SEQUESTERING BY GLOBAL SYMMETRIES IN CALABI-YAU STRING MODELS

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- Soft scalar masses in supergravity
- Sequestering and global symmetries
- Calabi-Yau heterotic string models
- Kähler potential and contact terms
- Structure of scalar masses

Based on works with C. Andrey

SOFT SCALAR MASSES IN SUPERGRAVITY

General structure of soft scalar masses

In a supergravity theory with Kähler potential K and superpotential W, the soft scalar masses induced for the visible superfields Q^α when the hidden superfields X^i break supersymmetry is given by:

$$m^2_{\alpha\bar\beta} = -\Big(R_{\alpha\bar\beta i\bar\jmath} - \frac{1}{3} g_{\alpha\bar\beta} g_{i\bar\jmath}\Big) F^i \bar F^{\bar\jmath}$$

This result can be equivalently rewritten in terms of the Kähler function $\Omega = -3\, e^{-K/3}$, and in particular the wave-function $\Omega_{\alpha\bar\beta}$ of the visible superfields as a function of the hidden superfields:

$$m^2_{\alpha\bar\beta} = 3\,\Omega^{-1}\Big(\Omega_{\alpha\bar\beta i\bar\jmath} - \Omega^{-1\bar\delta\gamma}\Omega_{\bar\delta\alpha i}\Omega_{\gamma\bar\beta\bar\jmath}\Big) F^i \bar F^{\bar\jmath}$$

$$= 3\,\Omega^{-1}\Big(\Omega_{\alpha\bar\beta}\big|_D - \Omega^{-1\bar\delta\gamma}\big|\,\Omega_{\bar\delta\alpha}\big|_F\,\Omega_{\gamma\bar\beta}\big|_{\bar F}\Big)$$

FLAVOR PROBLEM AND SEQUESTERING

Supersymmetric flavor problem

The flavor structure of $m^2_{\alpha\bar{\beta}}$ is a priori generic, and this causes problems at the phenomenological level. One should then find some mechanism that naturally forces $m^2_{\alpha\bar{\beta}}$ to be approximately flavor-universal.

Sequestering

Randall, Sundrum 1999
Anisimov, Dine, Graesser, Thomas 2002

If the visible and hidden sectors are separated by an extra dimension, the classical contribution $m^2_{\alpha\bar{\beta}}$ is very restricted, whereas the quantum correction $\Delta m^2_{\alpha\bar{\beta}}$ is flavor-universal and insensitive to UV details.

In the minimal case with only the gravity multiplet in the bulk $m^2_{\alpha\bar{\beta}} = 0$.
In the general case involving also vector multiplets in the bulk $m^2_{\alpha\bar{\beta}} \neq 0$, but its form is very special and one may hope to force it to vanish.

APPROXIMATE GLOBAL SYMETRIES

Contact terms and global symmetries

Schmaltz, Sundrum 2006
Kachru, McAllister, Sundrum 2007

For generic sequestered models, we expect a Kähler function

$$\Omega \simeq Q^{\alpha}\bar{Q}^{\bar{\alpha}} + X^{i}\bar{X}^{\bar{\imath}} + \frac{1}{2}M^{-2}\sum_{a}\Big(c^{a}_{\alpha\bar{\beta}}Q^{\alpha}\bar{Q}^{\bar{\beta}} + c^{a}_{i\bar{\jmath}}X^{i}\bar{X}^{\bar{\jmath}}\Big)^{2}$$

This induces soft scalar masses given by

$$m^{2}_{\alpha\bar{\beta}} \simeq M^{-2}\Big[-c^{a}_{\alpha\bar{\beta}}c^{a}_{i\bar{\jmath}}F^{i}\bar{F}^{\bar{\jmath}} + (c^{a}c^{b})_{\alpha\bar{\beta}}c^{a}_{i\bar{\jmath}}F^{i}X^{\bar{\jmath}}c^{b}_{p\bar{q}}X^{p}\bar{F}^{\bar{q}}\Big]$$

If there are global symmetries associated to the currents $J^{a}_{X} = c^{a}_{i\bar{\jmath}}X^{i}\bar{X}^{\bar{\jmath}}$, the Ward identities $D^{2}J^{a}_{X} \simeq 0$ imply that $J^{a}_{X}|_{F} \simeq 0$ and $J^{a}_{X}|_{D} \simeq 0$, or:

$$c^{a}_{i\bar{\jmath}}F^{i}\bar{X}^{\bar{\jmath}} \simeq 0 \qquad c^{a}_{i\bar{\jmath}}F^{i}\bar{F}^{\bar{\jmath}} \simeq 0$$

In such a situation the Goldstino direction is restricted and one finds:

$$m^{2}_{\alpha\bar{\beta}} \simeq 0$$

HETEROTIC STRING MODELS

Heterotic M-theory on a Calabi-Yau

Horava,Witten 1996
Lukas, Ovrut, Stelle, Waldram 1999

Let us consider a generic heterotic string model based on a Calabi-Yau internal manifold M and a stable holomorphic vector bundle E over it. This also arises from M-theory on $M \times S^1/Z_2$ with two sequestered branes, in the weekly coupled limit where the size of S^1/Z_2 is small.

The 4D effective theory can be lifted to a 5D theory with localized brane sectors containing visible and hidden matter fields Q^α and X^i, and a bulk sector containing in particular Kähler moduli fields T^A.

Each non-minimal Kähler modulus T^a comes along with a heavy vector multiplet V^a, which couples non-trivially and induces contact terms in the effective Kähler function Ω once integrated out.

DERIVATION OF THE EFFECTIVE THEORY

Reduction of the weakly coupled heterotic string

The light 4D fields arise from the possible zero-modes of the 10D fields. The Q^α, X^i come from harmonic 1-forms in $H^1(M, E_Q)$, $H^1(M, E_X)$, while the T^A come from harmonic $(1,1)$-forms in $H^{1,1}(M)$:

$$Q^\alpha \Leftrightarrow u_\alpha \qquad X^i \Leftrightarrow u_i \qquad T^A \Leftrightarrow \omega_A$$

The effective K for the light fields may be derived by working out their kinetic terms by reduction on M and comparing with the general structure of supergravity theories. The effective W can be imagined to be arbitrary.

Discarding rather than integrating out heavy non-zero modes associated to non-harmonic forms is justified only whenever:

$$\mathrm{tr}(u_\alpha \wedge \bar{u}_{\bar{\beta}}) \ \text{ and } \ \mathrm{tr}(u_i \wedge \bar{u}_{\bar{\jmath}}) \ \text{ harmonic } \ \Leftrightarrow \ \omega_A$$

EFFECTIVE KAHLER POTENTIAL

General result for matter fields and Kähler moduli

Candelas, de la Ossa 1990
Paccetti Correia, Schmidt 2008
Andrey, Scrucca 2011

The effective Kahler potential is found to be

$$K = -\log\left[d_{ABC} J^A J^B J^C\right]$$

where

$$J^A = T^A + \bar{T}^A - c^A_{\alpha\bar{\beta}} Q^\alpha \bar{Q}^{\bar{\beta}} - c^A_{i\bar{\jmath}} X^i \bar{X}^{\bar{\jmath}}$$

The numerical quantities defining this result are:

$$d_{ABC} = \int \omega_A \wedge \omega_B \wedge \omega_C$$

$$c^A_{\alpha\bar{\beta}} = \int \omega^A \wedge \mathrm{tr}(u_\alpha \wedge \bar{u}_{\bar{\beta}}) \qquad c^A_{i\bar{\jmath}} = \int \omega^A \wedge \mathrm{tr}(u_i \wedge \bar{u}_{\bar{\jmath}})$$

CANONICAL PARAMETRIZATON

Canonical basis

With a suitable basis for the harmonic forms and fields, which defines an overall modulus T and some relative moduli T^a, one may rewrite K as:

$$K = -\log\left(J^3 - \frac{1}{2}JJ^aJ^a + \frac{1}{6}d_{abc}J^aJ^bJ^c\right)$$

where

$$J = T + \bar{T} - \frac{1}{3}Q^\alpha\bar{Q}^{\bar{\alpha}} - \frac{1}{3}X^i\bar{X}^{\bar{\imath}}$$

$$J^a = T^a + \bar{T}^a - c^a_{\alpha\bar{\beta}}Q^\alpha\bar{Q}^{\bar{\beta}} - c^a_{i\bar{\jmath}}X^i\bar{X}^{\bar{\jmath}}$$

Contact terms

The leading terms in the Kähler function for $J^a \ll J$ are

$$\Omega \simeq -3\,J + \frac{1}{2}J^{-1}J^aJ^a - \frac{1}{6}d_{abc}J^{-2}J^aJ^bJ^c$$

STRUCTURE OF SOFT SCALAR MASSES

Structure of soft terms

One may study the following reference point, around which the canonical parametrizaton is particularly convenient:

$$T \simeq \frac{1}{2} \qquad T^a \simeq 0 \qquad Q^\alpha \simeq 0 \qquad X^i \simeq 0$$

Using the computed K and imagining a generic W, one obtains:

$$m^2_{\alpha\bar\beta} \simeq -c^a_{\alpha\bar\beta} c^a_{i\bar\jmath} F^i \bar F^{\bar\jmath} - \Big(\frac{1}{3}\delta_{\alpha\bar\beta}\delta_{ab} + (d_{abc}c^c - c^a c^b)_{\alpha\bar\beta}\Big) F^a \bar F^{\bar b}$$
$$- c^a_{\alpha\bar\beta} F^a \bar F + \mathrm{c.c.}$$

This vanishes identically if the Goldstino direction is suitably constrained:

$$m^2_{\alpha\bar\beta} \simeq 0 \;\Leftrightarrow\; F^a \simeq 0 \;\text{ and }\; c^a_{i\bar\jmath} F^i \bar F^{\bar\jmath} \simeq 0$$

SEQUESTERING BY GLOBAL SYMMETRIES

Approximate global symmetries

Andrey, Scrucca 2011

The Goldstino direction can be guaranteed to point in a direction for which $m^2_{\alpha\bar{\beta}} \simeq 0$ by postulating that the following transformations represent two approximate symmetries not only of K but also of W:

$$\delta^1_a T^b = i\delta^b_a \;\Leftrightarrow\; F^a \simeq 0$$
$$\delta^2_a X^i = -ic^a_{\bar{\jmath}i} X^j \;\Leftrightarrow\; c^a_{i\bar{\jmath}} F^i \bar{F}^{\bar{\jmath}} \simeq 0$$

Clearly δ^1_a always form a group $U(1)^{\#}$ and give exact symmetries of K. However δ^2_a only form a group H if $c^a_{i\bar{\jmath}}$ generate a closed algebra and only represent exact symmetries of K if d_{abc} is a symmetric invariant of this algebra. We conclude that:

Sequestering by symmetries possible for some Calabi-Yau models

PARTICULAR CASE OF ORBIFOLDS

Symmetric structure in orbifolds

Andrey, Scrucca 2010

One special class of models where one is automatically in business is provided by orbifold constructions. In the untwisted sector, one finds that the forms u_α, u_i and ω_A are not only harmonic but actually constant. The formula for K is then always reliable and moreover one finds:

$$c^a_{\alpha\bar\beta}, c^a_{i\bar\jmath}: \quad \text{generators of some } H \subset SU(3)$$

$$d_{abc}: \quad \text{symmetric invariant of this } H \subset SU(3)$$

The scalar manifold is symmetric, and H belongs to the stability group. As a result, $U(1)^\# \times H$ is an exact symmetry of K, and imposing it also to W leads to vanishing masses. We thus conclude that

Sequestering by symmetries possible for all orbifold models

CONCLUSIONS

- Under some assumptions, the Kähler potential of heterotic models can be fully computed. The resulting soft scalar masses are found to vanish for suitably oriented Goldstino directions.
- The Goldstino direction can be forced to align along such special directions by relying on some global symmetries, but this appears to be possible only under some extra assumptions.
- A special class of models where this mechanism can always work is that of orbifold models. But it might be possible to put it at work also for other special classes of Calabi-Yau models.