma^{MN}\psi_N\vec{t} \\ & \left. - \frac{i}{4\sqrt{3}}\bar{\psi}_P\gamma^{MNPQ}\psi_Q\hat{F}_{MN} + 4C + 2i\bar{\lambda}\gamma^M\psi_M\right) \\ & + \rho\text{-dep. } \lambda\text{-indep.}\end{aligned}$$

Notation:

$$W^M = \frac{1}{12} \epsilon^{MNPQR} \partial_N B_{PQR} + \frac{1}{4} \bar{\psi}_P \vec{\tau} \gamma^{PMQ} \psi_Q \vec{Y}$$

$$\hat{F}_{MN} = \partial_M A_N - \partial_N A_M + i(\sqrt{3}/2) \bar{\psi}_M \psi_N$$

$$\vec{H}_{MN} = \mathcal{D}_M \vec{V}_N - \mathcal{D}_N \vec{V}_M$$

The derivatives \mathcal{D}_M are $SU(2)_R$ and super-Lorentz covariant.

In particular:

$$\mathcal{D}_M \vec{Y} = \partial_M \vec{Y} + \vec{V}_M \times \vec{Y}$$

$$\mathcal{D}_M \vec{V}_N = D_M(\hat{\omega}) \vec{V}_N + \vec{V}_M \times \vec{V}_N$$

$$\mathcal{D}_M \psi_N = D_M(\hat{\omega}) \psi_N - \frac{i}{2} \vec{V}_M \vec{\tau} \psi_N$$

The \mathbf{Z}_2 parities of \mathcal{M} and \mathcal{T} are:

\mathcal{M}	e_M^A	ψ_M	A_M	\vec{t}	v_{AB}	\vec{V}_M	λ	C
+	$e_\mu^a, e_5^{\dot{5}}$	ψ_μ^1, ψ_5^2	A_5	$t^{1,2}$	$v_{a\dot{5}}$	$V_\mu^3, V_5^{1,2}$	λ^1	C
-	$e_\mu^{\dot{5}}, e_5^a$	ψ_μ^2, ψ_5^1	A_μ	t^3	v_{ab}	$V_\mu^{1,2}, V_5^3$	λ^2	

\mathcal{T}	\vec{Y}	B_{MNP}	ρ	N
+	$Y^{1,2}$	$B_{\mu\nu\rho}$	ρ^1	N
-	Y^3	$B_{\mu\nu 5}$	ρ^2	

At the fixed-points, the even components of \mathcal{M} form an $N = 1$ intermediate multiplet $I = (e_\mu^a, \psi_\mu^1; a_\mu, b_a, t^2 + it^1, \lambda^1, S)$ with

$$\begin{aligned} S &= C + \frac{1}{2} e_\mu^5 \bar{\lambda}^1 \psi_\mu^2 - \frac{1}{2} \mathcal{D}_5 t^3 \\ a_\mu &= -\frac{1}{2} (V_\mu^3 + 4 v_{\mu 5}) - \frac{2}{\sqrt{3}} e_\mu^5 \hat{F}_{\mu 5} \\ b_a &= v_{a 5} \end{aligned}$$

plus an $N = 1$ chiral mult. $T = (\pi e_\mu^5 + i(2\pi/\sqrt{3})A_5, \pi \psi_\mu^2; F_T)$ with $q_T = 0$ and

$$F_T = \pi [V_5^1 - 4 e_5^5 t^2] + i\pi [V_5^2 + 4 e_5^5 t^1]$$

Similarly, the even components of \mathcal{T} form an $N = 1$ chiral multiplet $S = (Y^2 + iY^1, \rho; F_S)$ with $q_S = 2$ and

$$F_S = [-2N + \mathcal{D}_5 Y^3] + i[-2W_5 + 12(Y^2 t^1 - Y^1 t^2)]$$

After gauge-fixing: I conformal gravity multiplet, $S^{1/3}$ chiral compensator multiplet, T radion chiral multiplet.

The Lagrangians for an $N = 1$ boundary chiral multiplet Φ and vector multiplet V with $q_\Phi = 2/3$ and $q_V = 0$ are:

$$\begin{aligned} \mathcal{L}_4^\Phi &= |\mathcal{D}_\mu \phi|^2 + i\bar{\chi} \mathcal{D} \chi + |F - 4\phi(t^2 - it^1)|^2 \\ &\quad + \frac{1}{6} |\phi|^2 (\mathcal{R} + 2i\bar{\psi}_\mu^1 \gamma^{\mu\nu\rho} D_\nu \psi_\rho^1) + \dots \\ \mathcal{L}_4^V &= -\frac{1}{4} G_{\mu\nu}^2 + i\bar{\lambda} \mathcal{D} \lambda + \frac{1}{2} D^2 + \dots \end{aligned}$$

The chiral $U(1)_R$ -covariant derivatives are given by

$$\mathcal{D}_\mu = D_\mu + i q (a_\mu + 3 b_\mu) (i\gamma^5)^F$$

with

$$\begin{aligned} q_\phi &= 2/3 & q_\chi &= -1/3 \\ q_{A_\mu} &= 0 & q_\lambda &= -1 \end{aligned}$$

The only non-trivial auxiliary field dependence in the boundary Lagrangian is through $a_\mu + 3 b_\mu$. All the other auxiliary fields can be integrated out through their equations of motion.

The fields C and λ act as Lagrangian multipliers and enforce the constraints $Y = 1$ and $\rho = 0$. After gauge-fixing $\vec{Y} = (0, 1, 0)^T$, the remaining decoupled auxiliary fields can be integrated out, keeping only the coupled combination:

$$V_\mu = -2(a_\mu + 3 b_\mu) = V_\mu^3 - 2 v_{\mu\dot{5}} - \frac{2}{\sqrt{3}} e_\dot{5}^5 \hat{F}_{\mu 5}$$

The resulting Lagrangian is:

$$\begin{aligned} \mathcal{L} &= \frac{1}{6} \Omega_{\dot{5}}(x^5) \left[\mathcal{R} + 2i \bar{\psi}_M \gamma^{MNP} D_N \psi_P + \frac{2}{3} V_\mu^2 \right] - \frac{1}{4} \hat{F}_{\mu\nu}^2 \\ &+ e_\dot{5}^5 \delta(x^5) \left[|\partial_\mu \phi|^2 + i \bar{\chi} \not{D} \chi - \frac{1}{4} G_{\mu\nu}^2 + i \bar{\lambda} \not{D} \lambda \right] \\ &+ \frac{1}{\sqrt{3}} \left(\partial_{\dot{5}} A_\mu + \frac{1}{\sqrt{3}} J_{\mu\dot{5}}(x^5) \right) V^\mu + \dots \end{aligned}$$

The **Kähler** kinetic function is defined as

$$\Omega_{\dot{5}}(x^5) = -\frac{3}{2} + e_{\dot{5}}^5 \delta(x^5) |\phi|^2$$

The current which couples to V_μ is the sum of

$$J_{\mu\dot{5}}^\Phi(x^5) = e_{\dot{5}}^5 \delta(x^5) \left[i(\phi^* \partial_\mu \phi - \text{c.c.}) - \frac{i}{2} \bar{\chi} \gamma_\mu \gamma^{\dot{5}} \chi + \dots \right]$$

$$J_{\mu\dot{5}}^V(x^5) = e_{\dot{5}}^5 \delta(x^5) \left[-\frac{3i}{2} \bar{\psi} \gamma_\mu \gamma^{\dot{5}} \psi + \dots \right]$$

$$J_{\mu\dot{5}}^T(x^5) = -\sqrt{3} e_{\dot{5}}^5 \partial_\mu A_5 + \dots$$

Note that:

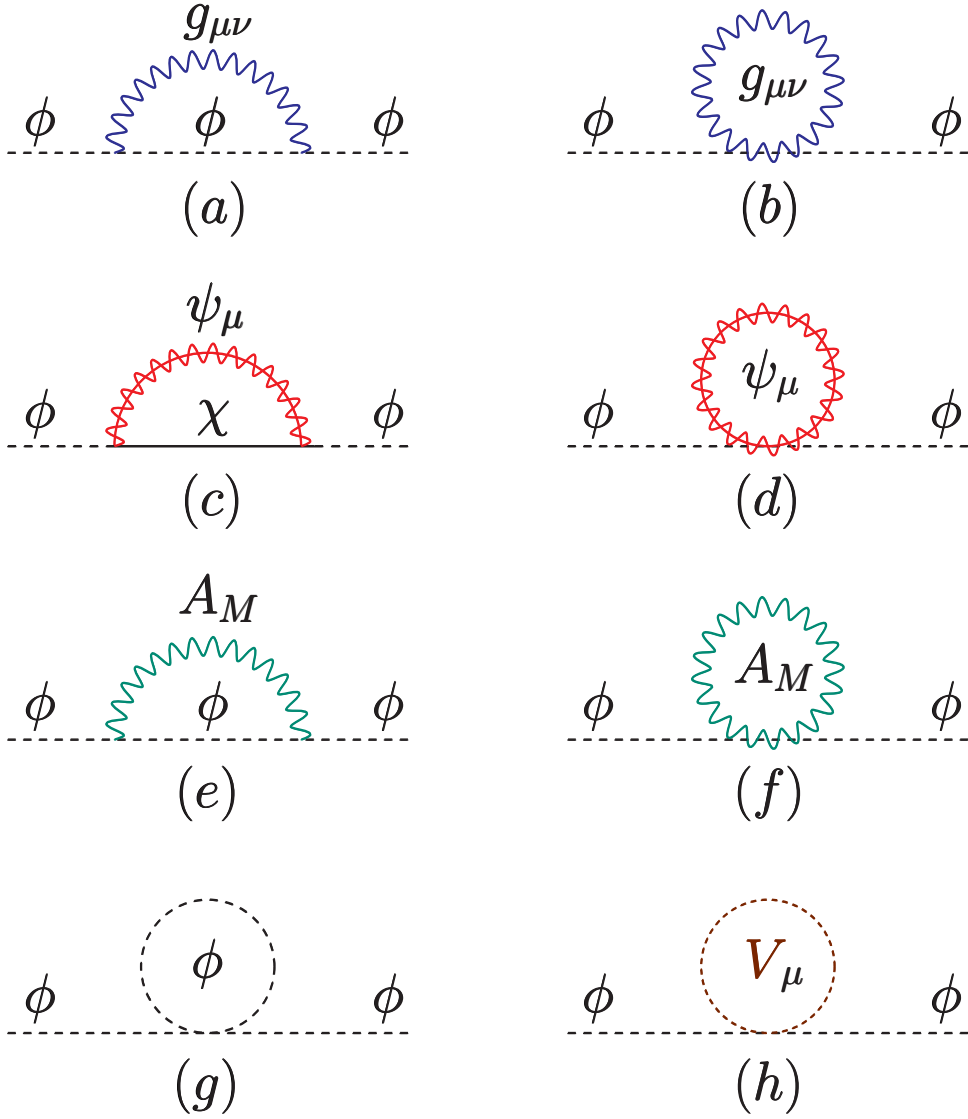
$$\partial_{\dot{5}} A_\mu + \frac{1}{\sqrt{3}} J_{\mu\dot{5}}^T(x^5) = -e_{\dot{5}}^5 \widehat{F}_{\mu 5}$$

Redefining V_μ through a shift to complete the squares, one finds finally:

$$\begin{aligned} \mathcal{L} = & \frac{1}{6} \Omega_{\dot{5}}(x^5) \left[\mathcal{R} + 2i \bar{\psi}_M \gamma^{MNP} D_N \psi_P + \frac{2}{3} \tilde{V}_\mu^2 \right] - \frac{1}{4} \widehat{F}_{\mu\nu}^2 \\ & + e_{\dot{5}}^5 \delta(x^5) \left[|\partial_\mu \phi|^2 + i \bar{\chi} \not{D} \chi - \frac{1}{4} G_{\mu\nu}^2 + i \bar{\lambda} \not{D} \lambda \right] \\ & - \frac{3}{4 \Omega_{\dot{5}}(x^5)} \left(\partial_{\dot{5}} A_\mu + \frac{1}{\sqrt{3}} J_{\mu\dot{5}}^T(x^5) \right)^2 + \dots \end{aligned}$$

Loop corrections

Consider as before the 1-loop correction to the mass of ϕ , which must vanish by **SUSY** non-renormalization theorem.



The result is:

$$\Delta m^2 = \frac{i}{6\pi R} \sum_{\alpha} \sum_{n=-\infty}^{\infty} \int \frac{d^4 p}{(2\pi)^4} \frac{N_{\alpha,n}}{p^2 - m_n^2}$$

with

$$N_{a,n} = 0$$

$$N_{b,n} = 5 p^2$$

$$N_{c,n} = 0$$

$$N_{d,n} = -8 p^2$$

$$N_{e,n} = p^2 - m_n^2$$

$$N_{f,n} = -p^2 + 4 m_n^2$$

$$N_{g,n} = -p^2 + m_n^2$$

$$N_{h,n} = 4 p^2 - 4 m_n^2$$

Low-energy theory

The low energy theory for $E \ll M_C$ is obtained by integrating out A_μ . Neglecting $\partial_\mu \sim E$ with respect to $\partial_5 \sim M_C$, its equation of motion is:

$$\partial_5 \left[\frac{1}{\Omega_5(x^5)} \left(\partial_5 A_\mu + \frac{1}{\sqrt{3}} J_{\mu 5}(x^5) \right) \right] = 0$$

The solution is

$$\partial_5 A_\mu = -\frac{1}{\sqrt{3}} \left(J_{\mu 5}(x^5) - \frac{\Omega_5(x^5)}{\Omega} J_\mu \right)$$

with

$$\Omega = \int_0^{2\pi} dx^5 e_5^{\dot{5}} \Omega_5(x^5) = -\frac{3}{2}(T + T^*) + |\phi|^2$$

and

$$J_\mu^\Phi = \int_0^{2\pi} dx^5 e_5^{\dot{5}} J_{\mu 5}^\Phi(x^5) = i(\Omega_\phi \partial_\mu \phi - \text{c.c.}) - \frac{i}{2} \Omega_{\phi\phi^*} \bar{\chi} \gamma_\mu \gamma^5 \chi + \dots$$

$$J_\mu^V = \int_0^{2\pi} dx^5 e_5^{\dot{5}} J_{\mu 5}^V(x^5) = -\frac{3i}{2} \bar{\lambda} \gamma_\mu \gamma^5 \lambda + \dots$$

$$J_\mu^T = \int_0^{2\pi} dx^5 e_5^{\dot{5}} J_{\mu 5}^T(x^5) = i(\Omega_T \partial_\mu T - \text{c.c.}) + \dots$$

Substituting back in the Lagrangian and integrating over x^5 , one finds:

$$\begin{aligned} \mathcal{L}^{\text{eff}} = & \frac{1}{6} \Omega \left[\mathcal{R} + 2i \bar{\psi}_\mu^1 \gamma^{\mu\nu\rho} D_\nu \psi_\rho^1 \right] - \frac{1}{4\Omega} J_\mu^2 \\ & + \Omega_{\phi\phi^*} \left[|\partial_\mu \phi|^2 + \bar{\chi} \not{D} \chi \right] + \left[-\frac{1}{4} G_{\mu\nu}^2 + i \bar{\lambda} \not{D} \lambda \right] + \dots \end{aligned}$$

LOOP EFFECTS IN SEQUESTERED MODELS

A generic sequestered model is defined by:

$$\Omega_{\mathbb{5}}(x^5) = -\frac{3}{2}M_{\mathbb{5}}^3 + \Omega_0 e_{\mathbb{5}}^5 \delta(x^5 - 0) + \Omega_{\pi} e_{\mathbb{5}}^5 \delta(x^5 - \pi)$$

We take:

$$\Omega_{0,\pi} = -3L_{0,\pi}M_{\mathbb{5}}^3 + \Phi_{0,\pi}\Phi_{0,\pi}^\dagger$$

The kinetic function of the effective theory is then

$$\Omega(\Omega_{0,\pi}, T + T^\dagger) = -\frac{3}{2}(T + T^\dagger)M_{\mathbb{5}}^3 + \Omega_0 + \Omega_{\pi}$$

and

$$M_{\mathbb{P}}^2 = (\text{Re } T + L_0 + L_{\pi})M_{\mathbb{5}}^3$$

The 1-loop correction to this has a divergent T -indep. (local) plus a finite T -dep. (non-local) parts. The relevant part is:

$$\Delta\Omega(\Omega_{0,\pi}, T + T^\dagger) = \sum_{m,n=0}^{\infty} \frac{c_{m,n} \Omega_0^m \Omega_{\pi}^n}{M_{\mathbb{5}}^{3(m+n)} (T + T^\dagger)^{2+m+n}}$$

The corresponding component effective action is $\Delta\Gamma = [\Delta\Omega]_D$.

In particular, when $F_{\pi} \neq 0$ and/or $F_T \neq 0$:

$$\text{Vac. energy: } c_{m,n} L_0^m L_{\pi}^n |F_T|^2, c_{m,n} L_0^m L_{\pi}^{n-1} |F_{\pi}|^2$$

$$\text{Soft masses: } c_{m,n} L_0^{m-1} L_{\pi}^n |F_T|^2, c_{m,n} L_0^{m-1} L_{\pi}^{n-1} |F_{\pi}|^2$$

To derive the $c_{m,n}$ s, one chooses one operator for each super-space term in $\Delta\Omega$, and computes its induced coefficient.

Strategy

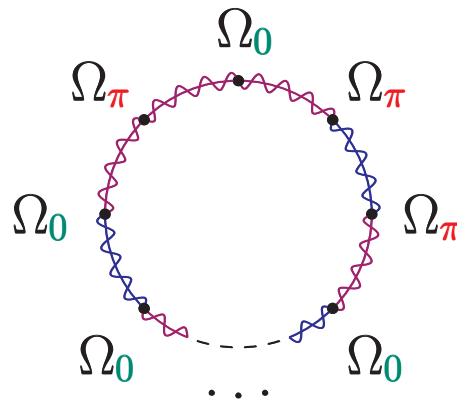
Crucial **trick**: use non-**SUSY** background with $F_T = 2\pi\epsilon \neq 0$. This corresponds to a $SU(2)_R$ Wilson line, and can be achieved in two ways: **SS** twist or constant boundary superpotentials. Only the gravitino **KK** modes are affected: $m_n = (n + \epsilon)/R$.

von Gersdorff, Quiros, Riotto;
Bagger, Feruglio, Zwirner

This leads to huge simplifications:

- One can use operators with scalars and no derivatives
 \Rightarrow Few diagrams, mostly with quartic couplings.
- The amplitudes must vanish in the **SUSY** limit $\epsilon \rightarrow 0$
 \Rightarrow All the information is in the gravitino diagrams.

In the end, there is a single type of diagram for each $c_{m,n}$:



The set of operator that we want to compute is given by the effective potential $\Delta V = -\partial_T \partial_{T^*} \Delta \Omega |F_T|^2$, function of R and

$$\alpha_{0,\pi} = \frac{\Omega_{0,\pi}}{6\pi R M_5^3} = -\frac{L_{0,\pi}}{2\pi R} + \frac{|\phi_{0,\pi}|^2}{6\pi R M_5^3} = -r_{0,\pi} + |\varphi_{0,\pi}|^2$$

The precise expression of the operators to be matched is:

$$\Delta V(\alpha_{0,\pi}, R, \epsilon) = \frac{-\epsilon^2}{4\pi^2 R^4} \sum_{m,n=0}^{\infty} c_{m,n} (2+m+n)(3+m+n) \alpha_0^m \alpha_\pi^n$$

Computation

The gravitino contribution to the full effective potential is:

$$\Delta W_\psi(\alpha_{0,\pi}, R, \epsilon) = -\frac{1}{2} \ln \det \left[\square_5 + (\alpha_0 \delta_0 + \alpha_\pi \delta_\pi) \square_4 \right]$$

The $\alpha_{0,\pi}$ -independent part is

$$\ln \det [\square_5] = 8 \operatorname{Re} \int \frac{d^4 p}{(2\pi)^4} \ln [F(pR, \epsilon)]$$

with

$$F(pR, \epsilon) = \prod_{n=-\infty}^{+\infty} (p + i m_n) = (\operatorname{Div.}) \sinh \pi(pR + i\epsilon)$$

The $\alpha_{0,\pi}$ -dependent part is

$$\begin{aligned} & \ln \det \left[1 + (\alpha_0 \delta_0 + \alpha_\pi \delta_\pi) \frac{\square_4}{\square_5} \right] \\ &= 8 \operatorname{Re} \int \frac{d^4 p}{(2\pi)^4} \ln \left| \begin{array}{cc} 1 - p\alpha_0 G_0(pR, \epsilon) & -p\alpha_\pi G_\pi(pR, \epsilon) \\ -p\alpha_0 G_\pi(pR, \epsilon) & 1 - p\alpha_\pi G_0(pR, \epsilon) \end{array} \right| \end{aligned}$$

with

$$G_0(pR, \epsilon) = \frac{1}{2\pi R} \sum_{n=-\infty}^{+\infty} \frac{e^{i0n}}{p + i m_n} = \frac{1}{2} \coth \pi(pR + i\epsilon)$$

$$G_\pi(pR, \epsilon) = \frac{1}{2\pi R} \sum_{n=-\infty}^{+\infty} \frac{e^{i\pi n}}{p + i m_n} = \frac{1}{2} \operatorname{csch} \pi(pR + i\epsilon)$$

Putting these two pieces together and simplifying one finds:

$$\begin{aligned} \Delta W_\psi(\alpha_{0,\pi}, R, \epsilon) \\ = \text{Div.} - \frac{1}{2\pi^6 R^4} \text{Re} \int_0^\infty dl l^3 \ln \left| 1 - \frac{1 + \alpha_0 l}{1 - \alpha_0 l} \frac{1 + \alpha_\pi l}{1 - \alpha_\pi l} e^{-2(l+i\pi\epsilon)} \right| \end{aligned}$$

The $\mathcal{O}(\epsilon^0)$ part cancels the contributions of other bulk fields.

The $\mathcal{O}(\epsilon^2)$ part yields the relevant potential ΔV that we need.

The $\mathcal{O}(\epsilon^{2n})$ terms map to D-terms with superderivatives.

Expanding $\Delta W_\psi|_{\epsilon^2}$ in powers of $\alpha_{0,\pi}$ and comparing with the general expression for ΔV , one extracts the coefficients $c_{m,n}$.

The first few ones are:

$$c_{0,0} = \frac{\zeta(3)}{4\pi^2}, \quad c_{1,0} = c_{0,1} = \frac{\zeta(3)}{6\pi^2}, \quad c_{1,1} = \frac{\zeta(3)}{6\pi^2}, \quad \dots$$

An independent and direct computation exploiting supergraph techniques leads to the same results.

Buchbinder et al.

Since we know the exact expression $\Delta W_\psi|_{\epsilon^2}$ for ΔV , we can do better and find the exact expression for $\Delta\Omega$ by solving the differential equation $\Delta V = -\epsilon^2 \partial_R^2 \Delta\Omega$. The result is:

$$\begin{aligned} \Delta\Omega(\Omega_{0,\pi}, T+T^\dagger) \\ = -\frac{9}{\pi^2} M_5^2 \int_0^\infty dx x \ln \left[1 - \frac{1 + \frac{\Omega_0}{M_5^2} x}{1 - \frac{\Omega_0}{M_5^2} x} \frac{1 + \frac{\Omega_\pi}{M_5^2} x}{1 - \frac{\Omega_\pi}{M_5^2} x} e^{-6(T+T^\dagger)M_5 x} \right] \end{aligned}$$

This shows in particular that all the $c_{m,n}$ s are positive.

Results

The results for the vacuum energy and soft masses are:

$$\delta\mathcal{E}^4 = -\frac{\zeta(3)}{16\pi^2} \left[\frac{1}{3} f_\pi \frac{|F_\pi|^2}{M_4^2} M_C^2 + \frac{3}{2} f_T |F_T|^2 M_C^4 \right]$$

$$\delta m_0^2 = -\frac{\zeta(3) M_C^2}{16\pi^2 M_4^2} \left[\frac{1}{6} g_\pi \frac{|F_\pi|^2}{M_4^2} + g_T |F_T|^2 M_C^2 \right]$$

These depend on the parameters $r_{0,\pi}$ through

$$M_4^2 = \frac{1}{1+r_0+r_\pi} M_P^2$$

and the normalized functions

$$f_\pi = \frac{4}{3\zeta(3)} \int_0^\infty dl l^2 e^{-2l} \frac{(1-r_0 l)/(1+r_\pi l)}{[(1+r_0 l)(1+r_\pi l) - (1-r_0 l)(1-r_\pi l)e^{-2l}]}$$

$$f_T = \frac{2}{3\zeta(3)} \int_0^\infty dl l^3 e^{-2l} \frac{(1-r_0^2 l^2)(1-r_\pi^2 l^2)}{[(1+r_0 l)(1+r_\pi l) - (1-r_0 l)(1-r_\pi l)e^{-2l}]^2}$$

$$g_\pi = \frac{8}{3\zeta(3)} \int_0^\infty dl l^3 e^{-2l} \frac{1}{[(1+r_0 l)(1+r_\pi l) - (1-r_0 l)(1-r_\pi l)e^{-2l}]^2}$$

$$g_T = \frac{4}{3\zeta(3)} \int_0^\infty dl l^4 e^{-2l} \frac{(1-r_\pi^2 l^2)[(1+r_0 l)(1+r_\pi l) + (1-r_0 l)(1-r_\pi l)e^{-2l}]}{[(1+r_0 l)(1+r_\pi l) - (1-r_0 l)(1-r_\pi l)e^{-2l}]^3}$$

For $r_{0,\pi} = 0$, $\delta\mathcal{E}^4$ and δm_0^2 are negative \Rightarrow not interesting.

For $r_{0,\pi} \neq 0$, $\delta\mathcal{E}^4$ and δm_0^2 can have any sign \Rightarrow interesting.

Three main cases for the dependence on R at fixed $L_{0,\pi}$:

- $L_0 = 0, L_\pi = 0$: $\delta\mathcal{E}^4$ unstable, $\delta m_0^2 \sim -(\delta\mathcal{E}^4)'$.
- $L_0 = 0, L_\pi \neq 0$: $\delta\mathcal{E}^4$ stable, $\delta m_0^2 \sim -(\delta\mathcal{E}^4)'$.
- $L_0 \neq 0, L_\pi \neq 0$: $\delta\mathcal{E}^4$ metastable, $\delta m_0^2 \not\sim -(\delta\mathcal{E}^4)'$.

PROTOTYPE MODEL

The goal is to achieve values of T , F_T , F_π , F_S such that:

- $\mathcal{E}^4 \sim 0 \Rightarrow$ tuning of P .
- $\delta^{\text{grav}} m_0^2 > 0 \Rightarrow$ needs $r_\pi \neq 0$.
- $\delta^{\text{grav}} m_0^2 \sim \delta^{\text{gau}} m_0^2 \Rightarrow$ indep. stab. mech.

One can try to combine localized kinetic terms with gaugino condensation, with:

$$\Omega = -\frac{3}{2}(T + T^\dagger)M_5^3 + \Phi_0\Phi_0^\dagger - 3L_\pi M_5^3 + \Phi_\pi\Phi_\pi^\dagger$$

$$P = \Lambda_\pi^3 + M_\pi^2\Phi_\pi + \Lambda^3 e^{-\alpha\Lambda T}$$

To have $\mathcal{E}^4 \sim 0$ we need $\Lambda_\pi^3 \sim M_\pi^2 M_P$. We then get:

$$M_C \sim \alpha\Lambda, \quad F_T \sim \frac{M_\pi^2}{\Lambda M_P}, \quad F_S \sim \frac{M_\pi^2}{M_P}, \quad F_\pi \sim M_\pi^2$$

To have $r_\pi \gg 1$ we need $L_\pi \gg (\alpha\Lambda)^{-1}$. In this limit:

$$f_\pi, g_\pi \rightarrow \frac{2\ln(2)}{3\zeta(3)} \frac{1}{r_\pi^2}, \quad f_T, g_T \rightarrow -\frac{3}{4}$$

$\delta^{\text{grav}} m_0^2$ becomes positive for $r_\pi \sim \alpha^{-1}$; OK with $\alpha \ll 1$.

$\delta^{\text{grav}} m_0^2$ is of the same order of magnitude as $\delta^{\text{gau}} m_0^2$ if:

$$\alpha^2 \frac{M_C^2}{16\pi^2 M_4^2} \sim \left(\frac{g^2}{16\pi^2} \right)^2 \Rightarrow \frac{M_C}{M_P} \sim \frac{g^2}{4\pi\sqrt{\alpha}}$$

OUTLOOK

- Bulk-to-boundary couplings now well understood and full theory under control.
- Radius-dependent quantum corrections to sfermion squared masses generally negative, but can become positive with sizable localized kinetic terms.
- Sequestered models can work, but radion dynamics plays a crucial.