SUPERSYMMETRY IN PARTICLE PHYSICS AND ITS SPONTANEOUS BREAKDOWN

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- Standard model of particle physics and beyond.
- Supersymmetry and implications.
- Supergravity and string theory.
- Constraints from supersymmetry breaking.

STANDARD MODEL OF PARTICLE PHYSICS

Scales of fundamental forces

- The electromagnetic force has a long range and is sizable at all lengths. It has no characteristic energy scale.
- The weak force has a short range and is sizable only below the Fermi length. Its characteristic energy scale is $\boldsymbol{M_{\mathrm{F}} \sim 10^{2}\ \mathrm{GeV}}$.
- The strong force has a more complex behavior. Its characteristic energy scale can be defined as the typical binding energy involved in hadrons: $\boldsymbol{M_{\mathrm{H}} \sim 1\ \mathrm{GeV}}$.
- The gravitational force has a long range but its coupling depends on the energy. Its characteristic energy scales are the Planck and Cosmological scales $\boldsymbol{M_{\mathrm{P}} \sim 10^{19}\ \mathrm{GeV}}$ and $\boldsymbol{M_{\Lambda} \sim 10^{-12}\ \mathrm{GeV}}$.

Structure of the standard model

The SM describes the electromagnetic, weak and strong interactions, with couplings $\alpha_{\mathbf{E}}$, $\alpha_{\mathbf{W}}$ and $\alpha_{\mathbf{S}}$. It ignores the gravitational interaction, whose effective coupling is $\alpha_{\mathbf{G}}(E) \sim (E/M_{\mathbf{P}})^2$.

It is a relativistic quantum field theory. It has a Lagrangian that involves a finite number of fields and parameters, and the structure of interactions is fixed by local gauge symmetries.

Particle content

Leptons:	$e^- \quad \mu^- \quad \tau^-$	Int. bos: $\gamma \quad W^\pm \quad Z^0$	Higgs: H
	$\nu_e \quad \nu_\mu \quad \nu_\tau$		
Quarks:	$u_\alpha \quad c_\alpha \quad t_\alpha$	Gluons: g_a	
	$\underbrace{d_\alpha \quad s_\alpha \quad b_\alpha}_{\text{flavor}}$		

Electroweak sector

Weinberg 1967
Salam 1968

The electromagnetic and weak interactions rest on a $SU(2) \times U(1)$ group. This allows 2 dimensionless couplings but forbids mass terms.

The mass terms are induced by partial spontaneous symmetry breaking: $SU(2) \times U(1) \to U(1)$. This is triggered at the classical level by a Higgs scalar, whose vev sets the scale $M_{\mathbf{F}}$.

The weak bosons but the photon get masses from gauge couplings of H.
The matter fermions get masses from extra Yukawa couplings with H.

Strong sector

Gross, Wilczek 1973
Politzer 1974

The strong interactions are based on an $SU(3)$ local gauge symmetry. This allows 1 dimensionless couplings constants. This symmetry remains unbroken and the gluons are massless.

The scale $M_{\mathbf{H}}$ arises in a more subtle way, through quantum effects, as the scale where these interactions become effectively strong.

Experimental perspective

- The SM has been verified with very good accuracy below $M_{\mathbf{F}}$. The Higgs particle has however not been observed until now: $m_H > \mathbf{115\ GeV}$.
- New experiments will soon allow to probe the SM beyond $M_{\mathbf{F}}$. This should lead to a clarification of the mechanism of electroweak symmetry breaking.

Theoretical perspective

- The SM is expected to be an effective theory valid at most up to $M_{\mathbf{P}}$, where gravitational interactions become important.
- The Higgs particle must be light enough for perturbation theory to be reliable: $m_H < \mathbf{1\ TeV}$.

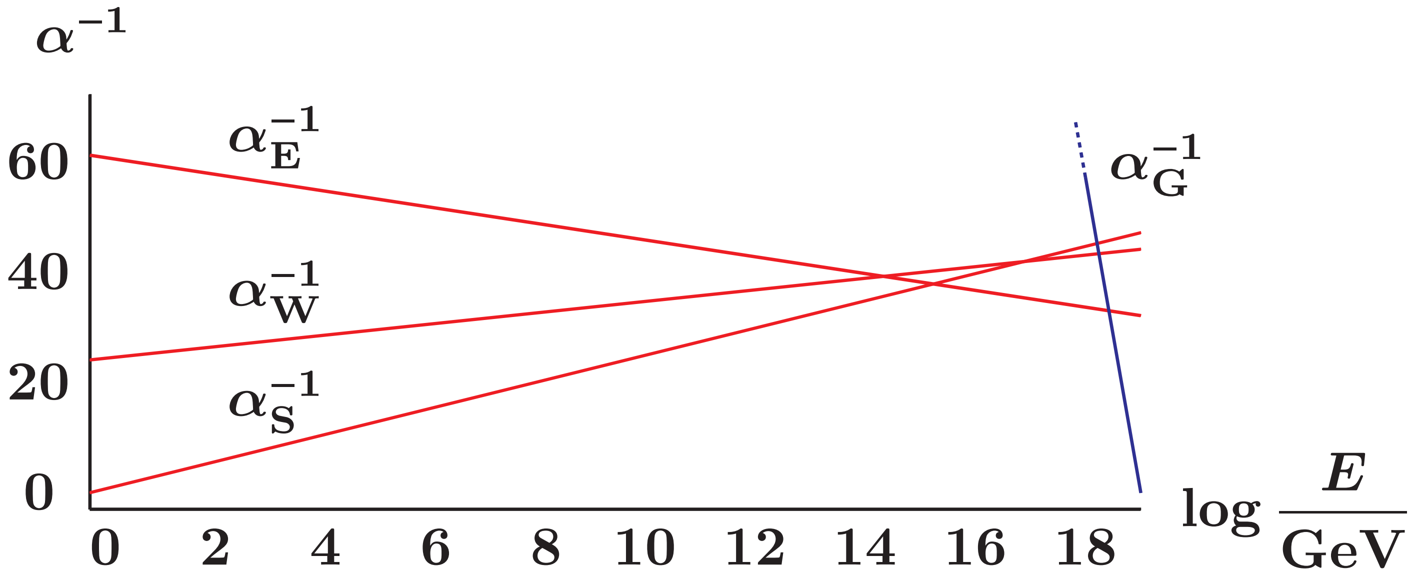
PHYSICS BEYOND THE STANDARD MODEL

Running of couplings

At the quantum level all the couplings become energy-dependent:

$$\alpha^{-1}_{\mathrm{E,W,S}}(E) = \alpha^{-1}_{\mathrm{E,W,S}} + \beta_{\mathrm{E,W,S}} \ln \frac{E}{M_{\mathrm{F}}} \qquad \alpha^{-1}_{\mathrm{G}}(E) = \left(\frac{E}{M_{\mathrm{P}}}\right)^{-2}$$

Extrapolating the values measured around M_{F} one finds:



Unification of gauge forces

Georgi, Glashow 1974

The three gauge forces are described in a very similar way in the SM. Moreover, their strengths become comparable at $M_{\mathrm{U}} \sim 10^{15-16}\ \mathrm{GeV}$. This suggests that a more fundamental theory might underly the SM, where theses gauge forces are unified.

Grand unified theories

- There exist candidates that are still field theories, but with a larger spontaneously broken local gauge symmetry and 1 coupling.
- They predict new particles with a mass of the order of M_{U}, and some constraints on the SM parameters.
- The most appealing examples are based on $SU(5)$ and $SO(10)$ groups.

Unification of gauge and gravitational forces

The proximity of $M_{\mathbf{U}}$ and $M_{\mathbf{P}}$ suggests that the gravitational force might also get unified with the gauge forces close to M_U.

Ideally, the ultimate theory should have $\mathbf{1}$ scale $M_{\mathbf{U}}$ and $\mathbf{1}$ coupling $\alpha_{\mathbf{U}}$, and all the other scales and parameters should be derived.

Fully unified theories

- The only candidates are string theories, where point particles are replaced by extended objects with typical size $M_{\mathbf{U}}^{-1}$.
- They predict infinitely many new particles, with mass of order $M_{\mathbf{U}}$. Their structure is complicated but constrained by consistency.
- Gravity is modified at $M_{\mathbf{U}} < M_{\mathbf{P}}$, where its effective strength is comparable to that of the gauge interactions: $(M_{\mathbf{U}}/M_{\mathbf{P}})^2 \sim \alpha_{\mathbf{U}}$. This implies that $M_{\mathbf{U}} \sim \alpha_{\mathbf{U}}^{\frac{1}{2}} M_{\mathbf{P}}$, which is roughly realized.

Hierarchy of scales

Assuming the existence of a fundamental scale close to $M_{\mathbf{P}}$, one may wonder how the much lower scales $M_{\mathbf{H}}$, $M_{\mathbf{F}}$ and $M_{\mathbf{\Lambda}}$ emerge.

- The hierarchy $M_{\mathbf{H}}/M_{\mathbf{P}}$ results from the slow quantum running of the dimensionless coupling $\alpha_{\mathbf{S}}$. $\Rightarrow$ Satisfactory.
- The hierarchy $M_{\mathbf{F}}/M_{\mathbf{P}}$ is achieved by a large tuning of the mass coupling μ^2 in the Higgs potential. $\Rightarrow$ Unsatisfactory.
- The hierarchy $M_{\mathbf{\Lambda}}/M_{\mathbf{P}}$ implies a huge tuning of cosmological constant parameter $\mathbf{\Lambda}$. $\Rightarrow$ Unsatisfactory.

New physics versus energy

We expect that two kinds of new physical features should show up around respectively $M_{\mathbf{F}}$ and $M_{\mathbf{P}}$.

SUPERSYMMETRY

Supersymmetry

Volkov, Akulov 1973
Wess, Zumino 1974

Supersymmetry is a unique extension of Poincaré spacetime symmetries. The new supertransformations mix bosons and fermions, and interfere with translations, rotations and boosts.

- It can be realized only on multiplets with the same number of bosons and fermions with equal masses.
- It limits quantum corrections, thanks to cancellations between bosonic and fermionic virtual partners.
- It is believed to play an important role concerning the consistency of any fundamental theory, and is predicted by string theory.

Minimal supersymmetric standard model

Dimopoulos, Georgi 1981

The MSSM is obtained by adding to the SM first a second Higgs field and then a superpartner for each ordinary field.

Particles

Leptons: $e^- \quad \mu^- \quad \tau^-$ $\quad$ Int. bos: $\gamma \; W^\pm \; Z^0$ $\quad$ Higgs: $H \; \phi_{1-2} \; \phi^\pm$

$\nu_e \quad \nu_\mu \quad \nu_\tau$

Quarks: $u_\alpha \quad c_\alpha \quad t_\alpha$ $\quad$ Gluons: g_a

$d_\alpha \quad s_\alpha \quad b_\alpha$

Sparticles

Sleptons: $\tilde{e}^- \quad \tilde{\mu}^- \quad \tilde{\tau}^-$ $\quad$ Chargini: $\chi^\pm_{1-2}$ $\quad$ Neutralini: χ^0_{1-4}

$\tilde{\nu}_e \quad \tilde{\nu}_\mu \quad \tilde{\nu}_\tau$

Squarks: $\tilde{u}_\alpha \quad \tilde{c}_\alpha \quad \tilde{t}_\alpha$ $\quad$ Gluini: $\tilde{g}_a$

$\tilde{d}_\alpha \quad \tilde{s}_\alpha \quad \tilde{b}_\alpha$

Supersymmetry breaking

Sparticles have not been observed experimentally. Supersymmetry must therefore be broken in such a way to induce a mass splitting with respect to particles, given by some scale $M_{\mathbf{B}}$.

The quantum corrections that tend to increase $M_{\mathbf{F}}$ are cut off at $M_{\mathbf{B}}$. The scale $M_{\mathbf{B}}$ should be close to $M_{\mathbf{F}}$ to solve the hierarchy naturalness problem.

The general paradigm is to add to the standard supersymmetric sector a breaking sector where supersymmetry is spontaneously broken, as well as a mediating sector that transmits the effect.

The effect of supersymmetry breaking can be parametrized by finitely many soft breaking terms, whose coefficients depend on the details of the breaking and mediation mechanisms:

$$\mathcal{L} = \mathcal{L}_{\mathbf{S}} + \mathcal{L}_{\mathbf{B}}$$

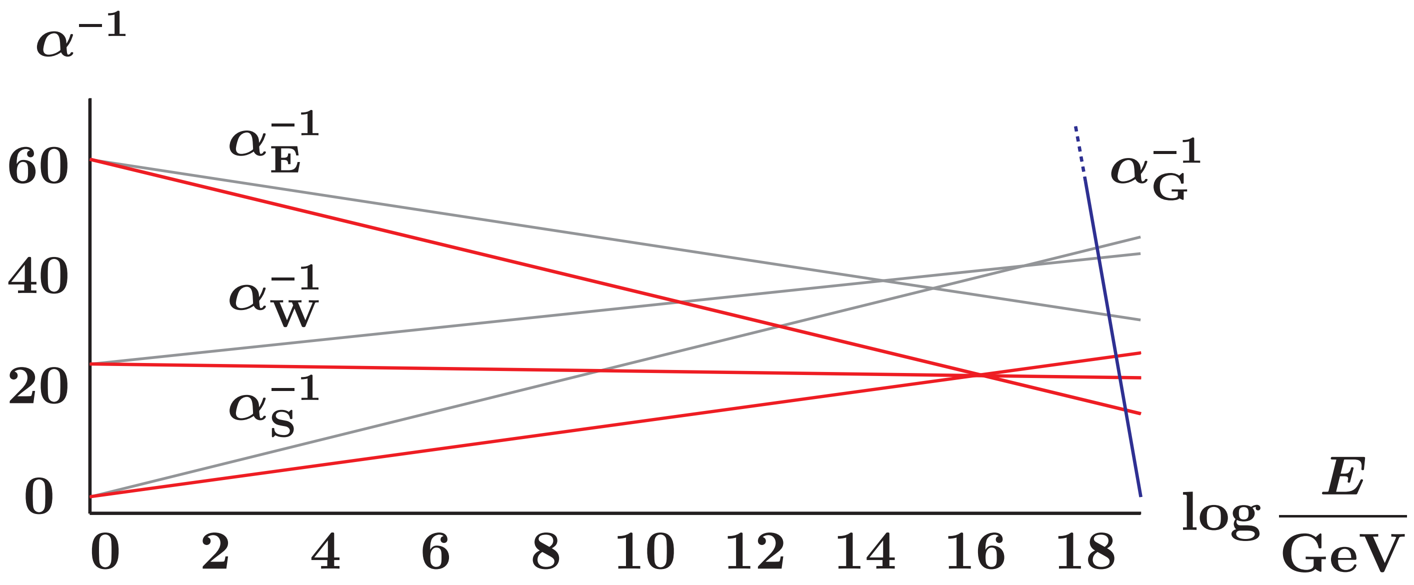
Phenomenological characteristics

The phenomenology of the MSSM can be studied as a function of the values of the soft terms. It works pretty well in general.

- The Higgs is predicted to be very light, with $m_H < 130\ \mathrm{GeV}$. This is barely compatible with the direct experimental bound $m_H > 115\ \mathrm{GeV}$.
- Sparticles imply potentially strong indirect effects also at energies below M_{F}, since M_{B} is close to it. These are harmless only if the soft terms posses some peculiar features.
- The lightest sparticle is stable, due to some discrete symmetry called R parity that is needed for proton stability, and represents a good candidate for the dark matter required by cosmology.

Impact on running of couplings

The presence of sparticles, besides particles, changes β in the running of gauge couplings. Extrapolating again the values around $M_{\mathbf{F}}$ one finds a more precise unification at $M_{\mathbf{U}} \sim \mathbf{10^{16-17}\ GeV}$:



SUPERGRAVITY AND STRING THEORY

Local supersymmetry

Freedman, van Nieuwenhuizen, Ferrara 1976

The Poincaré group of global spacetime symmetries can be promoted to a local symmetry. This gives rise in a very elegant way to Einstein's theory of gravity.

The supersymmetric extension of the Poincaré group can be promoted in a similar way to a local symmetry. This gives rise to supergravity.

Graviton: h

Gravitino: χ

Supergravity represents the union of supersymmetry and gravity, and emerges also as an effective description of string theory below M_{U}.

Gravity mediated supersymmetry breaking

Arnowitt, Chamseddine, Nath 1982
Barbieri, Ferrara, Savoy 1982

The standard and breaking sectors unavoidably interact through gravity. Supergravity represents therefore a natural mediation sector.

There exist effective interactions suppressed by powers of $M_{\mathbf{P}}^{-1}$ that mix the standard and breaking sectors. If supersymmetry is spontaneously broken at some scale $M_{\mathbf{S}}$, soft terms are induced:

$$M_{\mathbf{B}} = \frac{M_{\mathbf{S}}^2}{M_{\mathbf{P}}} \qquad\qquad M_{\mathbf{B}} \sim M_{\mathbf{F}} \;\text{ if }\; M_{\mathbf{S}} \sim \sqrt{M_{\mathbf{F}} M_{\mathbf{P}}}$$

Delicate points

- The potential sets both $M_{\mathbf{B}}$ and $M_{\mathbf{\Lambda}}$. Its is then unnatural to have $M_{\mathbf{\Lambda}} \ll M_{\mathbf{B}}$. $\Rightarrow$ Flatness.
- The scalars mediate new forces and affect nucleosynthesis. They have to be stabilized with $m > M_{\mathbf{B}}$. $\Rightarrow$ Stability.

Limitation on validity

At $M_{\mathbf{P}}$, the quantum theory gets out of control. Supergravity models are thus effective theories valid only up to $M_{\mathbf{P}}$, or better $M_{\mathbf{U}} = \alpha_{\mathbf{U}}^{\frac{1}{2}} M_{\mathbf{P}}$.

String theory

Green, Schwarz 1985
Dixon, Harvey, Witten 1985

String theory predicts supersymmetry and also extra space dimensions. In viable models, the former is broken at some scale $M_{\mathbf{B}}$ close to $M_{\mathbf{F}}$ and the latter are compactified at some scale $M_{\mathbf{C}}$ close to $M_{\mathbf{U}}$.

- The quantum theory is under control perturbatively, and below $M_{\mathbf{U}}$ it effectively reduces to a supergravity model.
- The compactification parameters and the coupling constant are dynamically fixed as vevs of light scalar fields called moduli.
- This neutral sector of the theory is a natural candidate for being the sector where supersymmetry is broken.

CONSTRAINTS FROM SUPERSYMMETRY BREAKING

SuperPoincaré algebra

Haag, Lopuszanski, Sohnius 1975

The structure of the superPoincaré algebra is essentially unique, and takes the following form:

$$[P_\mu, P_\nu] = 0$$

$$[P_\mu, M_{\rho\sigma}] = i\left(\eta_{\mu\rho} P_\sigma - \eta_{\mu\sigma} P_\rho\right)$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = i\left(\eta_{\nu\rho} M_{\mu\sigma} - \eta_{\mu\rho} M_{\nu\sigma} - \eta_{\nu\sigma} M_{\mu\rho} + \eta_{\mu\sigma} M_{\nu\rho}\right)$$

$$[Q_\alpha, P_\mu] = 0 \qquad [\bar{Q}_{\dot{\alpha}}, P_\mu] = 0$$

$$[Q_\alpha, M_{\rho\sigma}] = \frac{1}{2} \sigma_{\rho\sigma\,\alpha}{}^{\beta} Q_\beta \qquad [\bar{Q}_{\dot{\alpha}}, M_{\rho\sigma}] = -\frac{1}{2} \bar{\sigma}_{\rho\sigma\,\dot{\alpha}}{}^{\dot{\beta}} \bar{Q}_{\dot{\beta}}$$

$$\{Q_\alpha, Q_\beta\} = 0 \qquad \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0$$

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^\mu{}_{\alpha\dot{\beta}} P_\mu$$

Superfields

Salam, Strathdee 1974
Ferrara, Wess, Zumino 1974

The representations can be constructed on superfields $\Phi(x, \theta, \bar{\theta})$, which depend on 4 commuting coordinates x^μ and 4 additional anticommuting coordinates $\theta^\alpha, \bar{\theta}^{\dot{\alpha}}$, with

$$
\begin{aligned}
P_\mu &= i\partial_\mu \\
M_{\mu\nu} &= \Sigma_{\mu\nu} + i(x_\mu \partial_\nu - x_\nu \partial_\mu) + \frac{1}{2}\sigma_{\mu\nu\,\alpha}{}^{\beta}\theta^\alpha \partial_\beta - \frac{1}{2}\bar{\sigma}_{\mu\nu\,\dot{\alpha}}{}^{\dot{\beta}}\bar{\theta}^{\dot{\alpha}}\bar{\partial}_{\dot{\beta}} \\
Q_\alpha &= i\partial_\alpha - \sigma^\mu{}_{\alpha\dot{\beta}}\,\bar{\theta}^{\dot{\beta}}\partial_\mu \qquad \bar{Q}_{\dot{\alpha}} = -i\bar{\partial}_{\dot{\alpha}} + \bar{\sigma}^\mu{}_{\dot{\alpha}\beta}\,\theta^\beta \partial_\mu
\end{aligned}
$$

A superfield is a finite series in powers of $\theta^\alpha, \bar{\theta}^{\dot{\alpha}}$, whose coefficients are fields depending only on x^μ, with equal masses but different spins.

Invariants can be constructed in two ways:

- $\int d^4x\, d^2\theta\, d^2\bar{\theta}\, L(V)$ for general vector superfields V.
- $\int d^4x\, d^2\theta\, L(\Phi)$ for constrained chiral superfields Φ.

Spontaneous supersymmetry breaking

O'Raifeartaigh 1975
Fayet Iliopoulos 1974

SuperPoincaré symmetry is spontaneously broken to Poincaré symmetry if on the vacuum $P_\mu = M_{\mu\nu} = \mathbf{0}$ but $Q_\alpha \neq \mathbf{0}$. Only scalars get vevs, and there is a massless Goldstino fermion η_α.

Fields of different spins within each supermultiplet get their masses split. However, there is a simple sum rule on the mass matrix.

Metastability

Gomez-Reino, Scrucca 2006

The sGoldstino scalar $\tilde{\eta}$, partner of the massless Goldstino fermion, is dangerous for metastability, because its mass is induced by spontaneous supersymmetry breaking and is severely constrained.

Effects of gravity

Cremmer, Julia, Scherk, Ferrara, Girardello, van Nieuwenhuizen 1979

In supergravity, the Goldstino η_α is absorbed by the gravitino χ^μ_α through a superHiggs mechanism. One finds qualitatively similar results.

MINIMAL SETUP: ONLY CHIRAL MULTIPLETS

Theories with chiral multiplets

Zumino 1979
Freedman, Alvarez-Gaumé 1981

The simplest superfield is the chiral one, with component fields (ϕ, ψ_α, F):

$$\begin{aligned}\Phi(x, \theta, \bar\theta) = \phi(x) + \sqrt{2}\,\theta^\alpha \psi_\alpha(x) + \theta^\alpha\theta^\beta\epsilon_{\alpha\beta}F(x) \\ + i\,\theta^\alpha\bar\theta^{\dot\beta}\sigma^\mu{}_{\alpha\dot\beta}\partial_\mu\phi(x) + \frac{i}{\sqrt{2}}\,\theta^\alpha\theta^\beta\bar\theta^{\dot\gamma}\epsilon_{\alpha\beta}\bar\sigma^\mu{}_{\dot\gamma\delta}\partial_\mu\psi^\delta(x) \\ + \frac{1}{4}\,\theta^\alpha\theta^\beta\bar\theta^{\dot\gamma}\bar\theta^{\dot\delta}\epsilon_{\alpha\beta}\epsilon_{\dot\gamma\dot\delta}\Box\phi(x)\end{aligned}$$

The most general two-derivative action is parameterized by a real Kähler potential $K(\Phi, \bar\Phi)$ and a holomorphic superpotential $W(\Phi)$:

$$S = \int d^4x\, d^2\theta\, d^2\bar\theta\, K(\Phi, \bar\Phi) + \int d^4x\, d^2\theta\, W(\Phi) + \mathrm{h.c.}$$

The action for the components fields is worked out by performing the θ and $\bar\theta$ integrals. The result depends on the derivatives of the functions K and W with respect to the chiral multiplets Φ^i and their conjugate $\bar\Phi^{\bar\imath}$.

Component Lagrangian

The Lagrangian is found to be:

$$\begin{aligned} L = &K_{i\bar{\jmath}}(\phi,\bar{\phi})\left(-\partial_\mu \phi^i \partial^\mu \bar{\phi}^{\bar{\jmath}} - \frac{i}{2}\psi^i \sigma^\mu \partial_\mu \bar{\psi}^{\bar{\jmath}} + \text{h.c.} + F^i \bar{F}^{\bar{\jmath}}\right) \\ &+ \frac{1}{2} K_{i\bar{\jmath}k}(\phi,\bar{\phi})\left(-\psi^i \psi^k \bar{F}^{\bar{\jmath}} + i\psi^i \sigma^\mu \bar{\psi}^{\bar{\jmath}} \partial_\mu \phi^k\right) + \text{h.c.} \\ &+ \frac{1}{4} K_{i\bar{\jmath}k\bar{l}}(\phi,\bar{\phi})\psi^i \psi^k \bar{\psi}^{\bar{\jmath}} \bar{\psi}^{\bar{l}} + \left(W_i(\phi)F^i - \frac{1}{2}W_{ij}(\phi)\psi^i \psi^j\right) + \text{h.c.} \end{aligned}$$

The supersymmetry transformations act as follows:

$$\begin{aligned} \delta\phi^i &= \sqrt{2}\,\epsilon\,\psi^i \\ \delta\psi^i &= \sqrt{2}\,\epsilon\,F^i + \sqrt{2}i\,\bar{\epsilon}\,\bar{\sigma}^\mu \partial_\mu \phi^i \\ \delta F^i &= \sqrt{2}i\,\bar{\epsilon}\,\bar{\sigma}^\mu \partial_\mu \psi^i \end{aligned}$$

The auxiliary fields F^i have an algebraic equation of motion:

$$F^i = -K^{i\bar{\jmath}}(\phi,\bar{\phi})\bar{W}_{\bar{\jmath}}(\bar{\phi}) + \frac{1}{2}K_{ijk}(\phi,\bar{\phi})\psi^j \psi^k$$

Dynamics of physical fields

The Lagrangian for the physical fields ϕ^i and ψ^i has the form $L = T - V$ where:

$$T = -g_{i\bar{\jmath}}\,\partial_\mu \phi^i \partial^\mu \bar{\phi}^{\bar{\jmath}} - \frac{i}{2} g_{i\bar{\jmath}}\, \psi^i \sigma^\mu \nabla_\mu \bar{\psi}^{\bar{\jmath}} + \mathrm{h.c.}$$

$$V = g^{i\bar{\jmath}} W_i \bar{W}_{\bar{\jmath}} - \frac{1}{2} \nabla_i W_j \psi^i \psi^j + \mathrm{h.c.} + \frac{1}{4} R_{i\bar{\jmath}k\bar{l}}\, \psi^i \psi^k \bar{\psi}^{\bar{\jmath}} \bar{\psi}^{\bar{l}}$$

This is a supersymmetric non-linear sigma model. The target space is a Kahler manifold. The scalar ϕ^i are its coordinates, whereas the fermions ψ^i are related to the tangent space. The geometry is specified by K:

$$g_{i\bar{\jmath}} = K_{i\bar{\jmath}} \quad \Gamma^i_{jk} = K^{i\bar{l}} K_{\bar{l}jk} \quad R_{i\bar{\jmath}k\bar{l}} = K_{i\bar{\jmath}k\bar{l}} - K_{ik\bar{s}} K^{\bar{s}r} K_{r\bar{\jmath}\bar{l}}$$

The target-space and space-time covariant derivative are:

$$\nabla_i V^j = \partial_i V^j + \Gamma^j_{ik} V^k \qquad \nabla_\mu \phi^i = \partial_\mu \phi^i$$

$$\nabla_i V_j = \partial_i V_j - \Gamma^k_{ij} V_k \qquad \nabla_\mu \psi^i = \partial_\mu \psi^i - \Gamma^i_{jk} \partial_\mu \phi^j \psi^k$$

Vacuum

The most general Poincaré-symmetric vacuum configuration is:

$$\phi^i = \text{const.}\,, \quad \psi^i = 0\,, \quad F^i = \text{const.}$$

Stationarity of the potential energy implies that:

$$\nabla_i W_j F^j = 0$$

Supersymmetry acts on this as $\delta\phi^i = 0$, $\delta\psi^i = \sqrt{2}\,\epsilon\,F^i$, $\delta F^i = 0$. The order parameter for supersymmetry breaking is thus the norm of F^i. The Goldstino is $\eta = \bar{F}_i\psi^i$ and the sGoldstino $\tilde{\eta} = \bar{F}_i\phi^i$.

The fluctuations of the fields ϕ^i and ψ^i have a common wave function matrix given by $Z_{i\bar{\jmath}} = g_{i\bar{\jmath}}$, $Z_{ij} = 0$, and mass matrices given by:

$$m^2_{\phi\, i\bar{\jmath}} = \nabla_i W_k\, g^{k\bar{l}}\, \nabla_{\bar{\jmath}} \bar{W}_{\bar{l}} - R_{i\bar{\jmath}k\bar{l}} F^k \bar{F}^{\bar{l}} \qquad m_{\psi\, ij} = -\nabla_i W_j$$

$$m^2_{\phi\, ij} = -\nabla_i \nabla_j W_k F^k \quad \bar{m}^2_{\phi\, \bar{\imath}\bar{\jmath}} = -\nabla_{\bar{\imath}} \nabla_{\bar{\jmath}} \bar{W}_{\bar{k}} \bar{F}^{\bar{k}} \qquad \bar{m}_{\psi\, \bar{\imath}\bar{\jmath}} = -\nabla_{\bar{\imath}} \bar{W}_{\bar{\jmath}}$$

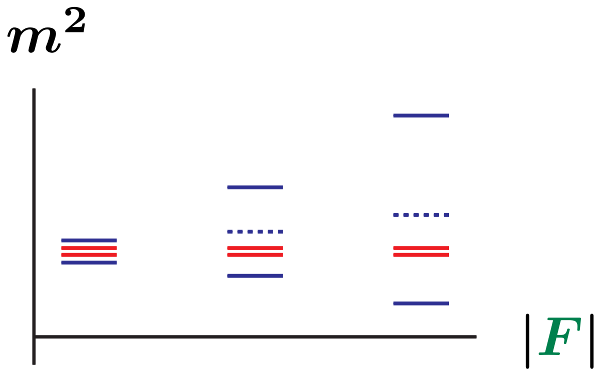
Physical masses

The full physical mass matrices for the $2n + 2n$ degrees of freedom are obtained after canonically normalizing the fields. One finds:

$$m^2_{\phi\,IJ} = \begin{pmatrix} m^2_{\phi\,i\bar{j}} & m^2_{\phi\,ij} \\ \bar{m}^2_{\phi\,\bar{i}\bar{j}} & m^2_{\phi\,\bar{i}j} \end{pmatrix} \quad m^2_{\psi\,IJ} = \begin{pmatrix} (m_\psi \bar{m}_\psi)_{i\bar{j}} & 0 \\ 0 & (\bar{m}_\psi m_\psi)_{\bar{i}j} \end{pmatrix}$$

For unbroken supersymmetry $F^i = 0$ and $m^2_{\phi\,IJ} = m^2_{\psi\,IJ}$. The masses are degenerate, and for each level there are two scalars and two fermions.

For broken supersymmetry $F^i \neq 0$ and $m^2_{\phi\,IJ} \neq m^2_{\psi\,IJ}$. In each group, the mean scalar and fermion mass shift and the two scalar masses split.



$$\Delta_{\phi\psi} : m^2_{\phi\,\mathrm{dia}} - m^2_\psi$$

$$\Delta_{\phi\phi} : m^2_{\phi\,\mathrm{off}}$$

Special features of mass spectrum

Grizaru, Rocek, Karlhede 1983
Gomez-Reino, Scrucca 2006

A first useful information concerns the shift between mean bosons and fermions masses. It can be extracted by taking the trace:

$$\mathrm{tr}[m_\phi^2] - \mathrm{tr}[m_\psi^2] = -2\, R_{i\bar{\jmath}}\, F^i \bar{F}^{\bar{\jmath}}$$

A second important information concerns the Goldstino and the mean sGoldstino masses. It can be extracted by looking in the direction F^i:

$$m_{\tilde{\eta}}^2 = -\frac{R_{i\bar{\jmath}k\bar{l}}\, F^i \bar{F}^{\bar{\jmath}} F^k \bar{F}^{\bar{l}}}{F^p \bar{F}_p}$$

$$m_\eta^2 = 0$$

We see that to achieve separation between partners and metastability, we need non-vanishing negative curvature. The effective theory then has a physical cut-off scale set by the curvature and is non-renormalizable.

Effects of gravity

Grizaru, Rocek, Karlhede 1983
Gomez-Reino, Scrucca 2006

In supergravity, assuming vanishing cosmological constant one finds:

$$\mathrm{tr}[m_\phi^2] - \mathrm{tr}[m_\psi^2] = -2\left(R_{i\bar{\jmath}} - \frac{1}{3}(n+1)\, g_{i\bar{\jmath}}\, M_{\mathrm{P}}^{-2}\right) F^i \bar{F}^{\bar{\jmath}}$$

and

$$m_{\tilde{\eta}}^2 = -\frac{\left(R_{i\bar{\jmath}k\bar{l}} - \frac{1}{3}\left(g_{i\bar{\jmath}}\, g_{k\bar{l}} + g_{i\bar{l}}\, g_{k\bar{\jmath}}\right) M_{\mathrm{P}}^{-2}\right) F^i \bar{F}^{\bar{\jmath}} F^k \bar{F}^{\bar{l}}}{F^p \bar{F}_p}$$

$$m_\chi^2 = \frac{1}{3}\, g_{i\bar{\jmath}}\, M_{\mathrm{P}}^{-2} F^i \bar{F}^{\bar{\jmath}}$$

We see that gravitational effects give a new negative contribution adding up to the curvature. They thus help, and we actually only need a curvature smaller than the critical value $\frac{2}{3} M_{\mathrm{P}}^{-2}$.

The minimal option is to use a would-be renormalizable theory with two sectors interacting only through gravity.

GENERAL SETUP: CHIRAL AND VECTOR MULTIPLETS

Theories with chiral and vector multiplets

Bagger, Witten 1982
Hull, Karlhede, Lindstrom, Rocek 1986

The general vector superfield has components $(\varphi, \psi_\alpha, F, \lambda_\alpha, A_\mu, D)$:

$$
\begin{aligned}
V(x,\theta,\bar\theta) = {} & 2\,\varphi(x) + \sqrt{2}\,\theta^\alpha \psi_\alpha(x) + \mathrm{h.c.} \\
& + \theta^\alpha\theta^\beta \epsilon_{\alpha\beta} F(x) + \mathrm{h.c.} - \theta^\alpha \bar\theta^{\dot\beta} \sigma^\mu{}_{\alpha\dot\beta} A_\mu(x) \\
& - i\,\theta^\alpha\theta^\beta\bar\theta^{\dot\gamma}\epsilon_{\alpha\beta}\Big(\bar\lambda_{\dot\gamma}(x) + \frac{1}{\sqrt{2}}\bar\sigma^\mu{}_{\dot\gamma\delta}\partial_\mu\psi^\delta(x)\Big) + \mathrm{h.c.} \\
& + \frac{1}{2}\,\theta^\alpha\theta^\beta\bar\theta^{\dot\gamma}\bar\theta^{\dot\delta}\epsilon_{\alpha\beta}\epsilon_{\dot\gamma\dot\delta}\Big(D(x) + \Box\varphi(x)\Big)
\end{aligned}
$$

Introducing a local invariance acting as $\delta V = \Lambda + \bar\Lambda$, this may be viewed as a gauge vector superfield in a Higgs phase, sum of a reduced vector multiplet $(\lambda_\alpha, A_\mu^\perp, D)$ and a chiral multiplet $(\varphi + iA^\parallel, \psi_\alpha, F)$.

The general case is thus described by chiral multiplets Φ^i interacting with gauge vector multiplets V^a coming with a local symmetry.

The most general two-derivative action involves a real Kähler potential $K(\Phi, \bar{\Phi}, V)$, a holomorphic superpotential $W(\Phi)$, some holomorphic Killing vectors $k_a^i(\Phi)$ and a holomorphic gauge kinetic matrix $f_{ab}(\Phi)$.

Component Lagrangian

One gets a supersymmetric gauged non-linear sigma model for the fields ϕ^i, ψ^i, A_μ^a and λ^a, while F^i and D^a are auxiliary fields. The symmetries are isometries of the target space, with $\delta\Phi^i = \Lambda^a k_a^i(\Phi)$. The gauge couplings and angles, matter charges and vector masses are:

$$h_{ab} = \mathrm{Re} f_{ab} \quad \theta_{ab} = \mathrm{Im} f_{ab} \quad q_{ai\bar{\jmath}} = i\nabla_i k_a^k g_{k\bar{\jmath}} \quad M_{ab}^2 = 2\, g_{i\bar{\jmath}} k_a^i \bar{k}_b^{\bar{\jmath}}$$

Supersymmetry breaking vacuum

Supersymmetry breaking is triggered by the Φ^i, and the V^a give only quantitative changes. In particular, at stationary points one finds:

$$M_{ab}^2 D^b - f_{ab}{}^c \theta_{cd} D^b D^d = 2\, q_{ai\bar{\jmath}} F^i \bar{F}^{\bar{\jmath}}$$

Physical masses and special features

Grizaru, Rocek, Karlhede 1983
Gomez-Reino, Scrucca 2007

The mass matrices at a stationary point breaking supersymmetry can be derived by proceeding as in the minimal case with chiral multiplets.

They display again two special features concerning their traces and their values along the supersymmetry breaking direction. One finds:

$$\mathrm{tr}[m^2_{\phi,A}] - \mathrm{tr}[m^2_{\psi,\lambda}] = -2\left(R_{i\bar{\jmath}} - h_{abi}h^{ac}h^{bd}h_{cd\bar{\jmath}}\right)F^i\bar{F}^{\bar{\jmath}} \\ +2\left(g^{i\bar{\jmath}}q_{ai\bar{\jmath}} - 2f_{ab}{}^{c}h^{bd}\theta_{cd}\right)D^a$$

and

$$m^2_{\tilde{\eta}} = -\frac{R_{i\bar{\jmath}k\bar{l}}\,F^i\bar{F}^{\bar{\jmath}}F^k\bar{F}^{\bar{l}}}{F^p\bar{F}_p} + \frac{M^2_{ab}\,D^aD^b}{F^p\bar{F}_p} \\ + \frac{h_{aci}h^{cd}h_{bd\bar{\jmath}}\,F^i\bar{F}^{\bar{\jmath}}D^aD^b}{F^p\bar{F}_p} + \frac{1}{4}\,\frac{h_{aci}g^{i\bar{\jmath}}h_{bd\bar{\jmath}}\,D^aD^bD^cD^d}{F^p\bar{F}_p}$$

$$m^2_{\eta} = 0$$

Effect of vector fields

Gomez-Reino, Scrucca 2006

The presence of the vector fields is generically to improve the situation, but only quantitatively, not qualitatively, much in the same way as gravity.

This is clear when the gauge symmetry is broken at a scale much higher that supersymmetry. The V^a then have a large supersymmetric mass $M^2_{ab} = 2\, g_{i\bar\jmath}\, k^i_a k^{\bar\jmath}_b$ and a small auxiliary field $D^a \simeq 2\, M^{-2ab} q_{bi\bar\jmath}\, F^i \bar F^{\bar\jmath}$.

At leading order, one then recovers then results of the same form as for just chiral multiplets, but with a shifted curvature, as can be verified also by integrating out the heavy V^a directly at the level of superfields.

$$R^{\rm eff}_{i\bar\jmath} = R_{i\bar\jmath} - 2\, q_{ai\bar\jmath}\, g^{k\bar l} q_{bk\bar l}\, M^{-2ab}$$
$$R^{\rm eff}_{i\bar\jmath k\bar l} = R_{i\bar\jmath k\bar l} - 2\left(q_{ai\bar\jmath}\, q_{bk\bar l} + q_{ai\bar l}\, q_{bk\bar\jmath}\right) M^{-2ab}$$

We need then a curvature smaller than the critical value $4\, q^2 M^{-2}$.

IMPLICATONS FOR STRING MODELS

Metastability for moduli

Covi, Gomez-Reino, Gross, Louis, Palma, Scrucca 2008

For the moduli sector of string models, one finds, in units where $M_{\mathrm{P}} = 1$:

$$K = -\log\big(S + \bar{S}\big) - \log\big(d_{ijk}(T + \bar{T})^i (T + \bar{T})^j (T + \bar{T})^k\big) + \cdots$$

$$W = \cdots$$

One may now check the value of the curvature and compare it to the critical value $\frac{2}{3}$ for metastability. One finds:

$$R = 2 + \cdots \ \text{along the field direction } S$$

$$R = \frac{2}{3} + \cdots \ \text{along a combination of } T^i$$

This means that S cannot dominate supersymmetry breaking, whereas T can dominate supersymmetry breaking only for certain choices of d_{ijk}. The situation where they both participate is instead allowed for any d_{ijk}.

Cremmer, Kounnas, Van Proeyen, Derendinger, Ferrara, de Witt, Girardello 1985
Fre, Trigiante, Van Proeyen 2002
Gomez-Reino, Louis, Scrucca 2009

Situation for extended supersymmetry

The moduli effective theory is very constrained by the higher-dimensional origin of these modes, which implies some features related to extended supersymmetry.

Already in the minimally extended case, the Kähler potential corresponds to Hyper-Kähler or Special-Kähler geometries, and the superpotential is induced by a gauging of isometries.

Metastability is harder to achieve. There exist no-go theorems for theories with only hyper multiplets or only abelian vector multiplets. But there are also a few positive examples using more general settings.

CONCLUSIONS

- General concepts like naturalness or unification suggest that a set of new ingredients should appear in a really fundamental theory of elementary particle physics.
- Supersymmetry is the most plausible and appealing new principle. It must however be spontaneously broken, and realizing this in viable way poses constraints.
- These constraints can be studied in full generality and can be used as a discrimination tool in the quest for the underlying fundamental theory.