CONSTRAINTS FOR THE EXISTENCE OF FLAT METASTABLE NON-SUSY VACUA IN SUGRA

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- Viable SUSY breaking in SUGRA theories.
- Constraints for minimal chiral SUGRA models.
- Constraints for gauge invariant SUGRA models.
- Interplay between F and D breaking effects.
- Factorizable scalar manifolds.
- Symmetric scalar manifolds.
- Implications for string moduli.

SUSY BREAKING AND SUGRA

In a viable SUGRA model, the vacuum state must be associated with a stationary point of the scalar potential where SUSY is spontaneously broken.

To get a realistic situation, there are however two additional conditions that must certainly be imposed:

- Flatness: The energy of the vacuum should be negligibly small, and reproduce the tiny value of the cosmological constant.
- Stability: The squared masses for small fluctuations around the vacuum should be positive.

The natural question is then whether these two conditions can be used to restrict the class of models of potential interest.

MINIMAL SUGRA MODELS

A model with chiral multiplets Φ^i is specified by a real Kähler potential K and a holomorphic superpotential W. It has a Kähler symmetry for which $(K, W) \to (K + F + \bar{F}, e^{-F} W)$, and depends only on:

$$G = K + \log W + \log \bar{W}$$

In the superconformal formulation, with a chiral compensator multiplet Φ, the Kähler symmetry becomes manifest, with $\Phi \to e^{F/3}\Phi$. One can then set $(K, W) \to (G, 1)$, and write the Lagrangian in the form:

$$\mathcal{L} = \int d^4\theta \left[-3\, e^{-G/3} \right] \Phi^\dagger \Phi + \left(\int d^2\theta\, \Phi^3 + \text{h.c.} \right)$$

The component action is obtained by freezing Φ to gauge fix the extra conformal symmetries.

The scalar fields ϕ^i behave as coordinates of a Kähler manifold, whose metric can be used to raise and lower chiral indices and is given by the second derivatives of G:

$$g_{i\bar{j}} = G_{i\bar{j}}$$

The auxiliary fields F^i are instead completely determined by the first derivatives of G:

$$F^i = -e^{G/2}\, G^i$$

The kinetic term is given by

$$T = g_{i\bar{j}}\, \partial_\mu \phi^i \partial^\mu \phi^{\bar{j}}$$

The potential has instead the form:

$$V = e^G \Big(G^k G_k - 3 \Big)$$

Cremmer, Julia, Scherk, Ferrara, Girardello, Van Nieuwenhuizen
Bagger, Witten

Vacuum

The flatness condition is $V = 0$ and it implies that:

$$G^k G_k - 3 = 0$$

The stationarity conditions can be written as $\nabla_i V = 0$ and imply:

$$G_i + G^k \nabla_i G_k = 0$$

The stability conditions amount finally to imposing

$$\begin{pmatrix} m^2_{i\bar{\jmath}} & m^2_{ij} \\ m^2_{\bar{\imath}\bar{\jmath}} & m^2_{\bar{\imath}j} \end{pmatrix} > 0$$

where the blocks $m^2_{i\bar{\jmath}} = \nabla_i \nabla_{\bar{\jmath}} V$ and $m^2_{ij} = \nabla_i \nabla_j V$ are given by:

$$m^2_{i\bar{\jmath}} = e^G \Big[g_{i\bar{\jmath}} + \nabla_i G_k \nabla_{\bar{\jmath}} G^k - R_{i\bar{\jmath}p\bar{q}}\, G^p G^{\bar{q}} \Big]$$

$$m^2_{ij} = e^G \Big[2\, \nabla_{(i} G_{j)} + G^k \nabla_{(i} \nabla_{j)} G_k \Big]$$

Supersymmetry is spontaneously broken, and the gravitino mass is:

$$m_{3/2} = e^{G/2}$$

The would-be Goldstino fermion is identified with the linear combination $\eta = f_i \psi^i$, where:

$$f_i = \frac{1}{\sqrt{3}} \frac{F_i}{m_{3/2}} = -\frac{1}{\sqrt{3}} G_i$$

For fixed $g_{i\bar{\jmath}}$, the quantities f_i can be treated as independent variables without any particular constraint.

Flatness condition

The flatness condition is the constraint that the Goldstino vector should have unit length:

$$g_{i\bar{\jmath}} f^i f^{\bar{\jmath}} = 1$$

Stability condition

The stability condition is more complicated and can be studied only model by model, by explicit diagonalization.

It is however possible to find simpler but weaker conditions for stability, which are necessary but not sufficient, by looking at particular directions in scalar field space.

In this case, there is only one special complex direction that appears in the problem and that we could use: the Goldstino direction G^i.

Looking at the two independent real directions $(G^i, G^{\bar{\imath}})$ and $(iG^i, -iG^{\bar{\imath}})$, one deduces the condition $m^2_{i\bar{\jmath}}\, G^i G^{\bar{\jmath}} > 0$. Using the flatness and the stationarity conditions, this gives:

$$R_{i\bar{\jmath}p\bar{q}}\, f^i f^{\bar{\jmath}} f^p f^{\bar{q}} < \frac{2}{3}$$

Constraints

Summarizing, a stationary point can lead to a satisfactory situation only if two simple flatness and stability conditions are satisfied at it.

It is convenient redefine the fields to locally switch to flat indices, with $g_{I\bar{J}} = \delta_{I\bar{J}}$. The two conditions become then simply:

Flatness: $\delta_{I\bar{J}}\, f^I f^{\bar{J}} = 1$

Stability: $R_{I\bar{J}P\bar{Q}}\, f^I f^{\bar{J}} f^P f^{\bar{Q}} < \frac{2}{3}$

The flatness condition fixes the overall amount of SUSY breaking. The stability condition requires the existence of directions with $R < 2/3$, and constrains the direction of SUSY breaking to be close to these.

Gomez-Reino, Scrucca

GAUGE INVARIANT SUGRA MODELS

A model with chiral multiplets Φ^i and vector multiplets V^a is specified by a real Kähler function G and a holomorphic gauge kinetic matrix H_{ab}.

In the superconformal formulation, the Lagrangian has the form:

$$\mathcal{L} = \int d^4\theta \left[-3\,e^{-G/3}\right] \Phi^\dagger\Phi + \left(\int d^2\theta\, \Phi^3 + \text{h.c.} \right)$$
$$+ \left(\int d^2\theta\, \frac{1}{4} H_{ab} W^{a\alpha} W^b_\alpha + \text{h.c.} \right)$$

The gauge transformations of the superfields are encoded in a set of holomorphic Killing vectors X^i_a:

$$\delta\Phi^i = \Lambda^a X^i_a \qquad \delta V^a = -i(\Lambda^a - \bar{\Lambda}^a)$$

The local charges are:

$$Q_{ai}{}^j = -\nabla_i X^j_a$$

The function G must be invariant: $\delta G = 0$. This implies:

$$G_a = -i\, X_a^i G_i = i\, X_a^{\bar{\imath}} G_{\bar{\imath}}$$

The first and second derivatives of these relations imply that:

$$X_{ai} + X_a^k \nabla_i G_k + G_k \nabla_i X_a^k = 0 \qquad X_{ai} = -i\, \nabla_i G_a$$

$$\nabla_i X_{a\bar{\jmath}} + \nabla_{\bar{\jmath}} X_{ai} = 0 \qquad Q_{ai\bar{\jmath}} = -\nabla_i \nabla_{\bar{\jmath}} G_a$$

The function H_{ab} must instead transform in such a way to cancel possible residual quantum anomalies: $\delta H_{bc} = i\, \Lambda^a A_{abc}$. This implies:

$$X_a^i \nabla_i H_{bc} = i\, A_{abc}$$

The scalar fields ϕ^i parametrize now a symmetric Kähler manifold. The metric for chiral indices is given by the second derivatives of G:

$$g_{i\bar{\jmath}} = G_{i\bar{\jmath}}$$

The vector fields A^a_μ gauge the symmetries associated to the isometries X^i_a. The real part of H_{ab} effectively acts as a metric for vector indices, while its imaginary part gives additional parameters:

$$h_{ab} = \mathrm{Re}\, H_{ab} \qquad \theta_{ab} = \mathrm{Im}\, H_{ab}$$

The auxiliary fields F^i and D^a are given by the first derivatives of G:

$$F^i = -e^{G/2}\, G^i \qquad D^a = -G^a$$

The kinetic terms are:

$$T = g_{i\bar{\jmath}} \Big(\partial_\mu \phi^i - X^i_a A^a_\mu\Big)\Big(\partial^\mu \phi^{\bar{\imath}} - X^{\bar{\imath}}_a A^{a\mu}\Big) - \frac{1}{4} h_{ab}\, F^a_{\mu\nu} F^{b\mu\nu} - \frac{1}{4} \theta_{ab}\, F^a_{\mu\nu} \tilde{F}^{b\mu\nu}$$

The potential has instead the form:

$$V = e^G \Big(G^k G_k - 3\Big) + \frac{1}{2}\, G^a G_a$$

Cremmer, Ferrara, Girardello, Van Proeyen
Bagger

Vacuum

The flatness condition is $V = 0$ and it implies that:

$$G^k G_k + \frac{1}{2} e^{-G} G^a G_a - 3 = 0$$

The stationarity conditions can be written as $\nabla_i V = 0$ and imply:

$$G_i + G^k \nabla_i G_k + e^{-G} \Big[G^a \Big(\nabla_i - \frac{1}{2} G_i \Big) G_a + \frac{1}{2} h_{abi} G^a G^b \Big] = 0$$

The stability conditions amount in this case to imposing the slightly weaker requirement:

$$\begin{pmatrix} m^2_{i\bar{j}} & m^2_{ij} \\ m^2_{\bar{i}\bar{j}} & m^2_{\bar{i}j} \end{pmatrix} \geq 0$$

The equality sign holds for the would-be Goldstone bosons, which are absorbed by the vector fields.

The blocks $m^2_{i\bar{\jmath}} = \nabla_i \nabla_{\bar{\jmath}} V$ and $m^2_{ij} = \nabla_i \nabla_j V$ are now given by the following more complicated expressions:

$$
\begin{aligned}
m^2_{i\bar{\jmath}} = e^G \Big[& g_{i\bar{\jmath}} - R_{i\bar{\jmath}p\bar{q}} G^p G^{\bar{q}} + \nabla_i G_k \nabla_{\bar{\jmath}} G^k \Big] \\
& + \Big[\frac{1}{2} \Big(G_i G_{\bar{\jmath}} - g_{i\bar{\jmath}} \Big) G^a G_a + \Big(G_{(i} h_{ab\bar{\jmath})} + h^{cd} h_{aci} h_{bd\bar{\jmath}} \Big) G^a G^b \\
& \quad - 2\, G^a G_{(i} \nabla_{\bar{\jmath})} G_a - 2\, G^a h^{bc} h_{ab(i} \nabla_{\bar{\jmath})} G_c \\
& \quad + h^{ab} \nabla_i G_a \nabla_{\bar{\jmath}} G_b + G^a \nabla_i \nabla_{\bar{\jmath}} G_a \Big]
\end{aligned}
$$

$$
\begin{aligned}
m^2_{ij} = e^G \Big[& 2\, \nabla_{(i} G_{j)} + G^k \nabla_{(i} \nabla_{j)} G_k \Big] \\
& + \Big[\frac{1}{2} \Big(G_i G_j - \nabla_{(i} G_{j)} \Big) G^a G_a + \Big(G_{(i} h_{abj)} + h^{cd} h_{aci} h_{bdj} \Big) G^a G^b \\
& \quad - \frac{1}{2} h_{abij} G^a G^b - 2 G^a G_{(i} \nabla_{j)} G_a - 2\, G^a h^{bc} h_{ab(i} \nabla_{j)} G_c \\
& \quad + h^{ab} \nabla_i G_a \nabla_j G_b + G^a \nabla_i \nabla_{\bar{\jmath}} G_a \Big]
\end{aligned}
$$

Supersymmetry is spontaneously broken, and the gravitino mass has the same expression as before:

$$m_{3/2} = e^{G/2}$$

The would-be Goldstino fermion is $\eta = f_i \psi^i + d_a \lambda^a$, where:

$$f_i = \frac{1}{\sqrt{3}} \frac{F_i}{m_{3/2}} = -\frac{1}{\sqrt{3}} G_i \qquad d_a = \frac{1}{\sqrt{6}} \frac{D_a}{m_{3/2}} = -\frac{1}{\sqrt{6}} e^{-G} G_a$$

Gauge symmetries are also spontaneously broken, and the vector mass matrix is:

$$M^2_{ab} = 2\, g_{i\bar{\jmath}} X^i_a X^{\bar{\jmath}}_b = 2\, g_{i\bar{\jmath}} \nabla^i G_a \nabla^{\bar{\jmath}} G_b$$

The would-be Goldstone bosons are $\sigma_a = v_{ai} \phi^i + v_{a\bar{\imath}} \phi^{\bar{\imath}}$, where:

$$v_{ai} = \frac{X_{ai}}{\sqrt{X^k_a X_{ak}}}$$

For fixed $g_{i\bar{\jmath}}$, X_a^i and h_{ab}, the quantities f_i and d_a can be thought as variables, but with some relations involving the parameters:

$$x_a^i = \frac{X_a^i}{m_{3/2}} \qquad m_{ab} = \frac{1}{2}\frac{M_{ab}}{m_{3/2}} \qquad q_{ai\bar{\jmath}} = \frac{Q_{ai\bar{\jmath}}}{m_{3/2}} \qquad a_{abc} = \frac{A_{abc}}{m_{3/2}}$$

There is a dynamical relation holding at stationary points, which is implied by stationarity along the directions X_a^i:

$$q_{ai\bar{\jmath}}\, f^i f^{\bar{\jmath}} - \sqrt{\frac{2}{3}}\left[2\, m_{ab}^2 + \left(3 f^i f_i - 1\right) h_{ab}\right] d^b + a_{abc}\, d^b d^c = 0$$

Kawamura

There is then a kinematical relation holding at any point, which is implied by gauge invariance of G:

$$d_a = -\frac{i}{\sqrt{2}}\, x_a^i f_i = \frac{i}{\sqrt{2}}\, x_a^{\bar{\imath}} f_{\bar{\imath}}$$

There is finally a kinematical bound, implied by this relation:

$$|d_a| \leq m_{aa}\sqrt{f^i f_i}$$

Flatness condition

The flatness condition is again simply the constraint that the Goldstino vector should have unit length:

$$g_{i\bar{\jmath}}\, f^i f^{\bar{\jmath}} + h_{ab}\, d^a d^b = 1$$

Stability condition

The stability condition is as before a complicated condition, which can be studied only model by model, by explicit diagonalization.

However, once again it is possible to find simpler but weaker conditions for stability, which are necessary but not sufficient, by looking at particular directions in scalar field space.

In this case, there are two kinds of special complex directions that appear: the projected Goldstino direction G^i and the Goldstone directions X_a^i.

Looking at the real directions $(G^i, G^{\bar\imath})$ and $(iG^i, -iG^{\bar\imath})$, one deduces the condition $m^2_{i\bar\jmath} G^i G^{\bar\jmath} \geq 0$. Using the flatness and the stationarity conditions, this yields:

$$R_{i\bar\jmath p\bar q}\, f^i f^{\bar\jmath} f^p f^{\bar q} + 2\left(h_{ab}h_{cd} - \frac{1}{2}\, h_{ab}{}^i h_{cdi}\right) d^a d^b d^c d^d$$
$$-\, 2\, h^{cd} h_{aci} h_{bd\bar\jmath}\, f^i f^{\bar\jmath}\, d^a d^b + \sqrt{\frac{3}{2}}\, a_{abc}\, d^a d^b d^c$$
$$-\, \frac{8}{3}\left(m^2_{ab} - \frac{1}{2}\, h_{ab}\right) d^a d^b \leq \frac{2}{3}$$

Looking at the real directions $(X^i_a, X^{\bar\imath}_a)$ and $(iX^i_a, -iX^{\bar\imath}_a)$, one finds that the former are flat directions whereas the latter imply the extra conditions $m^2_{i\bar\jmath}\, X^i_a X^{\bar\jmath}_a \geq 0$, which have however a complicated form.

No extra useful condition

Constraints

As before, a stationary point can lead to a satisfactory situation only if two simple flatness and stability conditions are satisfied at it.

It is convenient redefine the fields to locally switch to flat indices, with $g_{I\bar{J}} = \delta_{I\bar{J}}$ and $h_{AB} = \delta_{AB}$. For simplicity, we also assume a constant and diagonal gauge kinetic function. The two conditions read then:

Flatness: $\delta_{I\bar{J}}\, f^I f^{\bar{J}} = 1 - \sum_A d_A^2$

Stability: $R_{I\bar{J}P\bar{Q}}\, f^I f^{\bar{J}} f^P f^{\bar{Q}} \leq \frac{2}{3} + \frac{8}{3} \sum_A \Big(m_A^2 - \frac{1}{2}\Big) d_A^2 - 2 \Big(\sum_A d_A^2\Big)^2$

The flatness condition fixes as before the amount of SUSY breaking. The stability condition constrains instead the directions of SUSY breaking.

The f_I represent the basic qualitative seed for SUSY breaking, whereas the d_A provide additional quantitative effects.

Indeed, the d_A are not independent from the f_I, but rather related to them as follows:

Dynamical relation: $d_A = \sqrt{\frac{3}{8}} \frac{q_{AI\bar{J}} f^I f^{\bar{J}}}{m_A^2 - \frac{1}{2} + \frac{3}{2} f^I f_I}$

Kinematical relation: $d_A = -\frac{i}{\sqrt{2}} x_A^I f_I$

Kinematical bound: $|d_A| \leq m_A \sqrt{f^I f_I}$

These 3 relations are gradually weaker and simpler, and can be used to set up 3 different types of analyses of the constraints.

The effect of vector multiplets is generically to alleviate the constraints and results in a lowering of the effective curvature for chiral multiplets. One needs then $\tilde{R} < 3/2$, which is a milder constraint.

Gomez-Reino, Scrucca

RELATIVE EFFECT OF F AND D BREAKING

It is useful to introduce the new variables:

$$\tilde{f}^I = \frac{f^I}{\sqrt{1 - \sum_B d_B^2}} \qquad \tilde{d}_A = \frac{d_A}{\sqrt{1 - \sum_B d_B^2}}$$

The constraints can then be rewritten as:

Flatness: $\delta_{I\bar{J}}\, \tilde{f}^I \tilde{f}^{\bar{J}} = 1$

Stability: $R_{I\bar{J}P\bar{Q}}\, \tilde{f}^I \tilde{f}^{\bar{J}} \tilde{f}^P \tilde{f}^{\bar{Q}} \le \frac{2}{3}\Big(1 + \Delta(\tilde{d}_A)\Big)$

where:

$$\Delta(\tilde{d}_A) = 4\sum_A m_A^2\, \tilde{d}_A^2 + 4\sum_A \Big(m_A^2 - 1\Big)\tilde{d}_A^2 \sum_B \tilde{d}_B^2$$

The dynamical relation between auxiliary fields becomes:

$$\tilde{d}_A \sqrt{1+\sum_B \tilde{d}_B^2}\left[1+m_A^2-\frac{3}{2}\frac{\sum_B \tilde{d}_B^2}{1+\sum_B \tilde{d}_B^2}\right] = \sqrt{\frac{3}{8}}\, q_{AI\bar{J}}\, \tilde{f}^I \tilde{f}^{\bar{J}}$$

The kinematical relation becomes instead:

$$\tilde{d}_A = -\frac{i}{\sqrt{2}}\, x_A^I\, \tilde{f}_I$$

Finally the kinematical bound implies:

$$|\tilde{d}_A| \leq m_A$$

Exploiting these relations to eliminate the $\tilde{d}_A$, one can arrive to constraints involving only the $\tilde{f}_I$ and a corrected effective curvature $\tilde{R}_{I\bar{J}P\bar{Q}}$, taking the simple form:

Flatness: $\delta_{I\bar{J}}\, \tilde{f}^I \tilde{f}^{\bar{J}} = 1$

Stability: $\tilde{R}_{I\bar{J}P\bar{Q}}\, \tilde{f}^I \tilde{f}^{\bar{J}} \tilde{f}^P \tilde{f}^{\bar{Q}} \leq \dfrac{2}{3}$

Exploiting the dynamical relation

Whenever $|\tilde{d}_A| \ll 1$, as for instance when $m_A \gg 1$ or $m_A \ll 1$, the dynamical relation can be linearized and:

$$\tilde{d}_A \simeq \sqrt{\frac{3}{8}}\,\frac{1}{1+m_A^2}\,q_{AI\bar{J}}\,\tilde{f}^I\,\tilde{f}^{\bar{J}}$$

Moreover, keeping only the leading term in Δ, one finds:

$$\Delta(\tilde{d}_A) \simeq \frac{3}{2}\sum_A \left[\frac{m_A}{1+m_A^2}\right]^2 \left|q_{AI\bar{J}}\,\tilde{f}^I\,\tilde{f}^{\bar{J}}\right|^2$$

The net effect of vector multiplets is then to change the effective curvature for chiral multiplets to:

$$\tilde{R}_{I\bar{J}P\bar{Q}} \simeq R_{I\bar{J}P\bar{Q}} - \sum_A \left[\frac{m_A}{1+m_A^2}\right]^2 q_{AI(\bar{J}}\,q_{AP\bar{Q})}$$

Whenever $|\tilde{d}_A| \sim 1$, as generically for $m_A \sim 1$, one can combine the dynamical relation and the kinematical bound to derive the upper bound:

$$|\tilde{d}_A|\sqrt{1+\sum_B \tilde{d}_B^2} \leq \sqrt{\frac{3}{8}}\,\frac{1+\sum_B m_B^2}{1+m_A^2+\Big(m_A^2-\frac{1}{2}\Big)\sum_B m_B^2}\,\Big|q_{AI\bar{J}}\,\tilde{f}^I\tilde{f}^{\bar{J}}\Big|$$

Dropping the negative term in Δ, one finds then:

$$\Delta(\tilde{d}_A) \leq \frac{3}{2}\sum_A\left[\frac{m_A\Big(1+\sum_B m_B^2\Big)}{1+m_A^2+\Big(m_A^2-\frac{1}{2}\Big)\sum_B m_B^2}\right]^2\Big|q_{AI\bar{J}}\,\tilde{f}^I\tilde{f}^{\bar{J}}\Big|^2$$

This can be used to get a simpler but weaker form of the constraints, where the net effect of vector multiplets is encoded in:

$$\tilde{R}_{I\bar{J}P\bar{Q}} \simeq R_{I\bar{J}P\bar{Q}}-\sum_A\left[\frac{m_A\Big(1+\sum_B m_B^2\Big)}{1+m_A^2+\Big(m_A^2-\frac{1}{2}\Big)\sum_B m_B^2}\right]^2 q_{AI(\bar{J}}\,q_{AP\bar{Q})}$$

Exploiting the kinematical relation

Useful results only when the theory has special properties.

Exploiting the kinematical bound

Using the kinematical bound, one can derive an upper bound for Δ. This is a complicated function of the m_A^2. But it can also be further bounded by above by a simple function of $m^2 = \sum_A m_A^2$:

$$\Delta(\tilde{d}_A) \leq \frac{m^4}{1-m^2}\,\theta\Big(\frac{1}{2}-m^2\Big) + 4\,m^6\,\theta\Big(m^2-\frac{1}{2}\Big)$$

This can be used to get a simpler but weaker form of the constraints, where the net effect of vector multiplets is encoded in:

$$\tilde{R}_{I\bar{J}P\bar{Q}} \simeq \Big[1+\frac{m^4}{1-m^2}\,\theta\Big(\frac{1}{2}-m^2\Big)+4\,m^6\,\theta\Big(m^2-\frac{1}{2}\Big)\Big]^{-1} R_{I\bar{J}P\bar{Q}}$$

FACTORIZABLE SPACES

Suppose that $\mathcal{M}$ is a product of N distinct 1-dimensional manifolds. The function K splits then into a sum of terms depending on a single field, while W can instead still be arbitrary:

In this situation, $g_{i\bar{\jmath}}$ and $R_{i\bar{\jmath}p\bar{q}}$ are diagonal and have only N non-zero components. Moreover, these are related by $R_{i\bar{\imath}i\bar{\imath}} = R_i\, g_{i\bar{\imath}}^2$, in terms of the curvature scalars R_i of the various submanifolds.

The flat-index curvature tensor entering the constraints takes then the following form:

$$R_{I\bar{J}P\bar{Q}} = \begin{cases} R_i\,, & \text{if } I = J = P = Q \\ 0\,, & \text{otherwise} \end{cases}$$

This simplifies enough to problem to allow for an exact solution.

The flatness and stability conditions simplify to

Flatness: $\sum_k \Theta_k^2 = 1$

Stability: $\sum_k R_k \Theta_k^4 < \frac{2}{3}$

in terms of the N real and positive variables

$$\Theta_i = |f_I|$$

These constraints admit solutions only if the curvatures satisfy

$$\sum_k R_k^{-1} > \frac{3}{2}$$

The allowed breaking directions fill then a cone around the axis

$$\Theta_i^0 = \sqrt{\frac{R_i^{-1}}{\sum_k R_k^{-1}}}$$

More precisely, one finds $\Theta_i \in \left[\Theta_i^-, \Theta_i^+\right]$, with:

$$\Theta_i^+ = \begin{cases} \sqrt{\dfrac{R_i^{-1} + \sqrt{\frac{2}{3} R_i^{-1} \left(\sum_{k\neq i} R_k^{-1}\right)\left(\sum_k R_k^{-1} - \frac{3}{2}\right)}}{\sum_k R_k^{-1}}} \,, & R_i^{-1} < \dfrac{3}{2} \\ 1 \,, & R_i^{-1} > \dfrac{3}{2} \end{cases}$$

$$\Theta_i^- = \begin{cases} \sqrt{\dfrac{R_i^{-1} - \sqrt{\frac{2}{3} R_i^{-1} \left(\sum_{k\neq i} R_k^{-1}\right)\left(\sum_k R_k^{-1} - \frac{3}{2}\right)}}{\sum_k R_k^{-1}}} \,, & \sum_{k\neq i} R_k^{-1} < \dfrac{3}{2} \\ 0 \,, & \sum_{k\neq i} R_k^{-1} > \dfrac{3}{2} \end{cases}$$

The relevance of each Φ_i for SUSY breaking depends thus on the size of the corresponding R_i^{-1} with respect to $3/2$.

SYMMETRIC SPACES

Suppose that $\mathcal{M}$ is a coset space G/H, where G is a group of global isometries and H a local stability group. The function K has then some special form, but W can still be arbitrary:

In this situation, $g_{i\bar{\jmath}}$ and $R_{i\bar{\jmath}p\bar{q}}$ are G-invariant and there are relations among their components. Moreover $R_{i\bar{\jmath}p\bar{q}}$ can always be related to $g_{i\bar{\jmath}}$, through some overall curvature scale $R_{\rm all}$.

Calabi, Vesentini

The flat-index curvature tensor entering the constraints has the special structure

$$R_{I\bar{J}P\bar{Q}} = R_{\rm all}\Big(G\text{-invariant combination of } H\text{-invariant } \delta\text{'s}\Big)$$

This again simplifies enough to problem to allow for an exact solution.

Generalized spheres

With $N = 1 + q$ fields Φ_i one can have:

$$\mathcal{M} = \frac{SU(1,1+q)}{U(1) \times SU(1+q)} \supset \frac{SU(1,1)}{U(1)}$$

The Riemann tensor in normal coordinates reads

$$R_{I\bar{J}P\bar{Q}} = \frac{1}{2} R_{\rm all} \Big(\delta_{I\bar{J}}\, \delta_{P\bar{Q}} + \delta_{I\bar{Q}}\, \delta_{P\bar{J}} \Big)$$

The flatness and stability conditions can then be rewritten as

Flatness: $\Theta^2 = 1$

Stability: $R_{\rm all}\, \Theta^4 < \dfrac{2}{3}$

in terms of just 1 real and positive variable

$$\Theta = \sqrt{\textstyle\sum_k |f_K|^2}$$

The situation is then as for 1 field with $R = R_{\rm all} \;\Rightarrow\; R_{\rm all}^{-1} > 3/2$.

Unitary Grassmannians

With $N = p\,(p+q)$ fields Φ_{ia} one can have:

$$\mathcal{M} = \frac{SU(p,p+q)}{U(1)\times SU(p)\times SU(p+q)} \supset \left(\frac{SU(1,1)}{U(1)}\right)^p$$

The Riemann tensor in normal coordinates reads

$$R_{IA\,\bar{J}\bar{B}\,PC\,\bar{Q}\bar{D}} = \frac{1}{2}R_{\rm all}\Big(\delta_{I\bar{J}}\,\delta_{P\bar{Q}}\,\delta_{A\bar{D}}\,\delta_{C\bar{B}} + \delta_{I\bar{Q}}\,\delta_{P\bar{J}}\,\delta_{A\bar{B}}\,\delta_{C\bar{D}}\Big)$$

The flatness and stability conditions reduce then simply to

Flatness: $\sum_k \Theta_k^2 = 1$

Stability: $\sum_k R_{\rm all}\,\Theta_k^4 < \frac{2}{3}$

in terms of the p real and positive variables

$$\Theta_i = \big|\mathrm{Eigenvalue}_i\big(f_{IA}\big)\big|$$

The situation is then as for p fields with $R_i = R_{\rm all} \Rightarrow R_{\rm all}^{-1} > 3/(2p)$.

Orthogonal Grassmannians

With $N = 2 + q$ fields Φ_i one can have:

$$\mathcal{M} = \frac{SO(2,2+q)}{SO(2) \times SO(2+q)} \supset \left(\frac{SU(1,1)}{U(1)}\right)^2$$

The Riemann tensor in normal coordinates reads

$$R_{I\bar{J}P\bar{Q}} = \frac{1}{2} R_{\rm all} \Big(\delta_{I\bar{J}}\,\delta_{P\bar{Q}} + \delta_{I\bar{Q}}\,\delta_{P\bar{J}} - \delta_{IP}\,\delta_{\bar{J}\bar{Q}}\Big)$$

The flatness and stability conditions reduce then to

Flatness: $\Theta_+^2 + \Theta_-^2 = 1$

Stability: $R_{\rm all}\left(\Theta_+^4 + \Theta_-^4\right) < \dfrac{2}{3}$

in terms of the 2 real and positive variables

$$\Theta_\pm = \frac{1}{\sqrt{2}} \sqrt{\sum_k |f_K|^2 \pm \sqrt{\Big(\sum_k |f_K|^2\Big)^2 - \Big|\sum_k (f_K)^2\Big|^2}}$$

The situation is then as for 2 fields with $R_i = R_{\rm all} \Rightarrow R_{\rm all}^{-1} > 3/4$.

MODULI IN STRING MODELS

In string models, a natural candidate for the breaking sector is that of the moduli M_i controlling the coupling strength and the compactification geometry, and the Wilson lines Z_a of the hidden gauge groups.

Kaplunovky, Louis

In the simplest models, the scalar manifold is factorizable and symmetric, because it emerges as a projection of a $D = 10$ SUSY theory. For a gauge group of rank s, this has the form

$$\mathcal{M} = \frac{SU(1,1)}{U(1)} \times \frac{SO(6,6+s)}{SO(6) \times SO(6+s)}\bigg|_{\text{proj}}$$

Narain

The first factor describes the dilaton modulus S, and is always present. The second factor is spanned by the Kähler and complex structure moduli T_p and U_q, as well as the Wilson lines Z_a, and depends on the projection.

Minimal modulus geometry

The minimal factor that can arise for a single modulus M_i is described by the following Kähler potential:

$$K_i = -\,n_i \ln\left(M_i + M_i^\dagger\right)$$

Witten

This corresponds to the simplest possible symmetric space:

$$\mathcal{M}_i = \frac{SU(1,1)}{U(1)}$$

This is the basic one-dimensional space with constant curvature that can appear, and the curvature scalar is given by:

$$R_i = \frac{2}{n_i}$$

Maximally symmetric enhancement

Certain moduli M_i can mix to some number q_i of related Wilson lines Z_{a_i}, and these $1 + q_i$ fields have then

$$K_i = -\,n_i \ln\Big(M_i + M_i^\dagger - \sum_{a_i} Z_{a_i}^\dagger Z_{a_i}\Big)$$

Ellis, Kounnas, Nanopoulos
Ferrara, Kounnas, Porrati

The corresponding scalar manifold is given by:

$$\mathcal{M}_i = \frac{SU(1,1+q_i)}{U(1)\times SU(1+q_i)} \supset \frac{SU(1,1)}{U(1)}$$

This behaves as 1 copy of the minimal geometry for the flatness and stability constraints, with curvature scale:

$$R_i = \frac{2}{n_i}$$

Unitary enhancement

A set of p_r moduli with the same n_r can get enhanced to p_r^2 moduli $M_{i_r j_r}$ and couple to some number $p_r q_r$ of related Wilson lines $Z_{i_r a_r}$. These $p_r(p_r + q_r)$ fields have then

$$K_r = -\,n_r \ln\det\Big(M_{i_r j_r} + M^\dagger_{i_r j_r} - \sum\nolimits_{a_r} Z^\dagger_{i_r a_r} Z_{j_r a_r}\Big)$$

Ferrara, Kounnas, Porrati

The corresponding scalar manifold is:

$$\mathcal{M}_r = \frac{SU(p_r, p_r + q_r)}{U(1) \times SU(p_r) \times SU(p_r + q_r)} \supset \left(\frac{SU(1,1)}{U(1)}\right)^{p_r}$$

This behaves as p_r copies of the minimal geometry for the flatness and stability constraints, with overall curvature scale given by:

$$R_r = \frac{2}{n_r}$$

Orthogonal enhancement

A pair of $\mathbf{2}$ moduli M_{1_r} and M_{2_r} with the same n_r can also mix in a more peculiar fashion to some number q_r of related Wilson lines Z_{a_r}. The $2+q_r$ involved fields have then:

$$K_r = -\,n_r \ln\Big(\big(M_{1_r} + M_{1_r}^\dagger\big)\big(M_{2_r} + M_{2_r}^\dagger\big) - \textstyle\sum_{a_r}\big(Z_{a_r} + Z_{a_r}^\dagger\big)^2\Big)$$

Derendinger, Kounnas, Petropoulos, Zwirner

The corresponding scalar manifold is in this case:

$$\mathcal{M}_r = \frac{SO(2,2+q_r)}{SO(2)\times SO(2+q_r)} \supset \left(\frac{SU(1,1)}{U(1)}\right)^2$$

This behaves as $\mathbf{2}$ copies of the minimal geometry for the flatness and stability constraints, with an overall curvature given by:

$$R_r = \frac{2}{n_r}$$

Flatness and stability constraints

The structure of the flatness and stability constraints is blind to symmetry enhancements and is controlled by the minimal geometry:

$$\mathcal{M}_{\min} = \frac{SU(1,1)}{U(1)} \times \frac{SU(1,1)}{U(1)} \times \cdots$$

The crucial parameters in the constraints are the numerical coefficients n_i, related to the curvature of the basic submanifolds by $R_i = 2/n_i$.

The curvature bound $\sum_k R_k^{-1} > 3/2$ implies a simple restriction on the coefficients n_i:

$$\sum_k n_k > 3$$

The Goldstino cone is also entirely specified in terms of the n_i, and restricts the direction of supersymmetry breaking:

$$\Theta_i : \begin{cases} \text{upper bound smaller than } 1 \text{ if } \; n_i < 3 \\ \text{lower bound larger than } 0 \text{ if } \; \sum_{k \neq i} n_k < 3 \end{cases}$$

Dilaton and volume moduli

The most relevant moduli are the dilaton S, controlling the coupling, and the global volume modulus T, controlling the size of the internal manifold. These universally occur in all models, with:

$$n_S = 1 \; , \quad n_T = 3$$

Taking each field separately, the curvature bound is always violated. To fulfill it, one would need corrections. These should be large for S, but could be small for T.

Keeping both fields, the curvature bound is instead fulfilled. However T must dominate over S, and the Goldstino angle θ is constrained to the range $[0, \pi/4]$.

This demonstrates in an extremely robust way that the scenario where S dominates over T is impossible to realize, at least in the controllable limit where both are large.

CONCLUSIONS AND OUTLOOK

- In a generic SUGRA model with chiral and vector multiplets, there exist necessary conditions for flatness and stability that strongly constrain the geometry and the SUSY breaking direction.
- When $\boldsymbol{F}$ breaking dominates, the constraints are simple and rather strong. What matter is the Kähler curvature.
- The effect of an additional $\boldsymbol{D}$ breaking to alleviate the constraints. What matter is then a smaller effective Kähler curvature.