SCALE INVARIANCE AND SPIN EFFECTS IN D-BRANE DYNAMICS

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- D-BRANES AND THEIR DYNAMICS.

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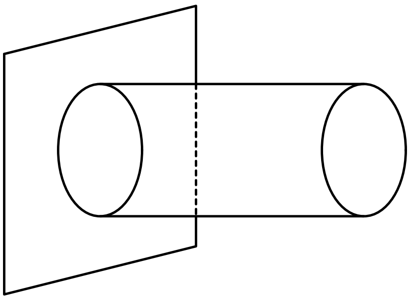
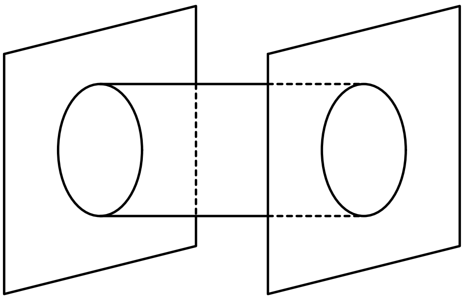
D-BRANES AND THEIR DYNAMICS

D-branes are non-perturbative objects defined as world-sheet boundaries with Dirichlet b.c..



Interaction with closed strings.



Interaction through the exchange of closed strings or fluctuation of open strings.

It is convenient to describe D-branes with a closed string state $|B\rangle$ implementing the b.c..

The couplings to closed strings can be extracted from the overlap of the corresponding state $|\Psi\rangle$ with $|B\rangle$

$$\langle\Psi\rangle = \langle B|\Psi\rangle$$

The interaction between D-branes is given by the correlation

$$\mathcal{A} = \langle B_1|\frac{1}{H}|B_2\rangle$$

The phase-shift for two // Dp-branes with $v_{1,2} = \tanh \pi\epsilon_{1,2}$ is (Bachas)

$$\mathcal{A}_{p-p} = \frac{V_p}{8i}(4\pi^2\alpha')^{4-p}\int_0^\infty \frac{dt}{(4\pi\alpha' t)^{\frac{8-p}{2}}} e^{-\frac{b^2}{4\pi\alpha' t}} \frac{\vartheta_1^4(i\frac{\epsilon}{2}|2it)}{\vartheta_1(i\epsilon|2it)\eta^9(2it)}$$

where $\epsilon = \epsilon_1 - \epsilon_2$, V_p is the volume and b the transverse distance.

At large distance, $b \gg l_s$, $\mathcal{A}_{p-p}$ is dominated by the exchange of massless closed strings and one finds

$$\mathcal{A}_{p-p} = V_p T_p^2 \frac{\frac{3}{4} + \frac{1}{4}\cosh 2\pi\epsilon - \cosh \pi\epsilon}{\sinh \pi\epsilon} G_{8-p}(b)$$

with

$$G_d(r) = \frac{1}{4\pi^{d/2}} \frac{\Gamma(\frac{d-2}{2})}{r^{d-2}}$$

This is the eikonal approximation of the phase-shift for two p-brane in SUGRA, with impact parameter b.

For $v \to 0$, $\mathcal{A}_{p-p} \sim v^3$ by SUSY (no force condition). Since

$$\text{Exchange of spin } s \text{ particle} \Rightarrow \cosh s\pi\epsilon$$

a cancellation occurs between the attractive dilaton and graviton exchange in the $NSNS$ sector and the repulsive (p+1)-form exchange in the RR sector (gravitational multiplet).

At short distance, $b \ll l_s$, $\mathcal{A}_{p-p}$ is dominated by loops of light open strings

$$\mathcal{A}_{p-p} = \frac{V_p}{2(4\pi)^{\frac{p}{2}}} \int_0^\infty \frac{dt}{t^{1+\frac{p}{2}}} e^{-(\frac{b}{2\pi\alpha'})^2 t} \frac{6 + 2\cosh 2\frac{\pi\epsilon}{2\pi\alpha'}t - 8\cosh \frac{\pi\epsilon}{2\pi\alpha'}t}{\sinh \frac{\pi\epsilon}{2\pi\alpha'}t}$$

This is the one-loop effective action for the SYM theory with 16 SUSY describing the open strings living on the two Dp-branes. For $b \neq 0$, the theory is broken to $U(1)$. By T-duality, the relative velocity corresponds to $E = \pi\epsilon$, and the particles running in the loop have $m = \frac{b}{2\pi\alpha'}$ and $e = \frac{1}{2\pi\alpha'}$.

Again, for $v \to 0$, $\mathcal{A}_{p-p} \sim v^3$ by SUSY (non-renorm. theorem). Since

$$\text{Loop of spin } s \text{ particle} \Rightarrow \cosh 2s\frac{\pi\epsilon}{2\pi\alpha'}t$$

a cancellation occurs between loops of spin 0 and 1 bosons and spin $\frac{1}{2}$ fermions (vector multiplet).

The two large and short distance limits match for $v \longrightarrow 0$. The reason is that for $v \longrightarrow 0$, $\mathcal{A}_{p-p}$ no longer depends on the scale l_s because the bosonic and fermionic oscillators cancel by SUSY. The exact non-relativistic potential is

$$V_{p-p} \sim \frac{v^4}{r^{7-p}}$$

The phase-shift for two // Dp and D(p+4)-branes is (Lifschytz)

$$\mathcal{A}_{p-p+4} = \frac{V_p}{8i}\left(4\pi^2\alpha'\right)^{-\frac{p(4-p)}{2}} \int_0^\infty \frac{dt}{(4\pi\alpha' t)^{\frac{4-p}{2}}} e^{-\frac{b^2}{4\pi\alpha' t}} \frac{\vartheta_1^2(i\frac{\epsilon}{2}|2it)\vartheta_2^2(i\frac{\epsilon}{2}|2it)}{\vartheta_1(i\epsilon|2it)\vartheta_2^2(0|2it)\eta^3(2it)}$$

At large distance, $b \gg l_s$, one finds

$$\mathcal{A}_{p-p+4} = V_p T_p T_{p+4} \frac{-\frac{1}{4} + \frac{1}{4}\cosh 2\pi\epsilon}{\sinh \pi\epsilon} G_{4-p}(b)$$

representing the eikonal approximation of the phase-shift in SUGRA.

For $v \longrightarrow 0$, $\mathcal{A}_{p-p+4} \sim v$ by SUSY (no force condition). Since

$$\text{Exchange of spin } s \text{ particle} \Rightarrow \cosh s\pi\epsilon$$

the cancellation occurs between the repulsive dilaton exchange and the attractive graviton exchange in the $NSNS$ sector, whereas the RR sector does not contribute (gravitational multiplet).

At short distance, $b \ll l_s$, one has

$$\mathcal{A}_{p-p} = \frac{V_p}{2(4\pi)^{\frac{p}{2}}} \int_0^\infty \frac{dt}{t^{1+\frac{p}{2}}} e^{-\left(\frac{b}{2\pi\alpha'}\right)^2 t} \frac{2 - 2\cosh \frac{\pi\epsilon}{2\pi\alpha'}t}{\sinh \frac{\pi\epsilon}{2\pi\alpha'}t}$$

which is the effective action for a SYM theory with 8 SUSY.

Again, for $v \longrightarrow 0$, $\mathcal{A}_{p-p+4} \sim v$ by SUSY (non-renorm. theorem). Since

$$\text{Loop of spin } s \text{ particle} \Rightarrow \cosh 2s\frac{\pi\epsilon}{2\pi\alpha'}t$$

the cancellation occurs occurs between loops of spin 0 bosons and spin $\frac{1}{2}$ fermions (hyper multiplet).

As before the two large and short distance limits match for $v \to 0$, because $\mathcal{A}_{p-p+4}$ does not depend on any scale in this limit. The exact non-relativistic potential is

$$V_{p-p+4} \sim \frac{v^2}{r^{3-p}}$$

The $\frac{v^4}{r^{7-p}}$ and $\frac{v^2}{r^{3-p}}$ potentials give only the universal part of the interactions in the Dp-Dp and Dp-D(p+4) systems.

Performing SUSY transformations, one can generate all the other spin-dependent leading interactions. This program can be carried out in the Green-Schwarz formalism, finding the scale-invariant interactions

$$V_{p-p} \sim \sum_{k=0}^{4} \frac{v^{4-k}}{r^{7-p+k}} \quad , \quad V_{p-p+4} \sim \sum_{k=0}^{2} \frac{v^{2-k}}{r^{3-p+k}}$$

BOUNDARY STATE IN THE G-S FORMALISM

Consider the Type II theories in the light-cone gauge. $X^+ = x^+ + p^+\tau$ whereas X^- is completely determined in terms of the transverse fields and after fixing the κ-symmetry, we are left with two left and right spinors, S^a e $\tilde{S}^a$, in the $\mathbf{8_s}$ representation of $SO(8)$.

The Fock space is constructed on a vacuum representing the algebra of S_0^a e $\tilde{S}_0^a$. The representation is $\mathbf{8_v} \oplus \mathbf{8_c}$ both for the left and the right parts, and

$$S_0^a|i\rangle = \frac{1}{\sqrt{2}}\gamma^i_{a\dot{a}}|\dot{a}\rangle \quad , \quad S_0^a|\dot{a}\rangle = \frac{1}{\sqrt{2}}\gamma^i_{a\dot{a}}|i\rangle$$
$$\tilde{S}_0^a|\tilde{i}\rangle = \frac{1}{\sqrt{2}}\gamma^i_{a\dot{a}}|\tilde{\dot{a}}\rangle \quad , \quad \tilde{S}_0^a|\tilde{\dot{a}}\rangle = \frac{1}{\sqrt{2}}\gamma^i_{a\dot{a}}|\tilde{i}\rangle$$

The light-cone coordinates $X^\pm$ automatically satisfy Dirichlet b.c., whereas the b.c. of the transverse coordinates X^i, $i = 1, 2, ..., 8$ can be chosen freely.

It is possible to define a Dp-brane-like configuration by choosing Neumann b.c. for $\mu = 1, 2, ..., p+1$ and Dirichlet b.c. for $I = p+2, ..., 8-p$. "Time" is temporarily identified with the 1 direction.

To recover the usual covariant description, it will be sufficient to perform the double analytic continuation $0 \leftrightarrow i\,1$ in the final results.

The 32 SUSY supercharges are

$$Q^a = \sqrt{2p^+}\oint d\sigma S^a \quad , \quad Q^{\dot{a}} = \frac{1}{\sqrt{p^+}}\gamma^i_{\dot{a}a}\oint d\sigma \partial X^i S^a$$
$$\tilde{Q}^a = \sqrt{2p^+}\oint d\sigma \tilde{S}^a \quad , \quad \tilde{Q}^{\dot{a}} = \frac{1}{\sqrt{p^+}}\gamma^i_{\dot{a}a}\oint d\sigma \bar{\partial} X^i \tilde{S}^a$$

and satisfy the $N = 2$ SUSY algebra.

The boundary state describing a Dp-brane is defined to implement the b.c. and preserve a combination of left and right SUSY.

We introduce the combinations

$$Q^a_{\pm} = \frac{1}{\sqrt{2}}\left(Q^a \pm iM_{ab}\tilde{Q}^b\right)$$
$$Q^{\dot{a}}_{\pm} = \frac{1}{\sqrt{2}}\left(Q^{\dot{a}} \pm iM_{\dot{a}\dot{b}}\tilde{Q}^{\dot{b}}\right)$$

satisfying the algebra

$$\{Q^a_+, Q^b_-\} = 2p^+\delta^{ab} \quad , \quad \{Q^{\dot{a}}_+, Q^{\dot{b}}_-\} = P^-\delta^{\dot{a}\dot{b}}$$
$$\{Q^a_+, Q^{\dot{a}}_-\} = \frac{1}{\sqrt{2}}\left[\gamma^i_{a\dot{a}}p^i + (M\gamma^i M^T)_{a\dot{a}}\tilde{p}^i\right]$$

and impose the BPS conditions

$$Q^a_+|B\rangle = 0 \quad , \quad Q^{\dot{a}}_+|B\rangle = 0 \;\Rightarrow\; Q^a_+, Q^{\dot{a}}_+ \text{ preserved}$$

$$Q^a_-|B\rangle \neq 0 \quad , \quad Q^{\dot{a}}_-|B\rangle \neq 0 \;\Rightarrow\; Q^a_-, Q^{\dot{a}}_- \text{ broken}$$

The bosonic b.c. imply

$$(\alpha^i_n + M_{ij}\tilde{\alpha}^j_{-n})|B\rangle = 0$$

with

$$M_{ij} = \begin{pmatrix} -I_{p+1} & 0 \\ 0 & I_{7-p} \end{pmatrix}$$

For the fermionic b.c., we make the ansatz

$$(S^a_n + iM_{ab}\tilde{S}^b_{-n})|B\rangle = 0$$

Consistency with the BPS conditions requires

$$(MM^T)_{ab} = \delta_{ab}$$

$$(M\gamma^i M^T)_{a\dot{a}} = M_{ij}\gamma^j_{a\dot{a}}$$

yielding

$$M_{ab} = (\gamma^1\gamma^2...\gamma^{p+1})_{ab}$$

$$M_{\dot{a}\dot{b}} = (\gamma^1\gamma^2...\gamma^{p+1})_{\dot{a}\dot{b}}$$

The solution for the boundary state is

$$|B\rangle = \exp \sum_{n>0} \left(\frac{1}{n} M_{ij} \alpha^i_{-n} \tilde{\alpha}^j_{-n} - i M_{ab} S^a_{-n} \tilde{S}^b_{-n} \right) |B_0\rangle$$

with the zero mode part

$$|B_0\rangle = M_{ij} |i\rangle |\tilde{j}\rangle - i M_{\dot{a}\dot{b}} |\dot{a}\rangle |\tilde{\dot{b}}\rangle$$

The complete boundary state in configuration space is

$$\begin{aligned} |B, \vec{x}\rangle &= (2\pi\sqrt{\alpha'})^{4-p} \delta^{(9-p)}(\vec{x}_0 - \vec{x}) |B\rangle \otimes |\vec{0}\rangle \\ &= (2\pi\sqrt{\alpha'})^{4-p} \int \frac{d^{9-p}q}{(2\pi)^{9-p}} e^{i\vec{q}\cdot\vec{x}} |B\rangle \otimes |\vec{q}\rangle \end{aligned}$$

Being BPS states, Dp-branes fill short supermultiplets on which the broken half of SUSY is realized. Performing an arbitrary broken SUSY transformation on $|B\rangle$, one obtains informations on the couplings of any components of the multiplet.

The state

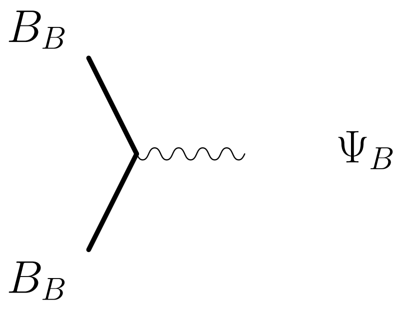
$$|B, \eta\rangle = e^{\eta Q^-} |B\rangle = \sum_{m=0}^{16} \frac{1}{m!} (\eta Q^-)^m |B\rangle$$

encodes the couplings to closed string states of a semi-classical current formed by an "in" and an "out" Dp-branes ($\eta = (\eta_a, \tilde{\eta}_{\dot{a}})$ and $Q^- = (Q^-_a, Q^-_{\dot{a}})$).

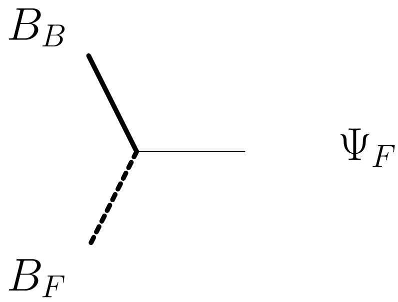
The sum corresponds to a multipole expansion, and terms with an even and odd number of Q^- are relevant for bosonic and fermionic currents, coupling to bosons and fermions respectively.

This is analog to what happens in SUGRA. The p-brane background has a Killing spinor and the SUSY transformations depend only on a projection of the parameter η.

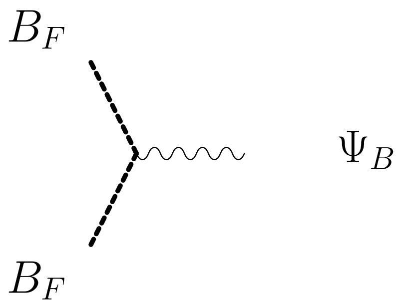
$$\langle \Psi_B | B \rangle =$$



$$\langle \Psi_F | Q^- | B \rangle =$$



$$\langle \Psi_B | Q^- Q^- | B \rangle =$$



For elastic scatterings, it suffice to consider even powers of Q^-. Moreover, in each

$$(\eta Q^-)^2 = (\eta_a Q^-_a + \tilde{\eta}_{\dot{a}} Q^-_{\dot{a}})^2$$

it will be enough to consider the SO(8) part $\eta_a Q^-_a \tilde{\eta}_{\dot{a}} Q^-_{\dot{a}}$, since the other simply reconstruct the covariant result.

Consider therefore the insertion of the operator

$$V_\eta = \eta_a Q^-_a \tilde{\eta}_{\dot{a}} Q^-_{\dot{a}}$$

which produces $|B\rangle_{(n)}$ corresponding to $2n$ SUSY transformations

$$|B\rangle_{(n)} = V^n_\eta |B\rangle$$

In this way, the boundary state $|B, \eta\rangle$ for a generic bosonic current is

$$|B, \eta\rangle = \sum_{n=0}^{8} \binom{2n}{n} \frac{V^n_\eta}{(2n)!} |B\rangle = \sum_{n=0}^{8} \frac{1}{(n!)^2} |B\rangle_{(n)}$$

For the zero mode part, $n \leq 4$ ($Q_0^{-a} \sim S_0^{-a}$, $Q_0^{-\dot{a}} \sim p^i \gamma^i_{\dot{a}a} S_0^{-a}$) and one obtains

$$|B_0\rangle_{(n)} = q_{i_1}...q_{i_n} \left[\eta_{[a_1}(\tilde{\eta}\gamma^{i_1})_{a_2}...\eta_{a_{2n-1}}(\tilde{\eta}\gamma^{i_n})_{a_{2n}]}\right] S_0^{-a_1}...S_0^{-a_{2n}}|B_0\rangle$$

Using the b.c. implemented by $|B_0\rangle$ and the antisymmetry of $[...]$, each S_0^- can be converted into $\sqrt{2}\, S_0$, all left-moving.

The S_0 satisfy the Fiertz identity

$$S_0^a S_0^b = \frac{1}{2}\delta^{ab} + \frac{1}{4}\gamma^{ij}_{ab} R_0^{ij}$$

in terms of the $SO(8)$ generators

$$R_0^{ij} = \frac{1}{4} S_0^a \gamma^{ij}_{ab} S_0^b$$

Using this property, $|B_0\rangle_{(n)} = V^n_{\eta 0}|B_0\rangle$ with

$$V^n_{\eta 0} = q_{i_1}...q_{i_n}\, \omega^{i_1...i_n}_{j_1...j_{2n}}(\eta) R_0^{j_1 j_2}...R_0^{j_{2n-1}j_{2n}}$$

and

$$\omega^{i_1...i_n}_{j_1...j_{2n}}(\eta) = \frac{1}{2^n}\left[\eta_{[a_1}(\tilde{\eta}\gamma^{i_1})_{a_2}...\eta_{a_{2n-1}}(\tilde{\eta}\gamma^{i_n})_{a_{2n}]}\right] \gamma^{j_1 j_2}_{a_1 a_2}...\gamma^{j_{2n-1}j_{2n}}_{a_{2n-1}a_{2n}}$$

The action of R_0^{ij} in the $\mathbf{8_v}$ and $\mathbf{8_c}$ representations is

$$R_0^{mn}|i\rangle = (\delta^{ni}\delta^{mj} - \delta^{mi}\delta^{nj})|j\rangle$$

$$R_0^{mn}|\dot{a}\rangle = -\frac{1}{2}\gamma^{mn}_{\dot{a}\dot{b}}|\dot{b}\rangle$$

and finally

$$|B_0\rangle_{(n)} = M^{(n)}_{ij}|i\rangle|\tilde{j}\rangle - iM^{(n)}_{\dot{a}\dot{b}}|\dot{a}\rangle|\tilde{\dot{b}}\rangle$$

with

$$M^{(n)}_{ij} = 2^n\, q_{i_1}...q_{i_n}\, \omega^{i_1...i_n}_{ik_1k_1...k_{n-1}k_{n-1}k_n}(\eta) M_{k_n j}$$

$$M^{(n)}_{\dot{a}\dot{b}} = \frac{1}{2^n}\, q_{i_1}...q_{i_n}\, \omega^{i_1...i_n}_{j_1...j_{2n}}(\eta)(\gamma^{j_1 j_2}...\gamma^{j_{2n-1}j_{2n}} M)_{\dot{a}\dot{b}}$$

For the oscillators, one could proceed similarly.

The generalization to moving Dp-branes is obtained through a Lorentz transformation. Assuming that "time" is the 1 direction, the boundary state for a Dp-brane moving along the 8 is

$$|B,\eta,\epsilon\rangle = e^{-i\pi\epsilon J^{18}}|B,\eta\rangle$$

For the zero mode part, the angular momentum is given by

$$J_{ij} = x_0^i p^j - x_0^j p^i - 2iR_0^{ij}$$

The bosonic part changes the momentum spectrum, whereas the fermionic one acts on $|B_0\rangle_{(n)}$. The net effect on $|B_0\rangle_{(n)}$ is

$$M_{ij}^{(n)} \longrightarrow M_{ij}^{(n)}(\epsilon) = (\Sigma(\epsilon)M^{(n)}\Sigma^T(\epsilon))_{ij}$$

$$M_{\dot{a}\dot{b}}^{(n)} \longrightarrow M_{\dot{a}\dot{b}}^{(n)}(\epsilon) = (\Sigma(\epsilon)M^{(n)}\Sigma^T(\epsilon))_{\dot{a}\dot{b}}$$

where $\Sigma(\epsilon)$ is the appropriate representation of the $SO(8)$ rotation

$$\Sigma_{ij}(\epsilon) = \begin{pmatrix} \cos\pi\epsilon & 0 & -\sin\pi\epsilon \\ 0 & I_6 & 0 \\ \sin\pi\epsilon & 0 & \cos\pi\epsilon \end{pmatrix}$$

$$\Sigma_{\dot{a}\dot{b}}(\epsilon) = \cos(\frac{\pi\epsilon}{2})\,\delta_{\dot{a}\dot{b}} - \sin(\frac{\pi\epsilon}{2})\,\gamma_{\dot{a}\dot{b}}^{18}$$

Again, for the oscillators one proceeds similarly.

Working exactly in ϵ would mask the role of SUSY. It is more convenient to expand the boost for $\epsilon \longrightarrow 0$, inserting the operator

$$V_\epsilon = -i\pi\epsilon J^{18}$$

whose zero mode part is

$$V_{\epsilon 0} = -2\pi\epsilon R_0^{18}$$

In this way

$$|B,\eta,\epsilon\rangle = \sum_{m=0}^{\infty}\frac{V_\epsilon^m}{m!}|B,\eta\rangle = \sum_{n=0}^{8}\sum_{m=0}^{\infty}\frac{V_\eta^n V_\epsilon^m}{(n!)^2 m!}|B\rangle$$

One-point functions

To obtain the couplings of a generic Dp-brane to massless closed strings, we compute the overlap of the corresponding states $|\Psi\rangle$ with $|B_0\rangle_{(n)}$

$$\Psi_{(n)} = \langle\Psi|B_0\rangle_{(n)}$$

The bosonic states are

$$|\Psi_{NSNS}\rangle = \xi_{mn}|m\rangle|\tilde{n}\rangle \quad , \quad \xi_{mn} \sim \delta_{mn}\,\phi + g_{mn} + b_{mn}$$

$$|\Psi_{RR}\rangle = C_{\dot{a}\dot{b}}|\dot{a}\rangle|\tilde{\dot{b}}\rangle \quad , \quad C_{\dot{a}\dot{b}} \sim \sum_k \frac{1}{k!} C^{(k)}_{m_1\ldots m_k} \gamma^{m_1\ldots m_k}_{\dot{a}\dot{b}}$$

One finds, apart from normalizations

$$\Psi^{NSNS}_{(n)} = q_{i_1}\ldots q_{i_n}\, \xi^{ij} \omega^{i_1\ldots i_n}_{ik_1k_1\ldots k_{n-1}k_{n-1}k_n}(\eta) M_{k_n j}$$

$$\Psi^{RR}_{(n)} = q_{i_1}\ldots q_{i_n} \sum_k \frac{1}{k!} C^{(k)}_{m_1\ldots m_k} \omega^{i_1\ldots i_n}_{j_1\ldots j_{2n}}(\eta) \mathrm{Tr}_S[\gamma^{m_1\ldots m_k}\gamma^{j_1j_2}\ldots\gamma^{j_{2n-1}j_{2n}} M]$$

From these one can read off all the couplings organized in a multipole expansion ($n = 0, 1, .., 4$). If $\mu, \nu, \ldots =$ Neumann and $I, J, \ldots =$ Dirichlet, using the symmetry properties of $\omega^{i_1\ldots i_n}_{j_1\ldots j_{2n}}(\eta)$ one finds

$$\Psi^{NSNS}_{(n)} \;\Rightarrow\; \begin{cases} \phi, g_{\mu\nu}, g_{IJ}, b_{\mu I} \quad , \quad n \text{ even} \\ g_{\mu I}, b_{\mu\nu}, b_{IJ} \quad , \quad n \text{ odd} \end{cases}$$

$$\Psi^{RR}_{(n)} \;\Rightarrow\; C_{(k)} \quad , \quad k = p+1-2n, \ldots, p+1+2n$$

n=0 (universal)

$$\Psi^{NSNS}_{(0)} = \xi_{ij} M^{ij}$$

$$\Psi^{RR}_{(0)} = \sum_k \frac{1}{k!} C^{(k)}_{m_1\ldots m_k} \mathrm{Tr}_S[\gamma^{m_1\ldots m_k} M]$$

These can be covariantized by introducing $M^{\mu\nu}$ with entry -1 for Neumann and $+1$ for Dirichlet directions, and $\mathcal{M} = \Gamma^0 ... \Gamma^p$. One finds

$$\Psi^{NSNS}_{(0)} = \xi_{\mu\nu} M^{\mu\nu}$$

$$\Psi^{RR}_{(0)} = \sum_k \frac{1}{k!} C^{(k)}_{\mu_1...\mu_k} \mathrm{Tr}_S[\gamma^{\mu_1...\mu_k} \mathcal{M}]$$

n=1 (dipole)

$$\Psi^{NSNS}_{(1)} = \xi_{ik} M^k{}_j \, (\eta\gamma^{ijl}\tilde{\eta}) q_l$$

$$\Psi^{RR}_{(1)} = \sum_k \frac{1}{k!} C^{(k)}_{m_1...m_k} \mathrm{Tr}_S[\gamma^{m_1...m_k}\gamma_{ij} M](\eta\gamma^{ijl}\tilde{\eta}) q_l$$

To covariantize, we introduce the Majorana-Weyl spinor ψ which in a chiral representation reads $\psi = \binom{\eta}{0}$ with $\eta = \binom{\eta^a}{\tilde{\eta}^{\dot{a}}}$. Defining

$$J^{\mu\nu\rho} = \bar{\psi}\Gamma^{\mu\nu\rho}\psi$$

one finds

$$\Psi^{NSNS}_{(1)} = \xi_{\mu\sigma} M^\sigma{}_\nu J^{\mu\nu\rho} q_\rho$$

$$\Psi^{RR}_{(1)} = \sum_k \frac{1}{k!} C^{(k)}_{\mu_1...\mu_k} \mathrm{Tr}_S[\Gamma^{\mu_1...\mu_k}\Gamma_{\mu\nu}\mathcal{M}] J^{\mu\nu\rho} q_\rho$$

n=2 (quadrupole)

$$\Psi^{NSNS}_{(2)} = \xi_{\mu\sigma} M^\sigma{}_\nu \, J^{\mu\rho\alpha} J_\rho{}^{\nu\beta} q_\alpha q_\beta$$

$$\Psi^{RR}_{(2)} = \sum_k \frac{1}{k!} C^{(k)}_{\mu_1...\mu_k} \mathrm{Tr}_S[\Gamma^{\mu_1...\mu_k}\Gamma_{\nu_1\nu_2}\Gamma_{\nu_3\nu_4}\mathcal{M}] J^{\nu_1\nu_2\alpha} J^{\nu_3\nu_4\beta} q_\alpha q_\beta$$

n=n (n-pole)

$$\Psi^{NSNS}_{(n)} = \xi_{\mu\sigma} M^\sigma{}_\nu J^{\mu\rho_1\alpha_1} ... J_{\rho_{n-1}}{}^{\nu\alpha_n} q_{\alpha_1} ... q_{\alpha_n}$$

$$\Psi^{RR}_{(n)} = \sum_k \frac{1}{k!} C^{(k)}_{\mu_1...\mu_k} \mathrm{Tr}_S[\Gamma^{\mu_1...\mu_k}\Gamma_{\nu_1\nu_2}...\Gamma_{\nu_{2n-1}\nu_{2n}}\mathcal{M}] J^{\nu_1\nu_2\alpha_1} ... J^{\nu_{2n-1}\nu_{2n}\alpha_n} q_{\alpha_1} ... q_{\alpha_n}$$

P-P INTERACTION

The phase-shift for two // Dp-branes with parameters η_i and ϵ_i is

$$\mathcal{A}_{p-p} = \frac{1}{16}\int_0^\infty dt\, \langle B_p, \eta_1, \epsilon_1, \vec{x}_1 | e^{-2\pi\alpha' t p^+ (P^- - p^-)} | B_p, \eta_2, \epsilon_2, \vec{x}_2 \rangle$$

where

$$P^- = \frac{1}{2p^+}\left[(p^i)^2 + \frac{1}{\alpha'}\sum_{n=1}^{\infty}(\alpha^i_{-n}\alpha^i_n + \tilde{\alpha}^i_{-n}\tilde{\alpha}^i_n + n\, S^a_{-n}S^a_n + n\, \tilde{S}^a_{-n}\tilde{S}^a_{-n})\right]$$

This can be rewritten as

$$\mathcal{A}_{p-p} = \frac{V_p\,(4\pi^2\alpha')^{4-p}}{16\sinh \pi\epsilon}\int_0^\infty dt \int \frac{d^{8-p}q}{(2\pi)^{8-p}}\, e^{i\vec{q}\cdot\vec{b}}\, e^{-\pi\alpha' t \vec{q}^2} Z_0(\eta_i,\epsilon_i) Z_{osc}(t,\eta_i,\epsilon_i)$$

with

$$Z_0(\eta_i,\epsilon_i) = \langle B_{p0}, \eta_1, \epsilon_1 | B_{p0}, \eta_2, \epsilon_2 \rangle$$

$$Z_{osc}(t,\eta_i,\epsilon_i) = \langle B_p, \eta_1, \epsilon_1 | e^{-2\pi\alpha' t p^+ P^-} | B_p, \eta_2, \epsilon_2 \rangle_{osc}$$

Case $\eta_i = 0$ and $\epsilon_i = 0$

The Dp-Dp system preserves $1/2$ of the SUSY and is therefore BPS. As a consequence

$$Z_0 = \mathrm{Tr}_V[1\!\!1] - \mathrm{Tr}_S[1\!\!1] = 8 - 8 = 0$$

$$Z_{osc}(t) = \prod_{n=1}^{\infty}\frac{(1-e^{-2\pi t n})^8}{(1-e^{-2\pi t n})^8} = 1$$

Integrating over the momentum and the modulus one finds

$$\mathcal{A}_{p-p} = V_{p+1}\, T_p^2\, (1-1)\, G_{9-p}(b)$$

This is zero but exact in α'.

Case $\eta_i = 0$ but $\epsilon_i \neq 0$

$$
\begin{aligned}
Z_0(\epsilon) &= \mathrm{Tr}_V[M^T(\epsilon_2)M(\epsilon_1)] - \mathrm{Tr}_S[M^T(\epsilon_2)M(\epsilon_1)] \\
&= 16\sin^4\frac{\pi\epsilon}{2} \sim v^4
\end{aligned}
$$

$$
Z_{osc}(t,\epsilon) = \prod_{n=1}^{\infty} \frac{|1-e^{i\pi\epsilon/2}e^{-2\pi tn}|^8}{|1-e^{i\pi\epsilon}e^{-2\pi tn}|^2(1-e^{-2\pi tn})^6} \sim 1
$$

After the analytic continuation $\epsilon \longrightarrow i\epsilon$, one finds

$$
\mathcal{A}_{p-p} = \frac{V_p}{8i}(4\pi^2\alpha')^{4-p}\int_0^\infty \frac{dt}{(4\pi\alpha' t)^{\frac{8-p}{2}}} e^{-\frac{b^2}{4\pi\alpha' t}} \frac{\vartheta_1^4(i\frac{\epsilon}{2}|2it)}{\vartheta_1(i\epsilon|2it)\eta^9(2it)}
$$

The behavior of $\mathcal{A}_{p-p}$ for $v \longrightarrow 0$ is completely determined by SUSY.

Notice that $Z_0(\epsilon)$ can be rewritten as a trace in a Type I theory

$$
Z_0(\epsilon) = \mathrm{Tr}_{S_0}[e^{V_{\epsilon 0}}]
$$

This is the analog of the integration over the fermionic zero modes in the path-integral representation of the open string vacuum amplitude.

The trace is 0 unless at least 8 zero modes S_0^a are inserted. The first $\neq 0$ is

$$
\begin{aligned}
t^{i_1\ldots i_8} &= \mathrm{Tr}_{S_0}[R_0^{i_1 i_2}R_0^{i_3 i_4}R_0^{i_5 i_6}R_0^{i_7 i_8}] \\
&= -\frac{1}{2}\epsilon^{i_1\ldots i_8} - \frac{1}{2}\left[\delta^{i_1 i_4}\delta^{i_2 i_3}\delta^{i_5 i_8}\delta^{i_6 i_7} + \text{perm.}\right] \\
&\quad + \frac{1}{2}\left[\delta^{i_2 i_3}\delta^{i_4 i_5}\delta^{i_6 i_7}\delta^{i_8 i_1} + \text{perm.}\right]
\end{aligned}
$$

Each $V_{\epsilon 0}$ provides 2 S_0^a and 1 ϵ, and expanding one check that $Z_0(\epsilon) \sim |v|^4$.

Expanding the whole e^{V_ϵ} in series

$$
Z_0(\epsilon) = \sum_{m=0}^{\infty}\frac{1}{m!}\mathrm{Tr}_{S_0}[V_{\epsilon 0}^m]
$$

$$
Z_{osc}(t,\epsilon) = \sum_{q=0}^{\infty}\frac{1}{q!}\langle B_p|V_\epsilon^q\, e^{-2\pi\alpha' t p^+ P^-}|B_p\rangle_{osc}
$$

We discard the effect of the boost on the bosonic zero modes.

We see that the first non-vanishing order in ϵ in $Z(t,\epsilon) = Z_0(\epsilon) Z_{osc}(t,\epsilon)$ receives a unique contribution from $m=4$ e $q=0$ ($\Leftrightarrow Z_{osc}(t,\eta_{1,2},\epsilon)=1$).

Therefore $Z(t,\epsilon) \to |v|^4$ and is independent of t, meaning that only BPS (massless) states contribute. The non-relativistic amplitude reduces to

$$\mathcal{A}_{p-p} = \frac{|v|^4}{8} V_{p+1}\, T_p^2\, G_{9-p}(r)$$

and is exact in α'.

Case $\eta_i \neq 0$ and $\epsilon_i \neq 0$

We can use the same strategy for the dependence on the SUSY parameters η_i. Expanding both in $\eta_{1,2}$ and ϵ one finds

$$Z_0(\eta_{1,2},\epsilon) = \sum_{n_1,n_2}^{n_1+n_2\leq 4} \sum_{m=0}^{\infty} \frac{1}{(n_1!)^2 (n_2!)^2 m!} \mathrm{Tr}_{S_0}[V_{\eta_1 0}^{n_1} V_{\eta_2 0}^{n_2} V_{\epsilon 0}^{m}]$$

$$Z_{osc}(t,\eta_{1,2},\epsilon) = \sum_{p_1,p_2}^{p_1+p_2\leq 8} \sum_{q=0}^{\infty} \frac{1}{(p_1!)^2 (p_2!)^2 q!} \langle B_p| V_{\eta_1}^{p_1} V_{\eta_2}^{p_2} V_{\epsilon}^{q}\, e^{-2\pi\alpha' t p^+ P^-} |B_p\rangle_{osc}$$

To get $Z_0(\eta_{1,2},\epsilon) \neq 0$ we need $n_1+n_2+m \geq 4$. The leading behavior is obtained by taking $n_1+n_2+m=4$ and $p_1,p_2,q=0$ ($\Leftrightarrow Z_{osc}(t,\eta_{1,2},\epsilon)=1$) and is independent of the modulus t. Each $V_{\eta 0}$ brings also a momentum q producing a derivative on the propagator G_{9-p}. The behavior is therefore

$$\mathcal{A}_{p-p}^{(n_1,n_2)} \sim \eta_1^{2n_1}\eta_2^{2n_2} |v|^{4-n_1-n_2} \partial^{n_1+n_2} G_{9-p}(r) \sim \eta_1^{2n_1}\eta_2^{2n_2} \frac{v^{4-n_1-n_2}}{r^{7-p+n_1+n_2}}$$

All these interactions are exact in α', that is scale-invariant. They can be written in terms of $t^{i_1\ldots i_8}$ and the tensor $\omega^{i_1\ldots i_n}_{j_1\ldots j_{2n}}(\eta)$.

Case $n_1 + n_2 = 0$

$$\mathcal{A}^{(0,0)}_{p-p} = \frac{V_{p+1}}{8 \cdot 4!} T_p^2 \, v_{m_1} v_{m_2} v_{m_3} v_{m_4} \, t^{1m_1 1m_2 1m_3 1m_4} \, G_{9-p}(r)$$

Case $n_1 + n_2 = 1$

$$\mathcal{A}^{(1,0)}_{p-p} = \frac{V_{p+1}}{8 \cdot 3!} T_p^2 \, v_{m_1} v_{m_2} v_{m_3} \, t^{i_1 i_2 1m_1 1m_2 1m_3} \, \omega^{j_1}_{i_1 i_2}(\eta_1) \, \partial_{j_1} G_{9-p}(r)$$

Case $n_1 + n_2 = 2$

$$\mathcal{A}^{(2,0)}_{p-p} = \frac{V_{p+1}}{8 \cdot 2!^3} T_p^2 \, v_{m_1} v_{m_2} \, t^{i_1 \ldots i_4 1m_1 1m_2} \, \omega^{j_1 j_2}_{i_1 \ldots i_4}(\eta_1) \, \partial_{j_1} \partial_{j_2} G_{9-p}(r)$$

$$\mathcal{A}^{(1,1)}_{p-p} = \frac{V_{p+1}}{8 \cdot 2!^2} T_p^2 \, v_{m_1} v_{m_2} \, t^{i_1 \ldots i_4 1m_1 1m_2} \, \omega^{j_1}_{i_1 i_2}(\eta_1) \, \omega^{j_2}_{i_3 i_4}(\eta_2) \, \partial_{j_1} \partial_{j_2} G_{9-p}(r)$$

Case $n_1 + n_2 = 3$

$$\mathcal{A}^{(3,0)}_{p-p} = \frac{V_{p+1}}{8 \cdot 3!^2} T_p^2 \, v_{m_1} \, t^{i_1 \ldots i_6 1m_1} \, \omega^{j_1 j_2 j_3}_{i_1 \ldots i_6}(\eta_1) \, \partial_{j_1} \partial_{j_2} \partial_{j_3} G_{9-p}(r)$$

$$\mathcal{A}^{(2,1)}_{p-p} = \frac{V_{p+1}}{8 \cdot 2!^2} T_p^2 \, v_{m_1} \, t^{i_1 \ldots i_6 1m_1} \, \omega^{j_1 j_2}_{i_1 \ldots i_4}(\eta_1) \, \omega^{j_3}_{i_5 i_6}(\eta_2) \, \partial_{j_1} \partial_{j_2} \partial_{j_3} G_{9-p}(r)$$

Case $n_1 + n_2 = 4$

$$\mathcal{A}^{(4,0)}_{p-p} = \frac{V_{p+1}}{8 \cdot 4!^2} T_p^2 \, t^{i_1 \ldots i_8} \, \omega^{j_1 \ldots j_4}_{i_1 \ldots i_8}(\eta_1) \, \partial_{j_1} \partial_{j_2} \partial_{j_3} \partial_{j_4} G_{9-p}(r)$$

$$\mathcal{A}^{(3,1)}_{p-p} = \frac{V_{p+1}}{8 \cdot 3!^2} T_p^2 \, t^{i_1 \ldots i_8} \, \omega^{j_1 j_2 j_3}_{i_1 \ldots i_6}(\eta_1) \, \omega^{j_4}_{i_7 i_8}(\eta_2) \, \partial_{j_1} \partial_{j_2} \partial_{j_3} \partial_{j_4} G_{9-p}(r)$$

$$\mathcal{A}^{(2,2)}_{p-p} = \frac{V_{p+1}}{8 \cdot 2!^4} T_p^2 \, t^{i_1 \ldots i_8} \, \omega^{j_1 j_2}_{i_1 \ldots i_4}(\eta_1) \, \omega^{j_3 j_4}_{i_5 \ldots i_8}(\eta_2) \, \partial_{j_1} \partial_{j_2} \partial_{j_3} \partial_{j_4} G_{9-p}(r)$$

P-P+4 INTERACTION

The phase-shift for two // Dp and D(p+4)-branes with parameters η_i, ϵ_i is

$$\mathcal{A}_{p-p+4} = \frac{1}{16}\int_0^\infty dt\,\langle B_p, \eta_1, \epsilon_1, \vec{x}_1 | e^{-2\pi\alpha' t p^+(P^- - p^-)} | B_{p+4}, \eta_2, \epsilon_2, \vec{x}_2\rangle$$

As before, this can be rewritten as

$$\mathcal{A}_{p-p+4} = \frac{V_p\,(4\pi^2\alpha')^{-\frac{p(4-p)}{2}}}{16\sinh\pi\epsilon}\int_0^\infty dt\int\frac{d^{4-p}q}{(2\pi)^{4-p}}\,e^{i\vec{q}\cdot\vec{b}}\,e^{-\pi\alpha' t\vec{q}^2}Z_0(\eta_i,\epsilon_i)Z_{osc}(t,\eta_i,\epsilon_i)$$

with

$$Z_0(\eta_i,\epsilon_i) = \langle B_{p0},\eta_1,\epsilon_1|B_{p+40},\eta_2,\epsilon_2\rangle$$

$$Z_{osc}(t,\eta_i,\epsilon_i) = \langle B_p,\eta_1,\epsilon_1|e^{-2\pi\alpha' t p^+ P^-}|B_{p+4},\eta_2,\epsilon_2\rangle_{osc}$$

Case $\eta_i = 0$ and $\epsilon_i = 0$

The Dp-D(p+4) system preserves 1/4 of the SUSY and is therefore BPS. As a consequence

$$Z_0 = \mathrm{Tr}_V[N] - \mathrm{Tr}_S[N] = (4-4) - 0 = 0$$

$$Z_{osc}(t) = \prod_{n=1}^{\infty}\frac{(1-e^{-2\pi tn})^4(1+e^{-2\pi tn})^4}{(1-e^{-2\pi tn})^4(1+e^{-2\pi tn})^4} = 1$$

where

$$N^{ij} = (M_p^T M_{p+4})^{ij} = \begin{pmatrix} I_{p+1} & 0 & 0 \\ 0 & -I_4 & 0 \\ 0 & 0 & I_{3-p}\end{pmatrix}$$

$$N_{\dot{a}\dot{b}} = (M_p^T M_{p+4})_{\dot{a}\dot{b}} = (\gamma^{p+2}...\gamma^{p+5})_{\dot{a}\dot{b}}$$

Integrating over the transverse momentum one finds

$$\mathcal{A}_{p-p+4} = V_{p+1}\,T_pT_{p+4}\,(1-1)\,G_{5-p}(b)$$

This is zero but exact in α'.

Case $\eta_i = 0$ but $\epsilon_i \neq 0$

$$Z_0(\epsilon) = \mathrm{Tr}_V[M_p^T(\epsilon_2)M_{p+4}(\epsilon_1)] - \mathrm{Tr}_S[M_p^T(\epsilon_2)M_{p+4}(\epsilon_1)]$$
$$= 16\cos^2\frac{\pi\epsilon}{2}\sin^2\frac{\pi\epsilon}{2} \sim v^2$$
$$Z_{osc}(t,\epsilon) = \prod_{n=1}^{\infty}\frac{|1-e^{i\pi\epsilon/2}e^{-2\pi tn}|^4|1+e^{i\pi\epsilon/2}e^{-2\pi tn}|^4}{|1-e^{i\pi\epsilon}e^{-2\pi tn}|^2(1-e^{-2\pi tn})^2(1+e^{-2\pi tn})^4} \sim 1$$

After the analytic continuation $\epsilon \to i\epsilon$

$$\mathcal{A}_{p-p+4} = \frac{V_p}{8i}(4\pi^2\alpha')^{-\frac{p(4-p)}{2}}\int_0^\infty \frac{dt}{(4\pi\alpha' t)^{\frac{4-p}{2}}}e^{-\frac{b^2}{4\pi\alpha' t}}\frac{\vartheta_1^2(i\frac{\epsilon}{2}|2it)\vartheta_2^2(i\frac{\epsilon}{2}|2it)}{\vartheta_1(i\epsilon|2it)\vartheta_2^2(0|2it)\eta^3(2it)}$$

Again, the behavior of $\mathcal{A}_{p-p+4}$ for $v \to 0$ is completely determined by SUSY.

$Z_0(\epsilon)$ can be rewritten as a trace in a Type I theory, with only 4 zero modes

$$Z_0(\epsilon) = \mathrm{Tr}_{S_0}[e^{V_{\epsilon 0}}N] = \mathrm{Tr}'_{S_0}[e^{V_{\epsilon 0}}]$$

The trace is 0 unless one inserts at least 4 zero modes S_0^a. The first $\neq 0$ is

$$t^{i_1\ldots i_4} = \mathrm{Tr}'_{S_0}\, R_0^{i_1i_2}R_0^{i_3i_4}$$
$$= 2\,\epsilon^{i_1\ldots i_4 p+2\ldots p+5}$$
$$+2\left[\delta^{i_1p+2}\delta^{i_2p+3}\delta^{i_3p+4}\delta^{i_4p+5} + N^{i_2i_4}\delta^{i_1i_3} + \mathrm{perm.}\right]$$

Since $V_{\epsilon 0}$ provides 2 S_0^a and 1 ϵ, expanding we recover $Z_0(\epsilon) \sim |v|^2$.

Expanding the whole e^{V_ϵ} in series

$$Z_0(\epsilon) = \sum_{m=0}^{\infty}\frac{1}{m!}\mathrm{Tr}'_{S_0}[V_{\epsilon 0}^m]$$
$$Z_{osc}(t,\epsilon) = \sum_{q=0}^{\infty}\frac{1}{q!}\langle B_p|V_\epsilon^q\, e^{-2\pi\alpha' t p^+P^-}|B_{p+4}\rangle_{osc}$$

we see that the first non-vanishing order in ϵ in $Z(t,\epsilon) = Z_0(\epsilon)Z_{osc}(t,\epsilon)$ receives a unique contribution with $m = 2$ e $q = 0$ ($\Leftrightarrow Z_{osc}(t,\eta_{1,2},\epsilon) = 1$).

Therefore $Z(t,\epsilon) \to |v|^2$ and is independent of t, meaning that only BPS (massless) states contribute. The non-relativistic amplitude reduces to

$$\mathcal{A}_{p-p+4} = \frac{|v|^2}{2} V_{p+1}\, T_p T_{p+4}\, G_{5-p}(r)$$

and is exact in α'.

Case $\eta_i \neq 0$ and $\epsilon_i \neq 0$

Expanding both in $\eta_{1,2}$ and ϵ

$$Z_0(\eta_{1,2},\epsilon) = \sum_{n_1,n_2}^{n_1+n_2\leq 12} \sum_{m=0}^{\infty} \frac{1}{(n_1!)^2(n_2!)^2 m!} \mathrm{Tr}'_{S_0}[V_{\eta_1 0}^{n_1} V_{\eta_2 0}^{n_2} V_{\epsilon 0}^{m}]$$

$$Z_{osc}(t,\eta_{1,2},\epsilon) = \sum_{p_1,p_2}^{p_1+p_2\leq 12} \sum_{q=0}^{\infty} \frac{1}{(p_1!)^2(p_2!)^2 q!} \langle B_p| V_{\eta_1}^{p_1} V_{\eta_2}^{p_2} V_{\epsilon}^{q}\, e^{-2\pi\alpha' t p^+ P^-} |B_{p+4}\rangle_{osc}$$

To get $Z_0(\eta_{1,2},\epsilon) \neq 0$ we need $n_1+n_2+m \geq 2$. The leading behavior is obtained by taking $n_1+n_2+m=2$ and $p_1,p_2,q=0$ ($\Leftrightarrow Z_{osc}(t,\eta_{1,2},\epsilon)=1$) and is independent of t. As before, each $V_{\eta 0}$ also brings a momentum q producing a derivative on the propagator G_{5-p}. The behavior is therefore

$$\mathcal{A}_{p-p+4}^{(n_1,n_2)} \sim \eta_1^{2n_1}\eta_2^{2n_2} |v|^{2-n_1-n_2} \partial^{n_1+n_2} G_{5-p}(r) \sim \eta_1^{2n_1}\eta_2^{2n_2} \frac{v^{2-n_1-n_2}}{r^{3-p+n_1+n_2}}$$

All these interactions are exact in α', that is scale-invariant. One finds

Case $n_1+n_2=0$

$$\mathcal{A}_{p-p+4}^{(0,0)} = \frac{V_{p+1}}{8\cdot 2!} T_p T_{p+4}\, v_{m_1} v_{m_2}\, t^{1m_1 1m_2}\, G_{5-p}(r)$$

Case $n_1+n_2=1$

$$\mathcal{A}_{p-p+4}^{(1,0)} = \frac{V_{p+1}}{8} T_p T_{p+4}\, v_{m_1}\, t^{i_1 i_2 1 m_1}\, \omega^{j_1}_{i_1 i_2}(\eta_1)\, \partial_{j_1} G_{5-p}(r)$$

Case $n_1+n_2=2$

$$\mathcal{A}_{p-p+4}^{(2,0)} = \frac{V_{p+1}}{8\cdot 2!^2} T_p T_{p+4}\, t^{i_1\ldots i_4}\, \omega^{j_1 j_2}_{i_1\ldots i_4}(\eta_1)\, \partial_{j_1}\partial_{j_2} G_{5-p}(r)$$

$$\mathcal{A}_{p-p+4}^{(1,1)} = \frac{V_{p+1}}{8} T_p T_{p+4}\, t^{i_1\ldots i_4}\, \omega^{j_1}_{i_1 i_2}(\eta_1)\, \omega^{j_2}_{i_3 i_4}(\eta_2)\, \partial_{j_1}\partial_{j_2} G_{5-p}(r)$$

MATRIX THEORY EFFECTIVE ACTION

The results for the D0-D0 amplitude can be made explicit and covariant. In $SO(9)$ notation, $\theta = \binom{\eta_a}{\tilde{\eta}_{\dot{a}}}$, one finds the following complete potential

$$
\begin{aligned}
V = \frac{1}{8} \Big[& v^4 \\
& + 2i\, v^2 v_m (\theta\gamma^{mn}\theta)\, \partial_n \\
& - 2 v_p\, v_q (\theta\gamma^{pm}\theta)(\theta\gamma^{qn}\theta)\, \partial_m\partial_n \\
& - \frac{4i}{9} v_i (\theta\gamma^{im}\theta)(\theta\gamma^{nl}\theta)(\theta\gamma^{pl}\theta)\, \partial_m\partial_n\partial_p \\
& + \frac{2}{63} (\theta\gamma^{ml}\theta)(\theta\gamma^{nl}\theta)(\theta\gamma^{pk}\theta)(\theta\gamma^{qk}\theta)\, \partial_m\partial_n\partial_p\partial_q \Big]\, G_9(r)
\end{aligned}
$$

Since this is scale-invariant, it has to be reproduced both in SUGRA and SYM.

Several explicit checks exists in the literature:

All terms in SUGRA : Plefka, Serone and Waldron

1^{st} term in SYM : Douglas, Kabat, Pouliot and Shenker

2^{nd} term in SYM : Kraus

3^{rd} term in SYM : McArthur

5^{th} term in SYM : Barrio, Helling and Polhemus

CONCLUSIONS

- SCALE INVARIANCE IN D-BRANE DYNAMICS $v \to 0$.
- ONE-LOOP SUGRA $\Leftrightarrow$ SYM EQUIVALENCE DICTATED BY SUSY. NON-TRIVIAL CHECKS AT TWO-LOOPS.
- SPIN-EFFECTS COMPUTABLE IN STRING THEORY. SAME PROBLEM IN SUGRA MORE DIFFICULT.