ANOMALIES IN STRING THEORY AND THEIR CANCELLATION

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- D-BRANES, O-PLANES AND RR FIELDS.
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D-BRANES, O-PLANES AND RR FIELDS

The spectrum of string theories splits into sectors that differ through boundary conditions:

$$\text{Open}: \ \text{NS}\,(\text{B}) + \text{R}\,(\text{F})$$

$$\text{Closed}: \text{NSNS}\,(\text{B}) + \text{NSR}\,(\text{F}) + \text{RNS}\,(\text{F}) + \text{RR}\,(\text{B})$$

The RR states are bispinors that can be decomposed into anti-symmetric tensors. These can be interpreted as forms: $C_{(n)}$.

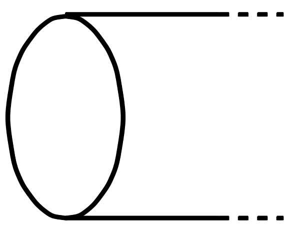
The perturbative string states are charged under the NSNS fields but not under the RR ones. There exist instead two kinds of non-perturbative objects with RR charge.

Polchinski

D-branes

D-branes are hyperplanes associated with boundaries of string WS with Neumann boundary conditions in the $\parallel$ directions and Dirichlet ones in the $\perp$ ones.

D :

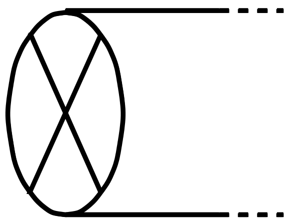


$\Rightarrow$ boundary

O-planes

O-planes are hyperplanes associated with crosscaps of string WS that implement an orientation change accompanied by a reflection acting as $+1$ in the $\parallel$ directions and as -1 in the $\perp$ ones.

O :



$\Rightarrow$ crosscap

These objects can have a variable number p of spatial dimensions and 1 temporal dimension. The WV of a Dp-brane or an Op-plane has thus $p+1$ dimensions.

The couplings of Dp-branes and Op-planes to the RR fields is:

$$S_p = -\mu_p \int_{p+1} C_{(p+1)}$$

with the charges $\mu_p^D = \sqrt{2\pi}$ and $\mu_p^O = -2^{p-4}\sqrt{2\pi}$.

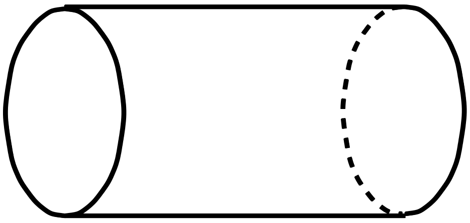
The RR potentials satisfy ${}^*H_{(p+2)} = H_{(8-p)}$. It follows that:

$$\mu_p = \text{electric for } C_{(p+1)} \text{ and magnetic for } C_{(7-p)}$$

$$\mu_{6-p} = \text{magnetic for } C_{(p+1)} \text{ and electric for } C_{(7-p)}$$

Therefore: Dp/Op interact electrically with Dp/Op and magnetically with D6-p/O6-p.

Interactions among D-branes e O-planes

$Z_A =$ 

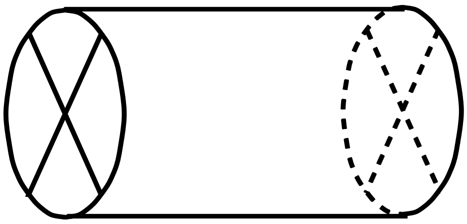
Loop : R + NS

Tree : RR + NSNS

$Z_M =$ 

Loop : R + NS

Tree : RR + NSNS

$Z_K =$ 

Loop : RR + NSNS

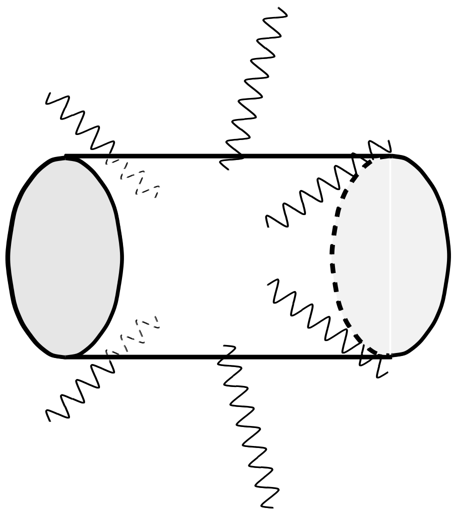
Tree : RR + NSNS

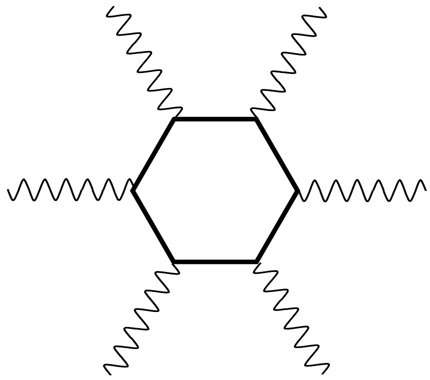
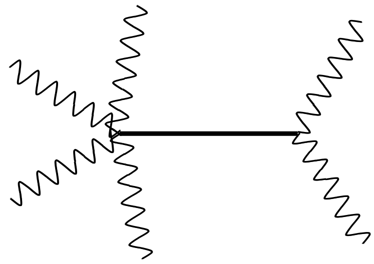
Anomalies

There is a precise correspondence between sectors in the two channels. In particular, the anomalous part of the loop channel always corresponds to the magnetic part of the tree channel.

The complete diagram cannot be anomalous, since it is finite. There must therefore be a cancellation mechanism that involves magnetic interactions.

The cancellation mechanism is:

$\mathcal{A} =$ 

$\Rightarrow$  $+ \ \Sigma$ 

This implies non-minimal couplings of D-branes and O-planes to RR fields:

$$S_p = -\mu_p \int_{p+1} \Big(C_{(p+1)} + Y_{(2)} \wedge C_{(p-1)} + Y_{(4)} \wedge C_{(p-3)} + \cdots \Big)$$

ANOMALIES AND INDICES

Anomalies can be interpreted as the non-invariance of the integration measure of the functional intergral.

Fujikawa

Gauge/gravitational anomalies in D dimensions are related to the chiral anomaly in $D+2$ dimensions. This is a topological index of the fiber bundles specifying the backgound.

Alvarez-Gaumé, Witten

Chiral anomalies

Consider a generic theory with a field φ interacting with A and ω, possessing a symmetry generated by Q, and define:

$$e^{-\Gamma^{\rm eff}(A,\omega)} = \int \mathcal{D}\varphi^\dagger \mathcal{D}\varphi \, e^{-S(\varphi,A,\omega)}$$

Under an infinitesimal transformation $\varphi \to \varphi + i\alpha Q \varphi$, $S \to S$ but:

$$\mathcal{D}\varphi^\dagger \mathcal{D}\varphi \to \mathcal{D}\varphi^\dagger \mathcal{D}\varphi \, e^{-i\alpha \mathrm{Tr}[Q]}$$

where Tr is on the spectrum of the kinetic operator K.

This translates into an anomalous variation $\delta\Gamma^{\rm eff} = i\alpha Z$, with:

$$Z = \mathrm{Tr}[Q]$$

Basic examples:

- Spinor in $D=2n$: $K=i\not{D}$ and $Q=\Gamma_{D+1}$.
- Tensor in $D=4n+2$: $K=\mathcal{D}=d+d^*$ and $Q=*_D$. (The kinetic operator is $K^2=\Box$. The rank is $D/2$)

When $\{Q,K\}=0$ and $Q^2=1$, as in these cases, it is easy to verify that:

$$
\begin{aligned}
Z &= \#(K=0,Q=+1)-\#(K=0,Q=-1) \\
&= \mathrm{index}(K)
\end{aligned}
$$

Chiral anomalies are thus indices, which depend on characteristic classes of the gauge and the tangent bundles. These can be computed using index theorems.

Alternatively, one can perform a direct calculation of the regularized trace:

$$
Z=\lim_{t\to 0}\mathrm{Tr}[Q\,e^{-tK^2}]
$$

This is the partition function of a SQM with hamiltonian K^2 and supercharge K, admitting a symmetry charge Q. Only zero-energy states contribute, and the result is independent of t. The appropriate SQM's are dimensional reductions of the NSM from $1+1$ to $0+1$ dimensions.

The most relevant characteristic classes are:

$$\mathrm{ch}(F) = \mathrm{tr}\,\exp i\tfrac{F}{2\pi} \qquad \Big(\mathrm{tr}\,\mathbb{1} + \tfrac{i}{2\pi}\,\mathrm{tr}\,F + \cdots\Big)$$

$$\widehat{A}(R) = \prod_{a=1}^{D/2} \frac{\frac{R_a}{4\pi}}{\sinh\frac{R_a}{4\pi}} \qquad \Big(1 + \tfrac{1}{48(2\pi)^2}\,\mathrm{tr}\,R\wedge R + \cdots\Big)$$

$$\widehat{L}(R) = \prod_{a=1}^{D/2} \frac{\frac{R_a}{2\pi}}{\tanh\frac{R_a}{2\pi}} \qquad \Big(1 - \tfrac{1}{6(2\pi)^2}\,\mathrm{tr}\,R\wedge R + \cdots\Big)$$

$$e(R) = \prod_{a=1}^{D/2} \frac{R_a}{2\pi} \qquad \Big(\tfrac{1}{(D/2)!\,(4\pi)^{D/2}}\,\mathrm{tr}\,R\wedge R\wedge\cdots\wedge R\Big)$$

For the two basic examples mentioned above, one finds:

$$\boxed{\begin{aligned} Z_\Gamma &= \mathrm{index}(i\not{D}) = \int \mathrm{ch}(F)\wedge\widehat{A}(R)\Big|_D \\ Z_* &= -\frac{1}{8}\,\mathrm{index}(\mathcal{D}) = -\frac{1}{8}\int \widehat{L}(R)\Big|_D \end{aligned}}$$

Gauge and gravitational anomalies

The anomaly $\mathcal{A}$ under a local symmetry is computed in a similar way. The subspaces with $Q = \pm 1$ contribute with opposite signs and one finds:

$$\mathcal{A} = \mathrm{Tr}[Q\,\delta]$$

$\mathcal{A}$ must satisfy the WZ consistency condition. This implies that it is the descent of a closed and gauge-invariant $D+2$-form $I(F,R)$. Defining $I = dI^{(0)}$ and $\delta I^{(0)} = dI^{(1)}$, the general solution is:

$$\boxed{\mathcal{A} = 2\pi i \int I^{(1)}}$$

One can verify that the form I from which $\mathcal{A}$ in D dimensions descends is given by Z in $D+2$ dimensions:

$$\boxed{\int I = Z}$$

INFLOW MECHANISM

It can happen that a consistent theory admits as vacuum a topological defect on which chiral zero-modes live. The anomaly that is localized on the WV must then be automatically cancelled thanks to the couplings with the rest of space time.

Callan, Harvey

This is the case of string vacua with D-branes and O-planes: they have a chiral spectrum but there cannot be anomalies. In general, there is a quantum anomaly on the WV, but this is cancelled by a classical inflow of anomaly from the bulk.

Classical anomalies can emerge from magnetic interactions. Consider some objects M_i in a space-time X, with RR couplings:

$$S_i = -\mu_i \int_{M_i} C \wedge Y_i$$

with $C = \sum_p C_{(p)}$ and $Y_i = Y_i(F, R)$.

These can be written as an integral on all of X by using the currents τ_{M_i}. Locally, $\tau_{M_i} \sim \delta(x^{d_i})\, dx^{d_i} \wedge \dots \wedge \delta(x^D)\, dx^D$, but globally τ_{M_i} is determined by $N(M_i)$.

The action for the RR fields then becomes:

$$S = -\frac{1}{2}\int_X H \wedge {}^*H + \sum_i \mu_i \int_X \tau_{M_i} \wedge H \wedge Y_i^{(0)}$$

The equations of motion and Bianchi identities read:

$$d^*H = \sum_i \mu_i \, \tau_{M_i} \wedge Y_i$$

$$dH = -\sum_i \mu_i \, \tau_{M_i} \wedge \bar{Y}_i$$

The Bianchi identity implies:

$$H = dC - \sum_i \mu_i \, \tau_{M_i} \wedge \bar{Y}_i^{(0)}$$

Since H must be invariant, C must transform as:

$$\delta C = \sum_i \mu_i \, \tau_{M_i} \wedge \bar{Y}_i^{(1)}$$

The RR complings are therefore anomalous:

$$\mathcal{A} = -i \sum_{i,j} \mu_i \, \mu_j \int_X \tau_{M_i} \wedge \tau_{M_j} \wedge \left(Y_i \wedge \bar{Y}_j\right)^{(1)}$$

The magnetic interaction of M_i with M_j produces an anomaly on their intersection M_{ij}. This follows from the property:

$$\tau_{M_i} \wedge \tau_{M_j} = \tau_{M_{ij}} \wedge e[N(M_{ij})]$$

Finally, the classical inflow of anomaly on any intersection M_{ij} can be written as $\mathcal{A}_{ij} = 2\pi i \int_{M_{ij}} I_{ij}^{(1)}$ with

$$\boxed{I_{ij} = -\frac{\mu_i \, \mu_j}{2\pi} Y_i \wedge \bar{Y}_j \wedge e[N(M_{ij})]}$$

Green, Harvey, Moore; Cheung, Yin

ANOMALY AND INFLOW FOR D-BRANES AND O-PLANES

Consider two Dp-branes and/or Op-planes superposed on $M \subset X$. The chiral fields on M can be read off from the corresponding partition functions:

DD : $Z_A \;\Rightarrow\;$ Chiral R spinor in the adjoint

DO : $Z_M \;\Rightarrow\;$ Chiral R spinor in the fundamental

OO : $Z_K \;\Rightarrow\;$ Self-dual RR tensor

These fields appear in multiplets obtained by reducing a single chiral field from X to M. Since $Q_D = Q_d\, Q_{D-d}$:

$$Q_D = + \;\rightarrow\; (Q_d, Q_{D-d}) = (+,+) \oplus (-,-)$$

$$Q_D = - \;\rightarrow\; (Q_d, Q_{D-d}) = (+,-) \oplus (-,+)$$

There can be anomalies only if the transverse space is curved, making the representations (, +) and (, −) inequivalent.

Anomaly from a reduced chiral spinor

The anomaly of a chiral spinor reduced from X to M is

$$\mathcal{A} = \lim_{t \to 0} \mathrm{Tr}\left[\Gamma^{D+1}\, \delta\, e^{-t(i\not{D})^2}\right]$$

Exponentiating δ, this can be written as $\mathcal{A} = 2\pi i Z^{(1)}$, where

$$Z = \lim_{t \to 0} \mathrm{Tr}\left[\Gamma^{D+1}\, e^{-t(i\not{D})^2}\right]$$

Mathematically, Z is the index of a spin complex:

$$Z = \mathrm{index}(i\not{D})$$

A spinor with chirality $\pm$ on all of X is a section of $S^{\pm}_{T(X)}$. On $M \subset X$, $T(X)$ decomposes into $T(M) \oplus N(M)$ and $S^{\pm}_{T(X)}$ into:

$$E^{\pm} = \left(S^{\pm}_{T(M)} \otimes S^{+}_{N(M)}\right) \oplus \left(S^{\mp}_{T(M)} \otimes S^{-}_{N(M)}\right)$$

Introducing also a gauge bundle, one obtains the complex

$$i\not{D}:\ \Gamma\Big[M, E^{+} \otimes V\Big] \to \Gamma\Big[M, E^{-} \otimes V\Big]$$

The index theorem yields:

$$\mathrm{index}(i\not{D}) = \int_M \mathrm{ch}(V)\,\mathrm{ch}(E^{+} \ominus E^{-})\,\frac{\mathrm{Td}[T(M^C)]}{e[T(M)]}$$

which leads to the result

$$Z = \int_M \mathrm{ch}(F) \wedge \frac{\widehat{A}(R)}{\widehat{A}(R')} \wedge e(R')$$

Physically, Z is a partition function. Given a SQM with $Q = i\not{D}$ and $(-1)^F = \Gamma^{D+1}$, Z becomes a Witten index:

$$Z = \mathrm{Tr}\left[(-1)^F\, e^{-tH}\right]$$

The appropriate SQM is obtained by reducing the NSM with N and D boundary conditions ($_{\mu,\nu,\ldots} \parallel M$; $_{i,j,\ldots} \perp M$):

$$x^{\mu} = x^{\mu} \qquad , \quad x^{i} = 0$$

$$\psi_1^{\mu} = \psi_2^{\mu} = \psi^{\mu} \ , \quad \psi_1^{i} = -\psi_2^{i} = \psi^{i}$$

The lagrangian is:

$$L = \frac{1}{2} g_{\mu\nu}\, \dot{x}^\mu \dot{x}^\nu + \frac{i}{2}\psi_\mu\Big(\dot{\psi}^\mu + \omega_\rho{}^\mu{}_\nu\, \dot{x}^\rho\, \psi^\nu\Big)$$
$$+\frac{i}{2}\psi_i\Big(\dot{\psi}^i + \omega_\rho{}^i{}_j\, \dot{x}^\rho\, \psi^j\Big) + \frac{1}{4} R_{\mu\nu ij}\psi^\mu\psi^\nu\psi^i\psi^j$$

with:

$$(-1)^F : (x,\psi) \rightarrow (x,-\psi)$$

Gauge fields can be incorporated in the standard way.

Because of $(-1)^F$, all the fields are periodic and

$$Z = \int_P \mathcal{D}x^\mu \int_P \mathcal{D}\psi^\mu \int_P \mathcal{D}\psi^i\, e^{-S}$$

For $t \rightarrow 0$, Z is dominated by constant trajectories:

$$x^\mu = x_0^\mu + \xi^\mu \quad , \quad x^i = 0$$
$$\psi^\mu = \psi_0^\mu + \lambda^\mu \, , \quad \psi^i = \psi_0^i + \lambda^i$$

It is sufficient to keep quadratic interactions with a maximal number of ψ_0. Considering also the gauge background, one finds:

$$L^{eff} = \frac{1}{2}\Big(\dot{\xi}_\mu\dot{\xi}^\mu + i\lambda_\mu\dot{\lambda}^\mu + i\lambda_i\dot{\lambda}^i + iR_{\mu\nu}\dot{\xi}^\mu\xi^\nu + R'_{ij}\lambda^i\lambda^j\Big)$$
$$+\frac{1}{2}R'_{ij}\psi_0^i\psi_0^j + iF$$

where

$$R_{\mu\nu} = \frac{1}{2}R_{\mu\nu\rho\sigma}(x_0)\psi_0^\rho\psi_0^\sigma \, , \quad R'_{ij} = \frac{1}{2}R_{ij\rho\sigma}(x_0)\psi_0^\rho\psi_0^\sigma$$
$$F = \frac{1}{2}F_{\mu\nu}(x_0)\psi_0^\mu\psi_0^\nu$$

The functional integral is easily computed:

$$Z = \int dx_0^\mu \int d\psi_0^\mu \,\mathrm{tr}\exp\{iFt\}\, \overbrace{\frac{\det_P(i\eta_{\mu\nu}\partial_\tau)}{\det_P(\eta_{\mu\nu}\partial_\tau^2 + iR_{\mu\nu}\partial_\tau)}}^{(2\pi t)^{-\frac{d}{2}}\prod_{a=1}^{d/2}\frac{R_a t/2}{\sinh R_a t/2}}$$

$$\underbrace{\det_P(i\eta_{ij}\partial_\tau + R'_{ij})}_{\prod_{a=1}^{d/2}\frac{\sinh R'_a t/2}{R'_a t/2}} \underbrace{\int d\psi_0^i \exp\left\{\frac{t}{2}R'_{ij}\psi_0^i\psi_0^j\right\}}_{\prod_{a=d/2}^{D/2} R'_a t}$$

Finally, one finds:

$$\boxed{Z = \int_M \mathrm{ch}(F) \wedge \frac{\widehat{A}(R)}{\widehat{A}(R')} \wedge e(R')}$$

Cheung, Yin; Scrucca, Serone

Anomaly from a reduced self-dual tensor

The anomaly of a self-dual tensor reduced from X to M is

$$\mathcal{A} = \frac{1}{4}\lim_{t\to 0}\mathrm{Tr}\left[*_D\, I\,\delta\, e^{-t\mathcal{D}^2}\right]$$

where I denotes a transverse reflection leaving fixed $M \subset X$ and $\mathcal{D} = d + d^*$ on all of X.

Exponentiating δ, this can be written as $\mathcal{A} = 2\pi i Z^{(1)}$, with

$$Z = -\frac{1}{8} \lim_{t\to 0} \mathrm{Tr}\left[*_D \, I \, e^{-t\mathcal{D}^2} \right]$$

Mathematically, Z is a G-index of the signature complex:

$$Z = -\frac{1}{8} \,\mathrm{index}_G(\mathcal{D})$$

More precisely:

$$\mathcal{D}: \; \Gamma\Big[X, {}^{+}\!\wedge T^*(X)\Big] \longrightarrow \Gamma\Big[X, {}^{-}\!\wedge T^*(X)\Big]$$
$$G : X \longrightarrow X \; \Big(I : (x^\mu, x^i) \longrightarrow (x^\mu, -x^i)\Big)$$

$G = \mathbf{Z}_2$ preserves the orientation since D and d must be even. It leaves $M \subset X$ fixed and acts as $+\mathbf{1}$ in $T(M)$ and $-\mathbf{1}$ in $N(M)$.

The G-index theorem yields:

$$\mathrm{index}_G(\mathcal{D}) = \int_M \frac{\mathrm{ch}(E^+ \ominus E^-)\,\mathrm{ch}(F^+ \ominus F^-)}{\mathrm{ch}(F)} \frac{\mathrm{Td}[T(M^C)]}{e[T(M)]}$$

where

$$E^\pm = {}^{\pm}\!\wedge T^*(M) \,, \quad F^\pm = {}^{\pm}\!\wedge N^*(M)$$
$$F = \oplus_i (-1)^i \wedge^i N^*(M)$$

Finally, one gets:

$$Z = -\frac{1}{8} \int_M \frac{\widehat{L}(R)}{\widehat{L}(R')} \wedge e(R')$$

Physically, Z is again a partition function. In this case, we need a SQM with $H = \mathcal{D}^2$ and a symmetry $\Omega = *_D$, in such a way that Z becomes a Witten index:

$$Z = -\frac{1}{8}\mathrm{Tr}\left[\Omega\, I\, e^{-tH}\right]$$

The needed SQM is the trivial dimensional reduction of the NSM,

$$L = \frac{1}{2} g_{MN}(x)\dot{x}^M \dot{x}^N + \frac{i}{2}\sum_{\alpha=1,2}\psi_{\alpha M}\left(\dot{\psi}_\alpha^M + \omega_M{}^M{}_N(x)\psi_\alpha^N\,\dot{x}^M\right)$$

$$+\frac{1}{4}R_{MNPQ}(x)\psi_1^M\psi_1^N\psi_2^P\psi_2^Q$$

with ($_{M,N,\dots} \in X$; $_{\mu,\nu,\dots} \parallel M$; $_{i,j,\dots} \perp M$):

$$\Omega : (x,\psi_1,\psi_2) \longrightarrow (x,-\psi_1,\psi_2)$$

$$I : (x^\mu, x^i; \psi_\alpha^\mu, \psi_\alpha^i) \longrightarrow (x^\mu, -x^i; \psi_\alpha^\mu, -\psi_\alpha^i)$$

Because of $\Omega\, I$, the periodicity of the fields is not the usual one and:

$$Z = -\frac{1}{8}\int_P \mathcal{D}x^\mu \int_A \mathcal{D}x^i \int_P \mathcal{D}\psi_1^\mu \int_A \mathcal{D}\psi_1^i \int_A \mathcal{D}\psi_2^\mu \int_P \mathcal{D}\psi_2^i\, e^{-S}$$

For $t \to 0$, Z is dominated by constant trajectories:

$$x^\mu = x_0^\mu + \xi^\mu \quad , \quad x^i = \xi^i$$

$$\psi_1^\mu = \psi_0^\mu + \lambda_1^\mu \, , \quad \psi_1^i = \lambda_1^i$$

$$\psi_2^\mu = \lambda_2^\mu \qquad , \quad \psi_2^i = \psi_0^i + \lambda_2^i$$

As before, it is sufficient to keep interactions that are quadratic in the fluctuations and contain a maximal number of ψ_0.

One finds:

$$
\begin{aligned}
L^{eff} = \frac{1}{2}\Big[& \dot\xi_\mu \dot\xi^\mu + \dot\xi_i \dot\xi^i + i\lambda_{1\mu}\dot\lambda_1^\mu + i\lambda_{1i}\dot\lambda_1^i + i\lambda_{2\mu}\dot\lambda_2^\mu + i\lambda_{2i}\dot\lambda_2^i \\
& + R_{\mu\nu}\left(i\,\dot\xi^\mu\xi^\nu + \lambda_2^\mu\lambda_2^\nu\right) + R'_{ij}\left(i\,\dot\xi^i\xi^j + \lambda_2^i\lambda_2^j\right)\Big] \\
& + \frac{1}{2}R'_{ij}\psi_0^i\psi_0^j
\end{aligned}
$$

where

$$
R_{\mu\nu} = \frac{1}{2}R_{\mu\nu\rho\sigma}(x_0)\psi_0^\rho\psi_0^\sigma\ , \quad R'_{ij} = \frac{1}{2}R_{ij\rho\sigma}(x_0)\psi_0^\rho\psi_0^\sigma
$$

The functional integral is easy to compute and yields:

$$
Z = -\frac{1}{8}\int dx_0^\mu \int d\psi_0^\mu \overbrace{\frac{\det_P(i\eta_{\mu\nu}\partial_\tau)\,\det_A(i\eta_{\mu\nu}\partial_\tau + R_{\mu\nu})}{\det_P(\eta_{\mu\nu}\partial_\tau^2 + iR_{\mu\nu}\partial_\tau)}}^{(\pi t)^{-\frac{d}{2}}\prod_{a=1}^{d/2}\frac{R_a t/2}{\tanh R_a t/2}}
$$

$$
\underbrace{\frac{\det_A(i\eta_{ij}\partial_\tau)\,\det_P(i\eta_{ij}\partial_\tau + R'_{ij})}{\det_A(\eta_{ij}\partial_\tau^2 + iR'_{ij}\partial_\tau)}}_{2^{-\cancel{\frac{D-d}{2}}}\prod_{a=1}^{d/2}\frac{\tanh R'_a t/2}{R'_a t/2}} \underbrace{\int d\psi_0^i \,\exp\left\{\frac{t}{2}R'_{ij}\psi_0^i\psi_0^j\right\}}_{2^{\cancel{\frac{D-d}{2}}}\prod_{a=d/2}^{D/2} R'_a t/2}
$$

Finally, the result is:

$$
\boxed{Z = -\frac{1}{8}\int_M \frac{\widehat{L}(R)}{\widehat{L}(R')}\wedge e(R')}
$$

Scrucca, Serone

Anomalous couplings

The anomalies on two coincident Dp-branes and/or Op-planes are

$$I_{DD} = \mathrm{ch}_{\mathbf{n}\otimes\bar{\mathbf{n}}}(F) \wedge \frac{\widehat{A}(R)}{\widehat{A}(R')} \wedge e(R')$$

$$I_{DO} = \mathrm{ch}_{\mathbf{n}\oplus\bar{\mathbf{n}}}(2F) \wedge \frac{\widehat{A}(R)}{\widehat{A}(R')} \wedge e(R')$$

$$I_{OO} = -\frac{1}{8}\frac{\widehat{L}(R)}{\widehat{L}(R')} \wedge e(R')$$

Assigning the RR couplings

$$S_{D,O} = \sqrt{2\pi}\int C \wedge Y_{D,O}$$

one finds the inflows

$$I_{DD} = -Y_D \wedge \bar{Y}_D \wedge e(R')$$

$$I_{DO} = -\left(Y_D \wedge \bar{Y}_O + Y_O \wedge \bar{Y}_D\right) \wedge e(R')$$

$$I_{OO} = -Y_O \wedge \bar{Y}_O \wedge e(R')$$

The cancellation of anomalies implies:

$$\boxed{\begin{aligned} Y_D &= \mathrm{ch}_{\mathbf{n}}(F) \wedge \sqrt{\frac{\widehat{A}(R)}{\widehat{A}(R')}} \\ Y_O &= -2^{p-4}\sqrt{\frac{\widehat{L}(R/4)}{\widehat{L}(R'/4)}} \end{aligned}}$$

STRING COMPUTATION OF ANOMALIES AND INFLOWS

Computation of the total anomaly

The anomalies of a theory are determined by one-loop diagrams in which one of the external particles has an unphysical polarization. The sum of these amplitudes measures the variation of the effective action.

The amplitudes to be considered are those with odd spin structure on the surfaces $\Sigma = A$, M and K. Their generating functional has the form:

$$\mathcal{A}_\Sigma = \int_0^\infty dt \left\langle J \, V^{\not{\text{phy}}} \, e^{-(S_0 + V^{\text{phy}})} \right\rangle_\Sigma$$

where $t \in [0, \infty[$ is the modulus of Σ, J is the supercurrent of the superconformal theory, and V denotes the vertex operator for a photon or a graviton.

This amplitude does not vanish, because the unphysical vertex with longitudinal polarization is not zero, but only BRST-trivial:

$$V^{\not{\text{phy}}} = \left[Q, \hat{V}^{\not{\text{phy}}}\right]$$

Since the physical vertices are BRST-invariant, $\left[Q, \hat{V}^{\text{phy}}\right] = 0$, one can move Q to act on J, which is then transformed into the energy momentum tensor:

$$\left[Q, J\right] = T$$

The insertion of T generates a deformation with respect to the variation of the metric. Since the theory is conformal, the result is a derivative with respect to the modulus:

$$\mathcal{A}_\Sigma = \int_0^\infty dt\,\frac{d}{dt}\Big\langle\,\hat{V}^{\mathrm{p\not{h}y}}\,e^{-(S_0+V^{\mathrm{phy}})}\,\Big\rangle_\Sigma$$

In consistent string models, the absence of divergences implies that $\mathcal{A}_A + \mathcal{A}_M + \mathcal{A}_K = 0$.

Inami, Kanno, Kubota; Polchinski, Cai

The dominant part of $\mathcal{A}_\Sigma$ can be computed by using effective vertices that are bilinear in ψ_0:

$$V^{\mathrm{phy}} \simeq V^{\mathrm{eff}}(F,R)$$
$$\hat{V}^{\mathrm{p\not{h}y}} \simeq V^{\mathrm{eff}}(F^{(1)},R^{(1)})$$

The role of the unphysical vertex is to generate the descent of the remaining partition function:

$$Z_\Sigma(t) = \Big\langle\,e^{-(S_0+V^{\mathrm{eff}})}\,\Big\rangle_\Sigma$$

The anomaly then takes the standard form $\mathcal{A}_\Sigma = 2\pi i \int I_\Sigma^{(1)}$, with:

$$\boxed{I_\Sigma = Z'_\Sigma(\infty) - Z'_\Sigma(0)}$$

This is the analogue of Fujikawa's method in string theory.

Scrucca, Serone

In the low-energy limit. the vertices simplify and $Z(t)$ becomes a topological index independent of t. Thus, $I_\Sigma = 0$ thanks to the cancellation between quantum anomalies ($Z_\Sigma(\infty)$) and classical inflows ($Z_\Sigma(0)$) associated to the same Σ.

The polynomials on D-branes and O-planes are given by:

$$I_{DD} = Z'_A = \frac{1}{4}\,\mathrm{Tr}'_R\left[(-1)^F\, e^{-tH}\right]$$

$$I_{DO} = Z'_M = \frac{1}{4}\,\mathrm{Tr}'_R\left[\Omega\, I\,(-1)^F\, e^{-tH}\right]$$

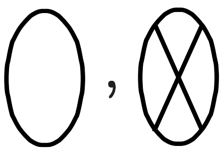
$$I_{OO} = Z'_K = \frac{1}{8}\,\mathrm{Tr}'_{RR}\left[\Omega\, I\,(-1)^{F+\tilde{F}}\, e^{-tH}\right]$$

These are indices receiving contributions only from modes that are constant along the spatial WS direction, with $H = 0$. In this way, one recovers the SQM introduced before.

Clearly, one obtains the same results as before for anomalies and anomalous couplings.

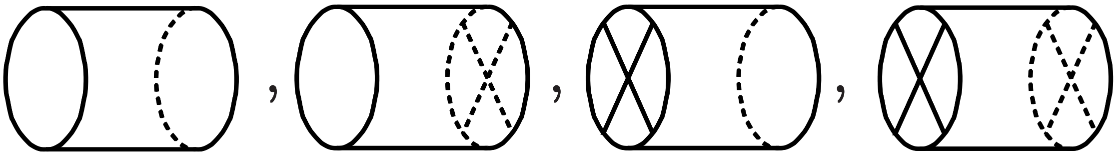
Direct computation of the RR couplings

The actual presence of the correct RR couplings can be verified by computing amplitudes on $\Sigma = D$ and C.



Li; Craps, Roose; Stefanski

Alternatively, one can compute magnetic RR interactions between D-branes and O-planes on $\Sigma = A$, M and K, and extract the charges by factorization.



Morales, Scrucca, Serone

Duality arguments

Some of the anomalous couplings have been predicted using various string dualities.

Bershadski, Sadov, Vafa; Dasgupta, Jatkar, Mukhi

CONCLUSIONS

- D-branes and O-planes have anomalous RR couplings that are crucial for consistency.
- The VM anomalies are canceleld through an inflow mechanism.
- Anomalies and inflows can be computed directly in string theory.