SUPERSYMMETRY AND SUPERGRAVITY

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- Symmetries and conservation laws
- Relativistic theories
- Supersymmetry and its implications
- Supersymmetric theories
- Implications in particle physics
- Supergravity

SYMMETRIES AND CONSERVED QUANTITIES

Symmetries in physics

Symmetry transformations must admit the notions of composition, inverse and identity, and should be parametrizable through a finite number of real parameters α_a. They must then form a Lie group.

These transformations are realized on the configuration space used to describe the system through a representation by linear operators $U(\alpha_a)$, which must satisfy the following properties:

$$
\begin{aligned}
&U(\alpha_a)\,U(\beta_a) = U(\gamma_a(\alpha_a, \beta_a)) \\
&U(\alpha_a)^{-1} = U(-\alpha_a) \\
&\mathbb{I} = U(0)
\end{aligned}
$$

Generators and algebra

For continuous transformations, the operators $U(\alpha_a)$ can be expressed in terms of a finite number of generators T^a as:

$$U(\alpha_a) = \exp\left\{i\alpha_a T^a\right\}$$

The fact that the operators $U(\alpha_a)$ form a group with a given composition law translates into the fact that the operators T^a span a Lie algebra with some structure constants:

$$[T^a, T^b] = i f^{ab}{}_c T^c$$

The generators T^a control infinitesimal transformations with parameters $\delta\alpha_a$, whose iterated application allows to construct finite transformations continuously connected to the identity:

$$U(\delta\alpha_a) = \mathbb{I} + i\,\delta\alpha_a T^a$$

Possible representations

A given group and its algebra admit various inequivalent representations on vector spaces of different dimensions. The linear operators can then be matrices on a finite vector space or/and differential operators on an infinite functional space.

Different representations can be characterized by the values taken by the Casimir operators commuting with all the generators.

Symmetries in quantum theories

In quantum theories, the configurations are generically described by a wave function or a field $\psi(x)$ with several components. The operators $U(\alpha_a)$ and T_a are then matrices of differential operators. Conservation of the probability implies that they should be unitary and hermitian:

$$U(\alpha_a)^\dagger = U(\alpha_a)^{-1} \;\Leftrightarrow\; (T^a)^\dagger = T^a$$

Invariances and conservation laws

The simplest criterium to characterize the symmetries of a theory is to consider the action functional I from which the wave equation for the field ψ can be derived. In fact, an infinitesimal transformation δ_a represents a symmetry of the theory if and only it leaves the action I invariant:

$$\delta_a I = 0$$

Each such invariance implies the presence of a family of symmetries with parameter α_a, and the existence of a conserved quantity associated to the value of the corresponding generator T^a:

$$\langle T^a \rangle : \text{ conserved } \Leftrightarrow \frac{d}{dt}\langle T^a \rangle = 0$$

Moreover, this implies that the scattering matrix S commutes with each such symmetry generator:

$$[T^a, S] = 0$$

SPACE-TIME SYMMETRIES

Changes of reference frame

The relativity principle states that the fundamental laws of physics must be invariant under changes of inertial reference frame that preserve the Minkowski interval in space-time.

This implies that any relativistic theory has to possess a universal group of symmetries whose infinitesimal transformations take the form

$$\delta x^{\mu} = -\epsilon^{\mu} + \omega^{\mu\nu} x_{\nu}$$

where ϵ^{μ} is arbitrary and $\omega^{\mu\nu}$ is antisymmetric, in such a way that:

ϵ^{μ} : translations in space (3) and time (1)

$\omega^{\mu\nu}$: rotations in space (3) and boosts (3)

Poincaré algebra

There are $\mathbf{10}$ independent generators corresponding to equally many conserved quantities, which are organized in two groups:

P_μ : linear momentum (3) and energy (1)

$M_{\mu\nu}$: angular moment (3) and center-of-mass momentum (3)

The structure constants of the algebra that these form are fixed by the commutator of two infinitesimal transformations:

$$[\delta_1, \delta_2] x^\mu = (\omega_1 \cdot \epsilon_2 - \omega_2 \cdot \epsilon_1)^\mu - (\omega_1 \cdot \omega_2 - \omega_2 \cdot \omega_1)^{\mu\nu} x_\nu$$

This result implies that the two groups of operators satisfy the following algebra:

$$[P_\mu, P_\nu] = 0$$
$$[P_\mu, M_{\rho\sigma}] = i(\eta_{\mu\rho} P_\sigma - \eta_{\mu\sigma} P_\rho)$$
$$[M_{\mu\nu}, M_{\rho\sigma}] = i(\eta_{\nu\rho} M_{\mu\sigma} - \eta_{\mu\rho} M_{\nu\sigma} - \eta_{\nu\sigma} M_{\mu\rho} + \eta_{\mu\sigma} M_{\nu\rho})$$

Representations and fields

The most general unitary representation is constructed on a field $\psi(x)$ with several components depending on the coordinates x^μ:

$$\psi(x) = \text{field with several components}$$

The general infinitesimal transformation takes the form

$$\delta\psi = \left(i\epsilon^\mu P_\mu + \frac{i}{2}\omega^{\mu\nu} M_{\mu\nu}\right)\psi$$

It combines matrix and differential representations:

$$P_\mu = i\partial_\mu$$
$$M_{\mu\nu} = \Sigma_{\mu\nu} + i(x_\mu\partial_\nu - x_\nu\partial_\mu)$$

One can construct an action that is automatically invariant by integrating over all space-time a scalar Lagrangian density:

$$I = \int d^4x\, \mathcal{L}[\psi(x)]$$

Possible options

The possible representations are characterized by two Casimir operators. Their values correspond to intrinsic properties of the described particle:

m : mass (real)

s : spin (integer or half-integer)

The number of components is $2s+1$, and the various possibilities for the matrices $\Sigma^{\mu\nu}$ can be constructed out of the Pauli matrices:

$$\sigma^\mu = (\mathbb{I}, \vec{\sigma}) \qquad \sigma^{\mu\nu} = \frac{1}{4}(\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu)$$

$$\bar{\sigma}^\mu = (\mathbb{I}, -\vec{\sigma}) \qquad \bar{\sigma}^{\mu\nu} = \frac{1}{4}(\bar{\sigma}^\mu \sigma^\nu - \bar{\sigma}^\nu \sigma^\mu)$$

The two simplest and most important cases are

- $s = 0$ (scalar) : $\Sigma^{\mu\nu} = 0$
- $s = \frac{1}{2}$ (spinor) : $\Sigma^{\mu\nu} = i\sigma^{\mu\nu}$ or $-i\bar{\sigma}^{\mu\nu}$

INTERNAL SYMMETRIES

Selection rules

A theory can optionally possess a group of internal symmetries, which preserve the coordinates and mix different fields of the same type.

One may a priori have an arbitrary group of transformations depending on some parameters α_a. This group must however be compact, in order for non-trivial finite-dimensional unitary representations to exist.

Algebra of internal symmetries

The n generators B^a then define conserved quantities:

$$B^a : \text{ charges (n)}$$

They moreover satisfy the algebra corresponding to the group:

$$[B^a, B^b] = i f^{ab}{}_c \, B^c$$

Representations

A generic unitary representation is constructed on a set of fields of the same type. The infinitesimal transformation takes the form:

$$\delta\psi = i\alpha_a B^a \psi$$

It involves a matrix representation:

$$B^a = \lambda^a$$

Possible options

The possible representations are characterized by a Casimir operator. Its value defines the selection properties of the described group of particles:

c : internal label (generally quantized)

The values of mutually commuting generators define additive charges:

q_k : additive charges

RELATIVISTIC THEORIES

General properties

Locality and relativistic invariance imply that for each particle there exists an antiparticle, with the same mass m and spin s, the same internal label c, but opposite values for the additive charges q_k:

$$\text{particles} \;\Leftrightarrow\; \text{antiparticles}$$

Consistency also forces particles of spin $s =$ integer to be bosons and particles of spin $s =$ half-integer to be fermions. In the two cases, the algebra $[N, a^\dagger] = a^\dagger$, $[N, a] = -a$ of creation, annihilation and number operators $a^\dagger$, a and $N = a^\dagger a$ is realized in two different ways:

$$\text{Bosons} \;:\; [a, a] = 0\,,\;\; [a^\dagger, a^\dagger] = 0\,,\;\; [a, a^\dagger] = 1$$

$$\text{Fermions} \;:\; \{a, a\} = 0\,,\;\; \{a^\dagger, a^\dagger\} = 0\,,\;\; \{a, a^\dagger\} = 1$$

Independence of space-time and internal symmetries

The space-time and the internal symmetries of a relativistic theory with a non-trivial S matrix must necessarily be independent, in the sense that the corresponding generators must commute:

$$[P_\mu, B^a] = 0$$
$$[M_{\mu\nu}, B^a] = 0$$

This limitation follows from the fact that the two sub-group of space-time symmetries generated by P_μ and $M_{\mu\nu}$ are non-compact, while the group of internal symmetries generated by B^a must be compact.

Interactions

Interactions between particles are described through non-quadratic terms in the Lagrangian, which translate into non-linearities in the equations of motion.

Free scalar field

The simplest bosonic case corresponds to a one-component scalar field ϕ with Lagrangian:

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2$$

This leads to the Klein-Gordon equation:

$$\Box\phi - m^2\phi = 0$$

Free spinor field

The simplest fermionic case corresponds to a two-component spinor field χ_α with Lagrangian:

$$\mathcal{L} = -\frac{i}{2}\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi + \text{h.c.} - \frac{1}{2}m\chi\chi + \text{h.c.}$$

This leads to the Dirac-Majorana equation:

$$i\bar{\sigma}^\mu\partial_\mu\chi + m\bar{\chi} = 0$$

Interacting scalar fields

The general Lagrangian describing several scalar fields ϕ^i in interaction depends on a positive kinetic metric Z_{ij} and a real potential V:

$$\mathcal{L} = -\frac{1}{2} Z_{ij}(\phi) \partial_\mu \phi^i \partial^\mu \phi^j - V(\phi)$$

The ground state defining the vacuum is $\phi_0^i =$ constant and such that:

$$V_i(\phi_0) = 0$$

Decomposing the fields as $\phi^i = \phi_0^i + \hat{\phi}^i$ and expanding in powers of the fluctuations one then finds:

$$\mathcal{L} = -\frac{1}{2} Z_{ij}^0 \partial_\mu \hat{\phi}^i \partial^\mu \hat{\phi}^j - \frac{1}{2} V_{ij}^0 \hat{\phi}^i \hat{\phi}^j + \text{interactions}$$

After a last linear transformation on the $\hat{\phi}^i$ that trivializes Z_{ij}^0 one can identify the square masses with the eigenvalues of the matrix

$$m_{\phi\,ij}^2 = V_{ij}^0$$

Interacting spinor fields

The general Lagrangian describing several spinor fields χ_α^i in interaction involves a positive kinetic metric $Z_{i\bar{\jmath}}$ and a real potential V:

$$\mathcal{L} = -\frac{i}{2} Z_{i\bar{\jmath}}(\chi,\bar{\chi})\bar{\chi}^j \bar{\sigma}^\mu \partial_\mu \chi^i + \text{h.c.} - V(\chi,\bar{\chi})$$

The ground state defining the vacuum is $\chi_0^i =$ constant such that:

$$V_i(\chi_0) = 0$$

Decomposing the fields as $\chi^i = \chi_0^i + \hat{\chi}^i$ and expanding in powers of the fluctuations one then finds:

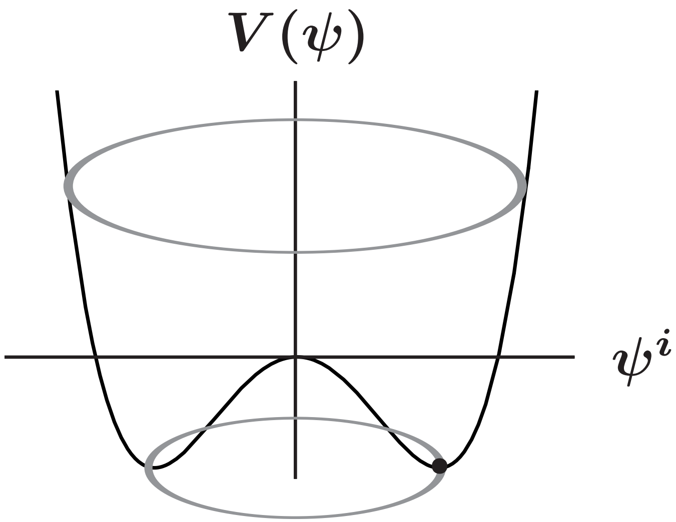
$$\mathcal{L} = -\frac{i}{2} Z^0_{i\bar{\jmath}} \hat{\bar{\chi}}^j \bar{\sigma}^\mu \partial_\mu \hat{\chi}^i + \text{h.c.} - \frac{1}{2} V^0_{ij} \hat{\chi}^i \hat{\chi}^j + \text{h.c.} + \text{interactions}$$

After a last linear transformation on the $\hat{\chi}^i$ that trivializes $Z^0_{i\bar{\jmath}}$ one can identify the masses with the diagonal values of the matrix:

$$m_{\chi\, ij} = V^0_{ij}$$

Spontaneous symmetry breaking and Goldstone modes

A symmetry generated by T^a in a theory with fields ψ^i is spontaneously broken by the ground state configuration ψ_0^i if $\delta^a \psi_0^i \neq 0$:



ψ_0^i such that $V_i^0 = 0$

$\delta^a \psi_0^i \neq 0 \;\Rightarrow\; T^a$ broken

$V_{ij}^0 \delta^a \psi_0^j = 0 \;\Rightarrow\; \hat{\sigma}^a$ massless

The space-time symmetries are always preserved, and therefore:

$$\phi_0^i = \text{constant} \qquad \chi_0^i = 0$$

The internal symmetries can instead be broken if $\delta^a \phi_0^i \neq 0$.

SUPERSYMMETRIES

Symmetries between bosons and fermions

A continuous symmetry relating bosons to fermions would be interesting from several points of views. But it seems impossible to realize in the context of ordinary symmetries based on Lie groups and Lie algebras.

The key point to define such a symmetry is to consider Lie supergroups whose elements depend both on real and Grassmannian parameters, and Lie superalgebras whose properties are defined by commutators and also anticommutators.

This generalization is completely natural, since it introduces symmetry generators which can be either bosonic or fermonic, very much like the degrees of freedom themselves.

Supergroups of transformations

A supergroup of transformations is realized on the configuration space of the system through a representation by linear operators $U(\alpha_a, \xi_\alpha)$ possessing the following properties:

$$U(\alpha_a, \xi_\alpha)\, U(\beta_a, \eta_\alpha) = U(\gamma_a(\alpha_a, \beta_a, \xi_\alpha, \eta_\alpha), \chi_\alpha(\alpha_a, \beta_a, \xi_\alpha, \eta_\alpha))$$
$$U(\alpha_a, \xi_\alpha)^{-1} = U(-\alpha_a, -\xi_\alpha)$$
$$\mathbb{I} = U(0, 0)$$

The parameters α_a, β_b, $\ldots$ and ξ_α, η_β, $\ldots$ are real and Grassmannian numbers, with the following multiplication properties:

$$\alpha_a \beta_b = \beta_b \alpha_a \;\Rightarrow\; \alpha_a^2 \neq 0\,, \quad \beta_b^2 \neq 0$$
$$\xi_\alpha \eta_\beta = -\eta_\beta \xi_\alpha \;\Rightarrow\; \xi_\alpha^2 = 0\,, \quad \eta_\beta^2 = 0$$
$$\alpha_a \eta_\beta = \eta_\beta \alpha_a$$

Generators and superalgebras

For continuous transformations, the operators $U(\alpha_a, \xi_\alpha)$ can be written in terms of some generators T^a and S^α as:

$$U(\alpha_a, \xi_\alpha) = \exp\left\{i(\alpha_a T^a + \xi_\alpha S^\alpha)\right\}$$

The fact that the $U(\alpha_a, \xi_\alpha)$ form a supergroup with a certain composition law translates into the fact that the T^a and S^α generate a superalgebra with certain structure constants:

$$\begin{aligned}
[T^a, T^b] &= i f^{ab}{}_c\, T^c \\
\{S^\alpha, S^\beta\} &= i\, g^{\alpha\beta}{}_c\, T^c \\
[T^a, S^\beta] &= i\, h^{a\beta}{}_\gamma\, S^\gamma
\end{aligned}$$

The generators T^a and S^α control again infinitesimal transformations, whose iterated application allows to construct finite transformations that are continuously connected to the identity.

SuperPoincaré algebra

There exists essentially only one possible extension of the usual space-time symmetries that is compatible with a non-trivial S matrix. Besides the bosonic generators P_μ, $M_{\mu\nu}$, this requires new fermonic generators Q_α, $\bar{Q}_{\dot{\alpha}}$, and has the following form:

$$
\begin{aligned}
&[P_\mu, P_\nu] = 0 \\
&[P_\mu, M_{\rho\sigma}] = i\big(\eta_{\mu\rho} P_\sigma - \eta_{\mu\sigma} P_\rho\big) \\
&[M_{\mu\nu}, M_{\rho\sigma}] = i\big(\eta_{\nu\rho} M_{\mu\sigma} - \eta_{\mu\rho} M_{\nu\sigma} - \eta_{\nu\sigma} M_{\mu\rho} + \eta_{\mu\sigma} M_{\nu\rho}\big) \\
&[Q_\alpha, P_\mu] = 0 && [\bar{Q}_{\dot{\alpha}}, P_\mu] = 0 \\
&[Q_\alpha, M_{\rho\sigma}] = i\sigma_{\rho\sigma\,\alpha}{}^{\beta} Q_\beta && [\bar{Q}_{\dot{\alpha}}, M_{\rho\sigma}] = -i\bar{\sigma}_{\rho\sigma\,\dot{\alpha}}{}^{\dot{\beta}} \bar{Q}_{\dot{\beta}} \\
&\{Q_\alpha, Q_\beta\} = 0 && \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0 \\
&\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^\mu{}_{\alpha\dot{\beta}} P_\mu
\end{aligned}
$$

The new transformations preserve m but change s by $\frac{1}{2}$.

Internal algebra

As before one may in addition have a group of internal symmetries, which can be arbitrary but must be compact, with an algebra of the type:

$$[B^a, B^b] = i f^{ab}{}_c B^c$$

Rest of the algebra

The space-time supersymmetries and the internal symmetries are not allowed to interfere, so that:

$$[P_\mu, B^a] = 0$$

$$[M_{\mu\nu}, B^a] = 0$$

$$[Q_\alpha, B^a] = 0 \qquad [\bar{Q}_\alpha, B^a] = 0$$

Possible generalizations

It is possible to introduce several independent supersymmetries. But this situation yields so strong constraints that it is not viable.

Representations and superfields

The most general unitary representation is constructed on superfields $\Psi(x, \theta, \bar{\theta})$, which depend on 4 bosonic coordinates x^μ and 4 fermionic coordinates $\theta^\alpha, \bar{\theta}^{\dot{\alpha}}$. The infinitesimal transformations are:

$$\delta\Psi = \left(i\epsilon^\mu P_\mu + \frac{i}{2}\omega^{\mu\nu} M_{\mu\nu} + i\xi^\alpha Q_\alpha + i\bar{\xi}^{\dot{\alpha}} \bar{Q}_{\dot{\alpha}}\right)\Psi$$

The generators have the general form

$$P_\mu = i\partial_\mu$$

$$M_{\mu\nu} = \Sigma_{\mu\nu} + i(x_\mu\partial_\nu - x_\nu\partial_\mu) + i\sigma_{\mu\nu\,\alpha}{}^{\beta}\theta^\alpha\partial_\beta - i\bar{\sigma}_{\mu\nu\,\dot{\alpha}}{}^{\dot{\beta}}\bar{\theta}^{\dot{\alpha}}\bar{\partial}_{\dot{\beta}}$$

$$Q_\alpha = i\partial_\alpha + \sigma^\mu{}_{\alpha\dot{\beta}}\,\bar{\theta}^{\dot{\beta}}\partial_\mu \qquad\qquad \bar{Q}_{\dot{\alpha}} = -i\,\bar{\partial}_{\dot{\alpha}} - \bar{\sigma}^\mu{}_{\dot{\alpha}\beta}\,\theta^\beta\partial_\mu$$

One can construct an action that is automatically invariant by integrating over the whole superspace-time a scalar Lagrangian density:

$$I = \int d^4x\, d^2\theta\, d^2\bar{\theta}\, l[\Psi(x, \theta, \bar{\theta})]$$

Possible options

The possible representations are characterized by two Casimir operators. Their values correspond to intrinsic properties of the described multiplet of particles:

$$m: \text{ common mass (real)}$$
$$s: \text{ minimal spin (integer or half-integer)}$$

A representation of the superPoincaré group therefore contains several representations of the Poincaré sub-group, with $m_k = m$ et $s_k \geq s$. For each particle there must then exist a superparticle, with the same m and q_k but s differing by $\frac{1}{2}$:

$$\text{particles } \Leftrightarrow \text{ superparticles}$$

This decomposition of the representations corresponds to the fact that a superfield admits a finite series expansion in powers of $\theta^\alpha, \bar{\theta}^{\dot{\alpha}}$, whose coefficients are ordinary fields depending only on x^μ.

General superfields

A general scalar superfield admits a series expansion involving a set of ordinary field components $(\phi, \chi_\alpha, f, \lambda_\alpha, A_\mu, d)$:

$$\begin{aligned}\Psi(x,\theta,\bar\theta) = \phi(x) &+ \sqrt{2}\,\theta^\alpha \chi_\alpha(x) + \text{h.c.} \\ &+ \theta^\alpha\theta^\beta \epsilon_{\alpha\beta} f(x) + \text{h.c.} - \theta^\alpha \bar\theta^{\dot\beta} \sigma^\mu{}_{\alpha\dot\beta} A_\mu(x) \\ &- i\,\theta^\alpha\theta^\beta\bar\theta^{\dot\gamma}\epsilon_{\alpha\beta}\Big(\bar\lambda_{\dot\gamma}(x) + \frac{1}{\sqrt{2}}\,\bar\sigma^\mu{}_{\dot\gamma\delta}\partial_\mu\chi^\delta(x)\Big) + \text{h.c.} \\ &+ \frac{1}{2}\,\theta^\alpha\theta^\beta\bar\theta^{\dot\gamma}\bar\theta^{\dot\delta}\epsilon_{\alpha\beta}\epsilon_{\dot\gamma\dot\delta}\Big(d(x) + \frac{1}{2}\Box\phi(x)\Big)\end{aligned}$$

This is by construction a representation of the superPoincaré group, but it is reducible. One can construct irreducible representations out of this by imposing supercovariant constraints.

Supercovariant derivatives

It is very useful to introduce the following supercovariant derivatives:

$$\mathcal{D}_\mu = \partial_\mu$$

$$\mathcal{D}_\alpha = \partial_\alpha + i\sigma^\mu{}_{\alpha\dot\beta}\,\bar\theta^{\dot\beta}\partial_\mu \qquad \bar{\mathcal{D}}_{\dot\alpha} = -\bar\partial_{\dot\alpha} - i\bar\sigma^\mu{}_{\dot\alpha\beta}\,\theta^\beta\partial_\mu$$

These derivatives commute or anticommute with the supertranslations P_μ, Q_α, $\bar Q_{\dot\alpha}$, and their commutator with $M_{\mu\nu}$ just reflects that they are vectors and spinors. Their action on a superfield therefore produces a new superfield with one extra Lorentz index.

Chiral, antichiral and vector superfields

The following superfields yield irreducible representations:

- Chiral or antichiral : $\bar{\mathcal{D}}_{\dot\alpha}\Psi = 0$ or $\mathcal{D}_\alpha\Psi = 0 \;\Rightarrow\; (\phi, \chi_\alpha, f)$
- Vector : remain part of $\Psi = \bar\Psi \;\Rightarrow\; (A_\mu, \lambda_\alpha, d)$

The simplest multiplet

A chiral superfield Ψ with components (ϕ, χ_α, f) has the expansion:

$$\begin{aligned}\Psi(x,\theta,\bar\theta) = \phi(x) &+ \sqrt{2}\,\theta^\alpha\chi_\alpha(x) + \theta^\alpha\theta^\beta\epsilon_{\alpha\beta}f(x)\\ &+ i\,\theta^\alpha\bar\theta^{\dot\beta}\sigma^\mu{}_{\alpha\dot\beta}\partial_\mu\phi(x) - \frac{i}{\sqrt{2}}\,\theta^\alpha\theta^\beta\bar\theta^{\dot\gamma}\epsilon_{\alpha\beta}\bar\sigma^\mu{}_{\dot\gamma\delta}\partial_\mu\chi^\delta(x)\\ &+ \frac{1}{4}\,\theta^\alpha\theta^\beta\bar\theta^{\dot\gamma}\bar\theta^{\dot\delta}\epsilon_{\alpha\beta}\epsilon_{\dot\gamma\dot\delta}\Box\phi(x)\end{aligned}$$

The general form of the Lagrangian can be parametrized by a real Kähler potential $K(\Psi,\bar\Psi)$ and a holomorphic superpotential $W(\Psi)$:

$$\begin{aligned}I = &\int d^4x\,d^2\theta\,d^2\bar\theta\,K\left[\Psi(x,\theta,\bar\theta),\bar\Psi(x,\theta,\bar\theta)\right]\\ &+\int d^4x\,d^2\theta\,W\left[\Psi(x,\theta,\bar\theta)\right] + \text{h.c.}\end{aligned}$$

The action in components is computed by integrating explicitly over the fermonic coordinates θ^α and $\bar\theta^{\dot\alpha}$, with the following rules:

$$\int d\theta\,1 = 0 \qquad \int d\theta\,\theta = 1 \qquad \int d\bar\theta\,1 = 0 \qquad \int d\bar\theta\,\bar\theta = 1$$

Free chiral multiplet

The simplest choice is $K = \bar{\Psi}\Psi$ and $W = \frac{1}{2} m \Psi^2$. This yields:

$$\mathcal{L} = -\partial_\mu \phi \, \partial^\mu \bar{\phi} - \frac{i}{2} \bar{\chi} \bar{\sigma}^\mu \partial_\mu \chi + \text{h.c.} + f \bar{f} \\ + m\left(\phi \, f - \frac{1}{2} \chi\chi + \text{h.c.}\right)$$

The infinitesimal supersymmetry transformation acts as

$$\begin{aligned} \delta\phi &= \sqrt{2}\, \xi \, \chi \\ \delta\chi &= \sqrt{2}\, \xi \, f + \sqrt{2} i \, \sigma^\mu \bar{\xi} \, \partial_\mu \phi \\ \delta f &= \sqrt{2} i \, \bar{\xi} \, \bar{\sigma}^\mu \partial_\mu \chi \end{aligned}$$

The corresponding equations of motion are:

$$\begin{aligned} \Box\phi &= -m \bar{f} \\ i \bar{\sigma}^\mu \partial_\mu \chi + m \bar{\chi} &= 0 \\ f &= -m \bar{\phi} \end{aligned}$$

Auxiliary and physical fields

The Lagrangian for the physical fields ϕ^i and χ^i is obtained by eliminating the auxiliary fields f^i. In this way one finds:

$$\mathcal{L} = -\partial_\mu \phi\, \partial^\mu \bar{\phi} - m^2 |\phi|^2 - \left(\frac{i}{2}\, \bar{\chi} \bar{\sigma}^\mu \partial_\mu \chi + \frac{1}{2} m\, \chi\chi \right) + \text{h.c.}$$

The supersymmetry algebra closes now only on mass-shell, and the new infinitesimal supersymmetry transformation reads:

$$\begin{aligned} \delta\phi &= \sqrt{2}\, \xi\, \chi \\ \delta\chi &= -\sqrt{2}\, \xi\, m \bar{\phi} + \sqrt{2} i\, \sigma^\mu \bar{\xi}\, \partial_\mu \phi \end{aligned}$$

The equations of motion correspondingly reduce to the usual Klein-Gordon and Dirac-Majorana wave equations:

$$\begin{aligned} \Box \phi - m^2 \phi &= 0 \\ i \bar{\sigma}^\mu \partial_\mu \chi + m \bar{\chi} &= 0 \end{aligned}$$

Interacting chiral multiplets

For several chiral multiplets and arbitrary K and W, one finds:

$$\begin{aligned}\mathcal{L} = {} & K_{i\bar{j}}(\phi,\bar{\phi})\Big[-\partial_\mu\phi^i\partial^\mu\bar{\phi}^j - \frac{i}{2}\,\bar{\chi}^j\bar{\sigma}^\mu\partial_\mu\chi^i + \text{h.c.} + f^i\bar{f}^j\Big] \\ & -\frac{1}{2}K_{i\bar{j}k}(\phi,\bar{\phi})\Big[\chi^i\chi^k\bar{f}^j + i\,\bar{\chi}^j\bar{\sigma}^\mu\chi^i\partial_\mu\phi^k\Big] + \text{h.c.} \\ & +\frac{1}{4}K_{i\bar{j}k\bar{l}}(\phi,\bar{\phi})\,\chi^i\chi^k\bar{\chi}^j\bar{\chi}^l + \Big(W_i(\phi)f^i - \frac{1}{2}W_{ij}(\phi)\chi^i\chi^j\Big) + \text{h.c.}\end{aligned}$$

The infinitesimal supersymmetry transformation acts as before:

$$\begin{aligned}\delta\phi^i &= \sqrt{2}\,\xi\,\chi^i \\ \delta\chi^i &= \sqrt{2}\,\xi\,f^i + \sqrt{2}i\,\sigma^\mu\bar{\xi}\,\partial_\mu\phi^i \\ \delta f^i &= \sqrt{2}i\,\bar{\xi}\,\bar{\sigma}^\mu\partial_\mu\chi^i\end{aligned}$$

The auxiliary fields f^i have again an algebraic equation of motion:

$$f^i = -K^{i\bar{j}}(\phi,\bar{\phi})\Big(\bar{W}_{\bar{j}}(\bar{\phi}) - \frac{1}{2}K_{r\bar{j}s}(\phi,\bar{\phi})\,\chi^r\chi^s\Big)$$

Dynamics of the physical fields

The Lagrangian for ϕ^i and χ^i is found to be:

$$
\begin{aligned}
\mathcal{L} = & -K_{i\bar{\jmath}}(\phi,\bar{\phi})\nabla_\mu\phi^i\nabla^\mu\bar{\phi}^j - \frac{i}{2}K_{i\bar{\jmath}}(\phi,\bar{\phi})\,\bar{\chi}^j\bar{\sigma}^\mu\nabla_\mu\chi^i + \text{h.c.} \\
& - K^{i\bar{\jmath}}\nabla_i W\nabla_{\bar{\jmath}}\bar{W}(\phi,\bar{\phi}) - \frac{1}{2}\nabla_i\nabla_j W(\phi,\bar{\phi})\,\chi^i\chi^j + \text{h.c.} \\
& + \frac{1}{4}R_{i\bar{\jmath}k\bar{l}}(\phi,\bar{\phi})\,\chi^i\chi^k\bar{\chi}^j\bar{\chi}^l
\end{aligned}
$$

This result is written in terms of a Kähler geometry associated to the manifold spanned by the scalar fields, defined by the derivatives of K:

$$
g_{i\bar{\jmath}} = K_{i\bar{\jmath}} \qquad \Gamma^i_{jk} = K^{i\bar{l}}K_{\bar{l}jk} \qquad R_{i\bar{\jmath}k\bar{l}} = K_{i\bar{\jmath}k\bar{l}} - K_{ik\bar{s}}K^{\bar{s}r}K_{r\bar{\jmath}\bar{l}}
$$

The covariant derivatives are

$$
\begin{aligned}
&\nabla_\mu\phi^i = \partial_\mu\phi^i \qquad \nabla_\mu\chi^i = \partial_\mu\chi^i - \Gamma^i_{jk}\partial_\mu\phi^j\chi^k \\
&\nabla_i W = W_i \qquad \nabla_i\nabla_j W = W_{ij} - \Gamma^k_{ij}W_k
\end{aligned}
$$

On-shell supersymmetry transformations

The supersymmetry transformations of the physical Lagrangian read:

$$\delta\phi^i = \sqrt{2}\,\xi\,\chi^i$$
$$\delta\chi^i = \sqrt{2}\,\xi\,F^i + \sqrt{2}i\,\sigma^\mu\bar{\xi}\,\nabla_\mu\phi^i - \Gamma^i_{jk}(\phi,\bar{\phi})\,\delta\phi^j\chi^k$$

where now

$$F^i = -K^{i\bar{\jmath}}\nabla_{\bar{\jmath}}\bar{W}(\phi,\bar{\phi})$$

Reparametrization and Kähler invariances

The theory is invariant under target-space reparametrizations acting as arbitrary redefinitions of the scalar fields $\phi^i \to f^i(\phi)$.

It is also trivially invariant under Kähler transformations generated by an arbitrary holomorphic function Σ and transforming

$$K \to K + \Sigma + \bar{\Sigma} \qquad W \to W$$

Vacuum state and fluctuations

The ground state corresponds to a configuration of the form:

$$\phi_0^i = \text{constant} \qquad \chi_0^i = 0 \qquad F_0^i = \text{constant}$$

Supersymmetry acts on this as $\delta\phi_0^i = 0$, $\delta\chi_0^i = \sqrt{2}\,\xi\, F_0^i$ and $\delta F_0^i = 0$.
It is therefore spontaneously broken if $F_0^i \neq 0$.

When $F_0^i \neq 0$, there is a massless Goldstino, in the field direction F_0^i.

The quadratic Lagrangian for the physical fluctuation fields $\hat{\phi}^i$ and $\hat{\chi}^i$ is:

$$\mathcal{L} = -g_{i\bar{\jmath}}\,\partial_\mu\hat{\phi}^i\nabla^\mu\hat{\bar{\phi}}^j - m^2_{\phi\, i\bar{\jmath}}\,\hat{\phi}^i\hat{\bar{\phi}}^j - \frac{1}{2}m^2_{\phi\, ij}\,\hat{\phi}^i\hat{\phi}^j + \text{h.c.}$$
$$- \frac{i}{2}g_{i\bar{\jmath}}\,\hat{\bar{\chi}}^j\bar{\sigma}^\mu\partial_\mu\hat{\chi}^i + \text{h.c.} - \frac{1}{2}m_{\chi\, ij}\,\hat{\chi}^i\hat{\chi}^j + \text{h.c.}$$

where $g_{i\bar{\jmath}} = K^0_{i\bar{\jmath}}$ and:

$$m^2_{\phi\, i\bar{\jmath}} = \nabla_i W^0_k \nabla_{\bar{\jmath}}\bar{W}^{0k} - R^0_{i\bar{\jmath}k\bar{l}}W^{0k}W^{0\bar{l}} \qquad m_{\chi\, ij} = \nabla_i W^0_j$$
$$m^2_{\phi\, ij} = \nabla_i\nabla_j W^0_k W^{0k}$$

Mass matrices

The physics is best described in terms of the quantities:

$$F_0^i = -W^{0i}$$
$$\mu_{ij}^0 = \nabla_i W_j^0 \qquad \lambda_{ijk}^0 = \nabla_i \nabla_j W_k^0$$

The mass matrices for the fluctuation fields $\hat{\phi}^i$ and $\hat{\chi}^i$ can then be written in the following form:

$$m^2_{\phi\, i\bar{\jmath}} = (\mu\bar{\mu})^0_{i\bar{\jmath}} - R^0_{i\bar{\jmath}k\bar{l}} F_0^k \bar{F}_0^{\bar{l}} \qquad m_{\chi\, ij} = \mu^0_{ij}$$
$$m^2_{\phi\, ij} = -\lambda^0_{ijk} F_0^k$$

The condition of stationarity of the potential implies that $m_{\chi\, ij} F_0^j = 0$. This shows that the Goldstino mode is $\hat{\eta} \propto F_{0i} \hat{\chi}^i$.

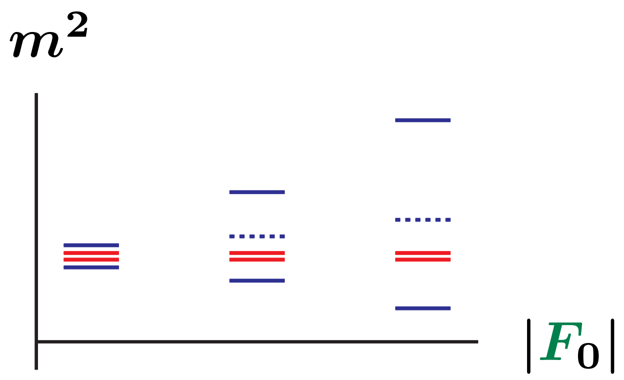
Physical masses

The physical square masses for the $2n + 2n$ degrees of freedom are given by:

$$m^2_{\phi\, IJ} = \begin{pmatrix} m^2_{\phi\, i\bar{j}} & m^2_{\phi\, ij} \\ \bar{m}^2_{\phi\, \bar{i}\bar{j}} & m^2_{\phi\, \bar{i}j} \end{pmatrix} \quad m^2_{\chi\, IJ} = \begin{pmatrix} (m_\chi \bar{m}_\chi)_{i\bar{j}} & 0 \\ 0 & (\bar{m}_\chi m_\chi)_{\bar{i}j} \end{pmatrix}$$

If $F^i_0 = 0$ (unbroken phase) one has $m^2_{\phi\, IJ} = m^2_{\chi\, IJ}$. The masses are degenerate and at each level there are two bosons and two fermions.

If $F^i_0 \neq 0$ (broken phase) one has $m^2_{\phi\, IJ} \neq m^2_{\chi\, IJ}$. In each multiplet, the masses of bosons and fermions split.



$\Delta_{\phi\chi} : m^2_{\phi\,\mathrm{dia}} - m^2_\chi$

$\Delta_{\phi\phi} : m^2_{\phi\,\mathrm{off}}$

Splitting scale: $M^2 \propto |R_0 F_0^2|$

Special features of the mass spectrum

A first useful information concerns the shift between mean boson and fermion masses. It can be extracted by taking the trace:

$$\mathbf{tr}[m_{\phi}^2] - \mathbf{tr}[m_{\chi}^2] = -2\, R^0_{i\bar{\jmath}}\, F_0^i \bar{F}_0^{\bar{\jmath}}$$

A second important information concerns the masses of the boson and fermion defined by the Goldstino direction F_0^i in field space:

$$m_{\tilde{\eta}}^2 = -R^0_{i\bar{\jmath}k\bar{l}}\, \frac{F_0^i \bar{F}_0^{\bar{\jmath}} F_0^k \bar{F}_0^{\bar{l}}}{|F_0|^2}$$

$$m_{\eta}^2 = 0$$

We see that to achieve separation between partners and metastability, we need non-vanishing negative curvature. The effective theory then has a physical cut-off scale set by the curvature and is non-renormalizable.

QUANTUM CORRECTIONS

Loop diagrams and mass corrections

The mass m of a particle receives quantum corrections Δm which can a priori be arbitrarily large. This poses a naturalness problem.

In certain instances, the problem is solved by the unbroken relativistic symmetries that are always present, which imply a cancellation between virtual particles and antiparticles for $p^* \gg m$. One then gets:

$$\Delta m \propto m$$

More in general, the problem is solved by broken supersymmetry with a splitting scale M, which implies a cancellation between virtual particles and superparticles for $p^* \gg M$. One then gets:

$$\Delta m \propto M$$

General properties of perturbative quantum corrections

It is possible to formulate a general characterization of the structure of perturbative quantum corrections, by using for example diagrammatic techniques.

In a relativistic theory, the kinetic metric Z_{ij} and the potential V both receive generic perturbative corrections, and these may be constrained to some extent only by internal symmetries:

$$\Delta Z_{ij} \neq 0 \quad \Delta V \neq 0$$

In a supersymmetric theory, the Kähler potential K generically receives perturbative corrections, but the superpotential W is instead free from any perturbative correction, independently of internal symmetries:

$$\Delta K \neq 0 \quad \Delta W = 0$$

IMPLICATIONS IN PARTICLE PHYSICS

The supersymmetric standard model

The minimal supersymmetric version of the standard model consists of a visible sector with the following content:

- $\mathbf{3}$ families of matter multiplets with spins $\frac{1}{2}$ and $\mathbf{0}$.
- $\mathbf{3}$ groups of radiation multiplets with spins $\mathbf{1}$ and $\frac{1}{2}$.
- $\mathbf{2}$ Higgs multiplets with spins $\mathbf{0}$ and $\frac{1}{2}$.

There must then be a hidden sector triggering the spontaneous breaking of supersymmetry, whose detailed form does not matter too much.

Finally, there must be a messenger sector of interactions that transmits the breaking effect from the hidden to the visible sector, in a way that is effectively suppressed at low energies.

Low-energy effective theory and soft terms

At energies much lower that the mediation scale, the effective theory is the visible sector supplemented by a finite set of relevant soft breaking terms encoding the splitting effects induced by supersymmetry breaking, with some typical scale M.

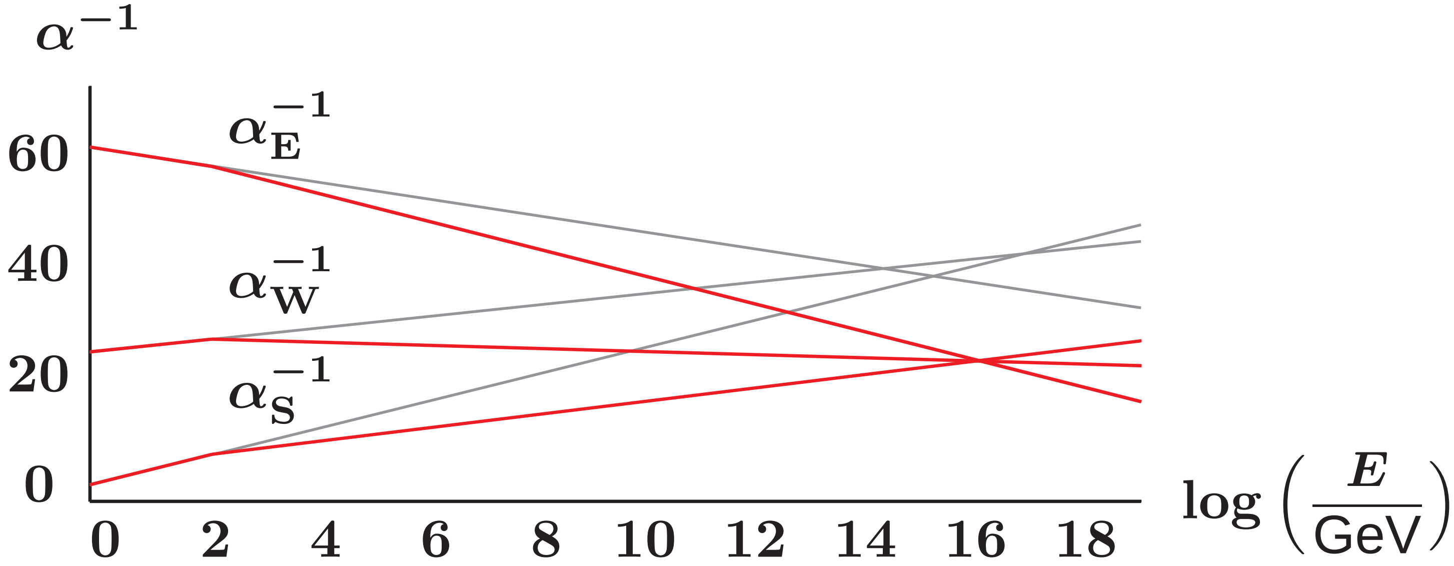
The phenomenology depends on the structure of the soft terms, which parametrize the features of this general situation. Superpartners induce important effects both directly as real particles and indirectly as virtual particles.

Contraints and discovery perspectives

The non-observation of superparticles so far implies $M \gtrsim \mathbf{10^{2\text{-}3}}$ GeV. The naturalness of the new model requires instead $M \lesssim \mathbf{10^{2\text{-}3}}$ GeV. One would therefore expect to discover something at $M \sim \mathbf{10^{2\text{-}3}}$ GeV.

Unification of fundamental interactions

Quantum effects induce a calculable dependence on the energy $\boldsymbol{E}$ for the couplings of the electromagnetic, weak and strong forces. Extrapolating the values measured at low energies, one then finds:



There is therefore an indirect evidence for both supersymmetry and the idea of unification.

GRAVITY

Local space-time symmetries

In general relativity, one requires all the reference frames to be on equal footing, even the accelerated ones. This leads to a gauging of Poincaré symmetries through the gravitational field.

One first introduces the vielbein e_μ^a and the spin connection s_μ^{ab} as gauge fields for P_a and M_{ab}. The torsion constraint then fixes $s_\mu^{ab} = s_\mu^{ab}(e)$, which can be viewed as the equation of motion of the spin connection.

The resulting theory is invariant under local diffeomorphisms and it is based on a single gravity field:

$$e_\mu^a : \; s = 2$$

Space-time geometry

Gravity describes the dynamics of the geometry of space-time, which is parametrized by the ordinary coordinates x^μ.

One may construct invariant actions by integrating a scalar density over all of space-time. This is done with the help of $e = \det e_\mu^a$, which furnishes an appropriate measure:

$$I = \int d^4x \, e(x) \, \mathcal{L}[\psi(x)]$$

GRAVITY THEORIES

Pure gravity

In Planck units, the dynamics of pure gravity is described by a Lagrangian involving a cosmological constant Λ:

$$\mathcal{L} = -\frac{1}{2}R - \Lambda$$

This leads to the modified Einstein equation:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - g_{\mu\nu}\Lambda = 0$$

This theory is unavoidably interacting and non-linear, reflecting the fact that any kind of energy induces a gravitational field. The curvature term is a kinetic energy, while the cosmological term is a constant potential energy.

Ground state and masses

The ground state is given by $e^a_{\mu 0}$ = maximally symmetric with $R = 4\Lambda$. There are then three different cases:

- $\Lambda = 0$: Minkowski space-time $\Leftrightarrow$ Poincaré symmetry
- $\Lambda > 0$: de Sitter space-time $\Leftrightarrow$ dS symmetry
- $\Lambda < 0$: Anti de Sitter space-time $\Leftrightarrow$ AdS symmetries

The global space-time symmetry always posses 10 generators P_a, M_{ab}. When $\Lambda \neq 0$ these satisfy a deformation of the Poincaré algebra where $[P_a, P_b] = \frac{2i}{3}\Lambda\, M_{ab}$.

The Lagrangian for the fluctuations of the metric is easily computed, and one finds that these have vanishing mass:

$$m_e^2 = 0$$

Scalars and spinors in interaction with gravity

The general Lagrangian for several scalar fields ϕ^i, spinor fields χ^i and the graviton e^a_μ interacting among themselves depends on three positive kinetic metrics Z, Z_{ij}, $Z_{i\bar{\jmath}}$ and a real potential V:

$$\mathcal{L} = -\frac{1}{2} Z(\phi, \bar{\phi}, \chi, \bar{\chi}) R - \frac{1}{2} Z_{ij}(\phi, \bar{\phi}, \chi, \chi) D_\mu \phi^i D^\mu \phi^j$$
$$- \frac{i}{2} Z_{i\bar{\jmath}}(\phi, \bar{\phi}, \chi, \bar{\chi})\, \bar{\chi}^j \bar{\sigma}^\mu D_\mu \chi^i + \text{h.c.} - V(\phi, \bar{\phi}, \chi, \bar{\chi})$$

The ground state defining the vacuum corresponds to $\phi^i_0 =$ constant, $\chi^i_0 = 0$ and $e^a_{\mu 0} =$ maximally symmetric.

The scalar and fermion masses are obtained as before by diagonalizing the second derivative of V after having trivialized Z_{ij} and $Z_{i\bar{\jmath}}$.

The graviton mass is instead identically vanishing, and the value taken by the cosmological constant is determined by the value of V after having trivialized Z.

SUPERGRAVITY

Local supersymmetry

In supergravity, also supersymmetry transformations must be promoted to local symmetries. This leads to a gauging of all the superPoincaré symmetries through a gravitational multiplet of fields.

One introduces the vielbein e_μ^a, the spin connection s_μ^{ab} and the gravitino ψ_μ^α as gauge fields for P_a, M_{ab} and Q_α. The torsion constraint then fixes $s_\mu^{ab} = s_\mu^{ab}(e, \psi, \bar{\psi})$.

The resulting theory is then invariant under local superdiffeomorphisms. These can be realized off-shell with a scalar and a vector auxiliary fields m and b^a. One finally needs to following multiplet of supergravity fields:

$$m: \; s = 0 \qquad b^a: \; s = 1 \qquad \psi_\mu^\alpha: \; s = \frac{3}{2} \qquad e_\mu^a: \; s = 2$$

Supergeometry

Supergravity is related to the geometry of superspace-time, parametrized by supercoordinates $Z^M = (x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$. To see this one introduces a superfielbein E_M^A and a superspin connection Ω_M^{AB}. One then imposes a maximal set of supertorsion constraints. One finally finds that:

$$E_M^A \ni e_\mu^a, \psi_\mu^\alpha \qquad \Omega_M^{AB} \ni m, b^a$$

One may construct invariant actions by integrating a scalar density over all of space-time. This is done with the help of $E = \det E_M^A$, which furnishes an appropriate measure:

$$I = \int d^4x \, d^2\theta \, d^2\bar{\theta} \, E(x, \theta, \bar{\theta}) \, l[\Psi(x, \theta, \bar{\theta})]$$

SUPERGRAVITY THEORIES

The simplest situation

The simplest situation arises when only the gravitational multiplet and chiral matter multiplets are present.

The general Lagrangian consists of a full superspace integral with the general density E of a real Kähler function Ω, plus a chiral superspace integral with a chiral density $\mathcal{E}$ of a holomorphic potential function P.

$$\mathcal{L} = \int d^2\theta\, d^2\bar\theta\, E(x,\theta,\bar\theta)\Omega\big[\Psi(x,\theta,\bar\theta),\bar\Psi(x,\theta,\bar\theta)\big] + \int d^2\theta\, \mathcal{E}(x,\theta,\bar\theta)P\big[\Psi(x,\theta,\bar\theta)\big] + \text{h.c.}$$

The comparison with the rigid case becomes most transparent if one parametrizes, in Planck units:

$$\Omega = -3\, e^{-K/3} \qquad P = W$$

Pure supergravity multiplet

For the supergravity multiplet, one can take $\Omega = -3$ and $P = \frac{1}{\sqrt{3}}\sqrt{-\Lambda}$.
The component Lagrangian is then found to be:

$$\mathcal{L} = -\frac{1}{2}R - \frac{i}{2}\bar{\psi}_\mu \bar{\sigma}^{\mu\nu\rho} D_\nu \psi_\rho + \text{h.c.} - \frac{1}{3}|m|^2 + \frac{1}{3} b^a b_a$$
$$- \frac{1}{\sqrt{3}}\sqrt{-\Lambda}\left[\bar{m} + \bar{\psi}_\mu \bar{\sigma}^{\mu\nu} \bar{\psi}_\nu\right] + \text{h.c.}$$

The supersymmetry transformations are:

$$\delta e_\mu^a = i\,\xi\,\sigma^a \bar{\psi}_\mu + \text{h.c.}$$
$$\delta \psi_\mu = 2\,D_\mu \xi + \frac{i}{3}\,e_\mu^a\,\sigma_a \bar{\xi}\,m + \frac{2i}{3}\,e_\mu^a\,\left(\eta_{ab} + i\,\sigma_{ab}\right)\xi\,b^b$$
$$\delta m = 8i\,\xi\,\sigma^{ab}\nabla_a \psi_b - i\,\xi\,\sigma^a \bar{\psi}_a m + i\,\xi\,\psi_a\,b^a$$
$$\delta b^a = \xi\,\sigma_1^{abc}\nabla_b \bar{\psi}_c + \frac{i}{4}\,\bar{\xi}\,\bar{\psi}^a\,m + \bar{\xi}\,\sigma_2^{abc}\bar{\psi}_b\,b_c + \text{h.c.}$$

The auxiliary fields m and b^a take the values:

$$m = -\sqrt{3}\,\sqrt{-\Lambda} \qquad b^a = 0$$

Physical field dynamics

The Lagrangian for the physical fields e_μ^a and ψ_μ is found by eliminating m and b^a. One gets:

$$\mathcal{L} = -\frac{1}{2}R - \frac{i}{2}\bar{\psi}_\mu \bar{\sigma}^{\mu\nu\rho} D_\nu \psi_\rho + \text{h.c.} - \frac{1}{\sqrt{3}}\sqrt{-\Lambda}\,\bar{\psi}_\mu \bar{\sigma}^{\mu\nu} \bar{\psi}_\nu + \text{h.c.} - \Lambda$$

The supersymmetry algebra closes now only on mass-shell, and the new infinitesimal supersymmetry transformations read:

$$\delta e_\mu^a = i\,\psi_\mu \sigma^a \bar{\xi} + \text{h.c.}$$

$$\delta \psi_\mu = 2\,D_\mu \xi - \frac{i}{\sqrt{3}}\sqrt{-\Lambda}\,e_\mu^a \sigma_a \bar{\xi}$$

The equations of motion are finally given by:

$$\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - g_{\mu\nu}\Lambda\right)(e) = T_{\mu\nu}(\psi, \bar{\psi})$$

$$i\bar{\sigma}^{\mu\nu\rho} D_\nu \psi_\rho + \frac{2}{\sqrt{3}}\sqrt{-\Lambda}\,\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu = 0$$

Ground state and masses

The ground state is $e^a_{\mu 0}$ = maximally symmetric with $R = 4\Lambda$, $\psi_{\mu 0} = 0$. There are then as before three different cases:

- $\Lambda = 0$: Minkowski space-time $\Leftrightarrow$ superPoincaré symmetry
- $\Lambda > 0$: de Sitter space-time $\Leftrightarrow$ dS symmetry
- $\Lambda < 0$: Anti de Sitter space-time $\Leftrightarrow$ superAdS symmetries

The surviving symmetry includes 10 bosonic generators P_a, M_{ab}, $\forall\Lambda$, and 4 fermionic generators Q, $\bar{Q}$, if $\Lambda \leq 0$. When $\Lambda < 0$ these satisfy a deformation of the superPoincaré algebra where $[P_a, P_b] = \frac{2i}{3}\Lambda\, M_{ab}$, $[P_a, Q] = \frac{i}{\sqrt{6}}\sqrt{-\Lambda}\,\sigma_a Q$ and $\{Q, \bar{Q}\} = 2\,\sigma^a P_a + \frac{\sqrt{8}}{\sqrt{3}}\sqrt{-\Lambda}\,\sigma^{ab} M_{ab}$.

The masses for the fluctuations fields are found to be:

$$m_e^2 = 0 \qquad m_\psi = \frac{1}{\sqrt{3}}\sqrt{-\Lambda}$$

Chiral and supergravity multiplets in interactions

In presence of several chiral multiplets with arbitrary $\boldsymbol{\Omega}$ and $\boldsymbol{P}$, one finds:

$$
\begin{aligned}
\mathcal{L} = {} & \Omega(\phi,\bar\phi)\Big[\frac{1}{6}R + \frac{i}{6}\bar\psi_\mu\bar\sigma^{\mu\nu\rho}D_\nu\psi_\rho + \text{h.c.} + \frac{1}{9}|m|^2 - \frac{1}{9}b^a b_a\Big] \\
& + \Omega_i(\phi,\bar\phi)\Big[-\frac{1}{3}f^i\bar m - \frac{i}{3}D_\mu\phi^i b^\mu + \frac{i}{3\sqrt{2}}\chi^i\psi_\mu b^\mu \\
& \qquad - \frac{i}{2}D_\mu\phi^i\psi_\mu\sigma^{\mu\nu\rho}\bar\psi_\nu + \frac{\sqrt{2}}{3}\chi^i\sigma^{\mu\nu}D_\mu\psi_\nu\Big] + \text{h.c.} \\
& + \Omega_{i\bar\jmath}(\phi,\bar\phi)\Big[-D_\mu\phi^i D^\mu\phi^j - \frac{i}{2}\bar\chi^j\bar\sigma^\mu D_\nu\chi^i + \text{h.c.} + f^i\bar f^j \\
& \qquad - \frac{1}{6}\chi^i\sigma_\mu\bar\chi^j b^\mu - \frac{1}{\sqrt{2}}D_\mu\phi^i\bar\psi_\nu\bar\sigma^\mu\sigma^\nu\bar\chi^j + \text{h.c.}\Big] \\
& - \frac{1}{2}\Omega_{i\bar\jmath k}(\phi,\bar\phi)\Big[\chi^i\chi^k\bar f^j + i\chi^i\sigma^\mu\bar\chi^j\partial_\mu\phi^k + \frac{i}{\sqrt{2}}\bar\chi^j\bar\sigma^\mu\psi_\mu\,\chi^i\chi^k\Big] + \text{h.c.} \\
& + \frac{1}{4}\Omega_{i\bar\jmath k\bar l}(\phi,\bar\phi)\,\chi^i\chi^k\bar\chi^j\bar\chi^l - P(\phi)\Big[\bar m + \bar\psi_\mu\bar\sigma^{\mu\nu}\bar\psi_\nu\Big] + \text{h.c.} \\
& + P_i(\phi)\Big[f^i - \frac{i}{\sqrt{2}}\chi^i\sigma^\mu\bar\psi_\mu\Big] + \text{h.c.} - \frac{1}{2}P_{ij}(\phi)\,\chi^i\chi^j + \text{h.c.}
\end{aligned}
$$

Supersymmetry transformations

The supersymmetry transformations are:

$$\delta e^a_\mu = i\,\xi\sigma^a\bar{\psi}_\mu + \text{h.c.}$$

$$\delta\psi_\mu = 2\,D_\mu\xi - \frac{i}{3}\,e^a_\mu\sigma_a\bar{\xi}\,m - \frac{2i}{3}\,e^a_\mu\xi\big(\eta_{ab} + \sigma_{ab}\big)b^b$$

$$\delta m = 8i\,\xi\sigma^{ab}D_a\psi_b - i\,\xi\sigma^a\bar{\psi}_a m + i\,\xi\psi_a b^a$$

$$\delta b^a = \xi\sigma_1^{abc}D_b\bar{\psi}_c + \frac{i}{4}\bar{\xi}\bar{\psi}^a m + \bar{\xi}\sigma_2^{abc}\bar{\psi}_b b_c$$

and

$$\delta\phi^i = \sqrt{2}\,\xi\,\chi^i$$

$$\delta\chi^i = \sqrt{2}\,\xi\,f^i + \sqrt{2}i\,\sigma^\mu\bar{\xi}\,D_\mu\phi^i$$

$$\delta f^i = \sqrt{2}i\,\bar{\xi}\,\bar{\sigma}^\mu D_\mu\chi^i + \frac{\sqrt{2}}{3}\,\xi\,\chi^i\bar{m} + \frac{1}{3\sqrt{2}}\,\bar{\xi}\,\bar{\sigma}_\mu\chi^i b^\mu$$

Auxiliary fields

The auxiliary fields m, b^a and f^i take the following values:

$$m = 9\Big[P(\phi) - \frac{1}{3}\Omega_i \Omega^{i\bar{\jmath}} \bar{P}_{\bar{\jmath}}(\phi, \bar{\phi})\Big]\Big(\Omega - \Omega_r \Omega^{r\bar{s}} \Omega_{\bar{s}}\Big)^{-1}(\phi, \bar{\phi})$$

$$b^a = -\frac{3}{2}\Big[i\,\Omega_i(\phi, \bar{\phi})\,\nabla^a \phi^i + \text{h.c.} + \Omega_{i\bar{\jmath}}(\phi, \bar{\phi})\chi^i \sigma^a \bar{\chi}^j$$
$$+\,\Omega_i(\phi, \bar{\phi})\,\chi^i \psi^a + \text{h.c.}\Big]\Omega^{-1}(\phi, \bar{\phi})$$

$$f^i = -\Big[\Omega^{i\bar{\jmath}}\Big(\Omega \bar{P}_{\bar{\jmath}} - 3\,\Omega_{\bar{\jmath}} P - \Omega_p \Omega^{p\bar{q}} \Omega_{\bar{q}} \bar{P}_{\bar{\jmath}} - \Omega_{\bar{\jmath}} \Omega_p \Omega^{p\bar{q}} \bar{P}_{\bar{q}}\Big)(\phi, \bar{\phi})$$
$$+\frac{1}{2}\Omega^{i\bar{\jmath}}\Omega_{\bar{\jmath}kl}(\phi, \bar{\phi})\chi^k \chi^l\Big]\Big(\Omega - \Omega_r \Omega^{r\bar{s}} \Omega_{\bar{s}}\Big)^{-1}(\phi, \bar{\phi})$$

Rescaling

It is convenient to perform a Weyl rescaling with $\lambda = -\frac{1}{2}\log(-\frac{1}{3}\Omega)$:

$$e_\mu^a \to e^{\lambda}(\phi, \bar{\phi})\, e_\mu^a \qquad \psi_\mu \to e^{\lambda/2}(\phi, \bar{\phi})\Big(\psi_\mu + \sqrt{2}i\,\lambda_{\bar{\jmath}}(\phi, \bar{\phi})\sigma_\mu \bar{\chi}^j\Big)$$

$$\phi^i \to \phi^i \qquad \chi^i \to e^{-\lambda/2}(\phi, \bar{\phi})\,\chi^i$$

Physical field dynamics

The Lagrangian for e_μ^a, ψ_μ, ϕ^i and χ^i in terms of $K = -3\log(-\frac{1}{3}\Omega)$ and $W = P$ is finally found to be:

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{2}R - \frac{i}{2}\bar{\psi}_\mu\bar{\sigma}^{\mu\nu\rho}\tilde{\nabla}_\nu\psi_\rho + \text{h.c.} \\
& - K_{i\bar{j}}(\phi,\bar{\phi})\tilde{\nabla}_\mu\phi^i\tilde{\nabla}^\mu\bar{\phi}^{\bar{j}} - \frac{i}{2}K_{i\bar{j}}(\phi,\bar{\phi})\,\bar{\chi}^j\bar{\sigma}^\mu\tilde{\nabla}_\mu\chi^i + \text{h.c.} \\
& - e^K\Big(K^{i\bar{j}}\tilde{\nabla}_i W\tilde{\nabla}_{\bar{j}}\bar{W} - 3|W|^2\Big)(\phi,\bar{\phi}) - \Big(\frac{1}{2}e^{K/2}\tilde{\nabla}_i\tilde{\nabla}_j W(\phi,\bar{\phi})\chi^i\chi^j \\
& + e^{K/2}W(\phi,\bar{\phi})\,\bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu + \frac{i}{\sqrt{2}}e^{K/2}\tilde{\nabla}_i W(\phi,\bar{\phi})\chi^i\sigma^\mu\bar{\psi}_\mu\Big) + \text{h.c.} \\
& - \frac{1}{\sqrt{2}}K_{i\bar{j}}(\phi,\bar{\phi})\tilde{\nabla}_\mu\phi^i\bar{\psi}_\nu\bar{\sigma}^\nu\sigma^\mu\bar{\chi}^j + \text{h.c.} \\
& + \frac{1}{4}\Big(R_{i\bar{j}k\bar{l}} - \frac{1}{2}K_{i\bar{j}}K_{k\bar{l}}\Big)(\phi,\bar{\phi})\,\chi^i\chi^k\bar{\chi}^{\bar{j}}\bar{\chi}^{\bar{l}} \\
& - \frac{1}{4}K_{i\bar{j}}(\phi,\bar{\phi})\,\chi^i\sigma_\nu\bar{\chi}^j\psi_\mu(\sigma^{\mu\nu\rho} - \eta^{\mu\rho}\sigma^\nu)\psi_\rho
\end{aligned}
$$

Geometry

The result is again written in terms of a Kähler geometry for the manifold spanned by the scalar fields, defined by the derivatives of K:

$$g_{i\bar{\jmath}} = K_{i\bar{\jmath}} \qquad \Gamma^i_{jk} = K^{i\bar{l}} K_{\bar{l}jk} \qquad R_{i\bar{\jmath}k\bar{l}} = K_{i\bar{\jmath}k\bar{l}} - K_{ik\bar{s}} K^{\bar{s}r} K_{r\bar{\jmath}\bar{l}}$$

The covariant derivatives are given by:

$$\begin{aligned}
\tilde{\nabla}_\mu \phi^i &= D_\mu \phi^i \\
\tilde{\nabla}_\mu \chi^i &= D_\mu \chi^i - \Gamma^i_{jk} \partial_\mu \phi^j \chi^k - \frac{1}{4}\big(K_j \partial_\mu \phi^j - \text{h.c.}\big)\chi^i \\
\tilde{\nabla}_\mu \psi_\nu &= D_\mu \psi_\nu + \frac{1}{4}\big(K_j \partial_\mu \phi^j - \text{h.c.}\big)\psi_\mu \\
\tilde{\nabla}_i W &= W_i + K_i W \\
\tilde{\nabla}_i \tilde{\nabla}_j W &= W_{ij} + K_i W_j + K_j W_i + (K_{ij} + K_i K_j) W \\
&\quad - \Gamma^k_{ij} (W_k + K_k W)
\end{aligned}$$

In this case the Lagrangian depends also on K and K_i.

On-shell supersymmetry transformations

The supersymmetry transformations of the physical Lagrangian read:

$$\delta e_\mu^a = i\,\xi\sigma^a\bar{\psi}_\mu + \text{h.c.}$$

$$\delta\psi_\mu = 2\,\tilde{\nabla}_\mu\xi + i\,M\sigma_\mu\bar{\xi} - \frac{i}{2}\sigma_{\mu\nu}\xi\,K_{i\bar{\jmath}}(\phi,\bar{\phi})\,\chi^i\sigma^\nu\bar{\chi}^j - \frac{1}{4}\big(K_j(\phi,\bar{\phi})\delta\phi^j - \text{h.c.}\big)\psi_\mu$$

and

$$\delta\phi^i = \sqrt{2}\,\xi\,\chi^i$$

$$\delta\chi^i = \sqrt{2}\,\xi\,F^i + \sqrt{2}i\,\sigma^\mu\bar{\xi}\,\tilde{\nabla}_\mu\phi^i - i\,\xi\,\bar{\sigma}^\mu\psi_\mu\,\chi^i - \Gamma^i_{jk}(\phi,\bar{\phi})\,\delta\phi^r\chi^s + \frac{1}{4}\big(K_j(\phi,\bar{\phi})\delta\phi^j - \text{h.c.}\big)\chi^i$$

where now

$$M = e^{K/2}W(\phi,\bar{\phi}) \qquad F^i = -e^{K/2}K^{i\bar{\jmath}}\tilde{\nabla}_{\bar{\jmath}}\bar{W}(\phi,\bar{\phi})$$

Reparametrization and Kähler invariances

The theory is invariant under target-space reparametrizations acting as arbitrary redefinitions of the scalar fields $\phi^i \to f^i(\phi)$.

It is also invariant under Kähler transformations generated by an arbitrary holomorphic function Σ, acting as chiral rotations on the fermion fields $\chi^i \to e^{\frac{1}{4}(\Sigma-\bar{\Sigma})}(\phi,\bar{\phi})\chi$, $\psi_\mu \to e^{-\frac{1}{4}(\Sigma-\bar{\Sigma})}(\phi,\bar{\phi})\psi_\mu$ and transforming:

$$K \to K + \Sigma + \bar{\Sigma} \qquad W \to e^{-\Sigma} W$$

One can then switch to a single invariant potential:

$$G = K + \log|W|^2$$

Indeed, performing a transformation with $\Sigma = \log W$ one finds:

$$\begin{array}{lll} K \to G & K_i \to G_i & K_{i\bar{\jmath}} \to G_{i\bar{\jmath}} \\ W \to 1 & \tilde{\nabla}_i W \to G_i & \tilde{\nabla}_i \tilde{\nabla}_j W \to \nabla_i G_j + G_i G_j \end{array}$$

Vacuum configuration

The ground state corresponds to a configuration of the form:

$$e^a_{\mu 0} = \text{maximally symmetric} \qquad \psi_{\mu 0} = 0 \qquad M_0 = \text{constant}$$

$$\phi^i_0 = \text{constant} \qquad \chi^i_0 = 0 \qquad F^i_0 = \text{constant}$$

Supersymmetry acts on this as $\delta e^a_\mu = 0$, $\delta \psi_{\mu 0} = \sqrt{2} \sigma_\mu \bar{\xi} M_0$, $\delta M_0 = 0$, $\delta \phi^i_0 = 0$, $\delta \chi^i_0 = \sqrt{2}\, \xi\, F^i_0$, $\delta F^i_0 = 0$. It is therefore spontaneously broken if $M_0 \neq 0$ and/or $F^i_0 \neq 0$. If $M_0 \neq 0$ but $F^i_0 = 0$, one however gets a deformed supersymmetry algebra in AdS space.

When $F^i_0 \neq 0$, the direction F^i_0 in field space is again special, as in the case of global supersymmetry. But since supersymmetry is now local, it does not lead to a physical massless Goldstino.

What happens is a superHiggs mechanism, through which the would-be Goldstino mode is absorbed by the gravitino and provides the additional degrees of freedom required for the latter to become massive.

Fluctuations

The quadratic Lagrangian for the fluctuation fields $\hat{e}_\mu^a$, $\hat{\psi}_\mu$, $\hat{\phi}^i$ and $\hat{\chi}^i$ is:

$$\mathcal{L} = -\frac{1}{2}R(\hat{e}) - \Lambda - g_{i\bar{\jmath}}\partial_\mu\hat{\phi}^i\partial^\mu\hat{\bar{\phi}}^{\bar{\jmath}} - m^2_{\phi\, i\bar{\jmath}}\hat{\phi}^i\hat{\bar{\phi}}^{\bar{\jmath}} - \frac{1}{2}m^2_{\phi\, ij}\hat{\phi}^i\hat{\phi}^j + \text{h.c.}$$
$$- \Big(\frac{i}{2}\hat{\bar{\psi}}_\mu\bar{\sigma}^{\mu\nu\rho}\partial_\nu\hat{\psi}_\rho + \frac{i}{2}g_{i\bar{\jmath}}\,\hat{\bar{\chi}}^{\bar{\jmath}}\bar{\sigma}^\mu\partial_\mu\hat{\chi}^i\Big) + \text{h.c.}$$
$$- \Big(m_\psi\hat{\bar{\psi}}_\mu\bar{\sigma}^{\mu\nu}\hat{\bar{\psi}}_\nu + \frac{1}{2}m_{\chi\, ij}\,\hat{\chi}^i\hat{\chi}^j + \frac{i}{\sqrt{2}}m_{*\, i}\,\hat{\chi}^i\sigma^\mu\hat{\bar{\psi}}_\mu\Big) + \text{h.c.}$$

where $g_{i\bar{\jmath}} = G^0_{i\bar{\jmath}}$, the cosmological constant reads

$$\Lambda = e^{G^0}\big(G^{0i\bar{\jmath}}G^0_iG^0_{\bar{\jmath}} - 3\big) \;\Leftrightarrow\; \Lambda = 0 \;\text{ if }\; |G^{0\prime}| = \sqrt{3}$$

and the masses are then given by:

$$m^2_{\phi\, i\bar{\jmath}} = e^{G^0}\big(\nabla_iG^0_k\nabla_{\bar{\jmath}}G^{0k} - R^0_{i\bar{\jmath}k\bar{l}}G^{0i}G^{0\bar{\jmath}} + g^0_{i\bar{\jmath}}\big)$$
$$m^2_{\phi\, ij} = e^{G^0}\big(\nabla_i\nabla_jG^0_kG^{0k} + 2\nabla_iG^0_j\big)$$
$$m_\psi = e^{G^0/2} \qquad m_{\chi\, ij} = e^{G^0/2}\big(\nabla_iG^0_j + G^0_iG^0_j\big) \qquad m_{*\, i} = e^{G^0/2}G^0_i$$

Superunitary gauge

The would-be Goldstino $\hat{\eta} = \frac{1}{\sqrt{3}} G_i^0 \hat{\chi}^i$ transforms as $\delta\hat{\eta}_0 = -\sqrt{6}\, m_\psi \xi$. One can then choose a unitary gauge for supersymmetry where it is zero. This is achieved with $\xi = \frac{1}{\sqrt{6}} m_\psi^{-1} \hat{\eta}$. The fermionic fields $\hat{\psi}_\mu$ and $\hat{\chi}^i$ are correspondingly redefined as:

$$\hat{\psi}_\mu \to \hat{\psi}_\mu + \frac{\sqrt{2}}{3}\, m_\psi^{-1} \partial_\mu (G_i^0 \hat{\chi}^i) + \frac{1}{3\sqrt{2}}\, \sigma_\mu (G_{\bar{\jmath}}^0 \hat{\bar{\chi}}^j)$$

$$\hat{\chi}^i \to \hat{\chi}^i \quad \text{such that} \quad G_i^0 \hat{\chi}^i = 0$$

The fluctuation Lagrangian can be rewritten entirely in terms of these new fields, and the masses change as follows:

$$m_\psi \to e^{G^0/2} \qquad m_{\chi\, ij} \to e^{G^0/2} \Big(\nabla_i G_j^0 + \frac{1}{3} G_i^0 G_j^0 \Big) \qquad m_{*\, i} \to 0$$

The net result is as anticipated that the would-be Goldstone mode has disappeared and one is left with a massive gravitino ψ_μ and $n-1$ chiral fermions $\hat{\chi}^i$ defined by the condition $G_i^0 \hat{\chi}^i = 0$.

Cosmological constant and mass matrices

The physics is best described in terms of the quantities

$$F_0^i = -e^{G^0/2} G^{0i} \qquad M_0 = e^{G^0/2}$$

$$\mu_{ij}^0 = e^{G^0/2} \nabla_i G_j^0 \qquad \lambda_{ijk}^0 = e^{G^0/2} \nabla_i \nabla_j G_k^0$$

The cosmological constant is:

$$\Lambda = |F_0|^2 - 3|M_0|^2 \;\Leftrightarrow\; \Lambda = 0 \;\text{ if }\; |M_0| = \frac{1}{\sqrt{3}}|F_0|$$

The masses of the fields $\hat{e}_\mu^a$, $\hat{\psi}_\mu$, $\hat{\phi}^i$ and $\hat{\chi}^i$ are then given by:

$$m_e^2 = 0 \qquad m_\psi = \frac{1}{\sqrt{3}}|F_0|$$

$$m^2_{\phi\, i\bar{j}} = (\mu\bar{\mu})^0_{i\bar{j}} - R^0_{i\bar{j}k\bar{l}} F_0^k \bar{F}_0^{\bar{l}} + \frac{1}{3} g^0_{i\bar{j}} |F_0|^2 \qquad m_{\chi\, ij} = \mu^0_{ij} + \frac{1}{\sqrt{3}} \frac{\bar{F}_{0i}\bar{F}_{0j}}{|F_0|}$$

$$m^2_{\phi\, ij} = -\lambda^0_{ijk} F_0^k + \frac{2}{\sqrt{3}} \mu^0_{ij} |F_0|$$

Physical masses

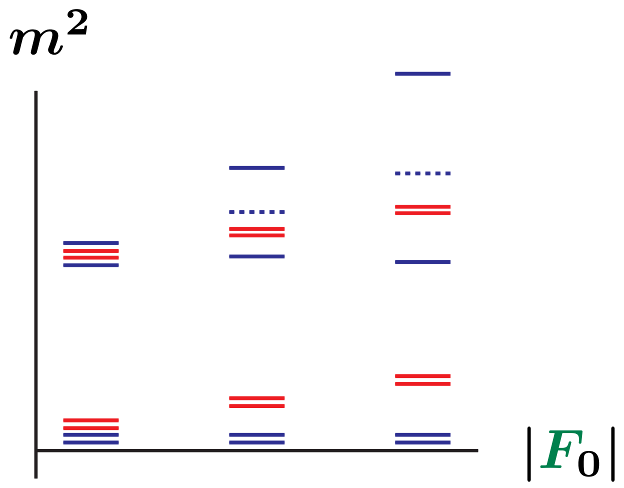
The physical square masses for the $2+2n+4+2(n-1)$ states are:

$$m^2_{e\,ab} = m^2_e \delta_{ab} \qquad m^2_{\psi\,xy} = |m_\psi|^2 \delta_{xy}$$

$$m^2_{\phi\,IJ} = \begin{pmatrix} m^2_{\phi\,i\bar{j}} & m^2_{\phi\,ij} \\ \bar{m}^2_{\phi\,\bar{i}\bar{j}} & m^2_{\phi\,\bar{i}j} \end{pmatrix} \qquad m^2_{\chi\,IJ} = \begin{pmatrix} (m_\chi \bar{m}_\chi)_{i\bar{j}} & 0 \\ 0 & (\bar{m}_\chi m_\chi)_{\bar{i}j} \end{pmatrix}$$

If $F_0^i = 0$ (unbroken phase) the masses are degenerate.

If $F_0^i \neq 0$ (broken phase) the masses split.



$\Delta_{\phi\chi} : m^2_{\phi\,\mathrm{dia}} - m^2_\chi$

$\Delta_{\phi\phi} : m^2_{\phi\,\mathrm{off}}$

$\Delta_{e\psi} : m^2_\psi$

Splitting scale: $M^2 \propto |R_0 F_0^2|, |\frac{1}{3} F_0^2|$

Special features of the mass spectrum

A first useful information concerns the shift between mean boson and fermion masses. It can be extracted by taking the trace:

$$\mathrm{tr}[m^2_e] + \mathrm{tr}[m^2_\phi] - \mathrm{tr}[m^2_\psi] - \mathrm{tr}[m^2_\chi] = -2R^0_{i\bar{\jmath}}F^i_0\bar{F}^{\bar{\jmath}}_0 + \frac{2}{3}(n-1)|F_0|^2$$

A second important information concerns the masses of the boson and fermion defined by the Goldstino direction F^i_0 in field space:

$$m^2_{\tilde{\eta}} = -R^0_{i\bar{\jmath}k\bar{l}}\frac{F^i_0\bar{F}^{\bar{\jmath}}_0F^k_0\bar{F}^{\bar{l}}_0}{|F_0|^2} + \frac{2}{3}|F_0|^2$$

$$m^2_\psi = \frac{1}{3}|F_0|^2$$

We see that to achieve separation between partners and metastability, we need small-enough curvature. The effective theory then has a physical cut-off scale set by the Planck scale.

CONCLUSIONS

- Supersymmetry is a totally natural and plausible generalization of the relativity principle, whose form happens to be almost uniquely fixed by consistency arguments.
- A large number of physicist believe that it may play a crucial role in a more fundamental and unified description of particle physics going beyond the standard model.
- The possibility that it manifests itself at experimentally accessible energies is indirectly suggested by naturalness arguments and the unification of extrapolated couplings.

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