SCALAR GEOMETRY AND MASSES IN CALABI-YAU STRING MODELS

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- Scalar masses in supergravity
- Effective theory of Calabi-Yau string models
- Geometry in the no-scale sector
- Average sgoldstino mass and metastability
- Soft sfermion masses and universality

SCALAR MASSES IN SUPERGRAVITY

General structure of soft scalar masses

In a supergravity theory with Kähler potential K and superpotential W, the scalar fields have a kinetic function and a potential given by:

$$Z_{I\bar{J}} = K_{I\bar{J}} \qquad V = e^{K}\Big[K^{I\bar{J}}\big(W_I + K_I W\big)\big(\bar{W}_{\bar{J}} + K_{\bar{J}}\bar{W}\big) - 3|W|^2\Big]$$

At a given vacuum where $\nabla_I V = 0$, susy breaking is controlled by $F_I = -e^{K/2}(W_I + K_I W)$ and $m_{3/2} = e^{K/2}|W|$. To get a vanishing cosmological constant $V = 0$ one then needs to adjust $|F| = \sqrt{3}\, m_{3/2}$. Finally, the masses are $m^2_{I\bar{J}} = \nabla_I \nabla_{\bar{J}} V$ and $m^2_{IJ} = \nabla_I \nabla_J V$.

The mass matrix depends on W and K, with susy and non-susy parts. But along certain particular directions it simplifies and its value is mostly controlled by the geometry defined by K and less by the form of W.

Average sGoldstino mass in the hidden sector

Gomez-Reino, Scrucca 2006
Denef, Douglas 2005

The average mass for the sgoldstino defined by the normalized direction f^I of susy breaking in the hidden sector is given by:

$$m^2_{\rm sgold} = 3\Big(R(f) + \frac{2}{3}\Big) m^2_{3/2}$$

in terms of the sectional curvature along f^I:

$$R(f) = -R_{I\bar{J}P\bar{Q}} f^I f^{\bar{J}} f^P f^{\bar{Q}}$$

A necessary condition for metastability is that $m^2_{\rm sgold}$ should be positive. This implies:

$$R(f) > -\frac{2}{3}$$

This represents a non-trivial constraint on K, even if W is allowed to be arbitrary.

Soft sfermion masses in the visible sector

Kaplunovsky, Louis, 1993
Brignole, Ibanez, Munoz 1993

The soft mass induced for the sfermions defined by a normalized direction v^I in the visible sector when susy is broken along a normalized direction f^I in the hidden sector is given by

$$m^2_{\rm sferm} = 3\Big(R(v,f) + \frac{1}{3}\Big)m^2_{3/2}$$

in terms of the bisectional curvature along v^I and f^I:

$$R(v,f) = -R_{I\bar{J}P\bar{Q}}\, v^I v^{\bar{J}} f^P f^{\bar{Q}}$$

For positivity and universality, one then needs:

$$R(\not{v},f) > -\frac{1}{3}$$

This represents once again a non-trivial constraint on K, even if W is allowed to be arbitrary.

EFFECTIVE THEORY OF CALABI-YAU STRING MODELS

Field content

The minimal chiral multiplets are the dilaton S, the Kähler moduli T^A and some matter fields Φ^α. They naturally split in visible and hidden sectors.

Effective Kähler potential

The Kähler potential controlling the kinetic energy is always dominated by a classical contributions of the form:

$$K = -\log\left(S + \bar{S}\right) - \log\left[Y\left(T^A + \bar{T}^A, \Phi^\alpha \bar{\Phi}^\beta\right)\right]$$

Effective superpotential

The superpotential controlling the potential energy can be dominated by non-classical contributions, and can thus a priori be quite arbitrary:

$$W = W\left(S, T^A, \Phi^\alpha\right)$$

Casas 1996
Brustein, de Alwis 2004
Gomez-Reino, Scrucca 2006

Dilaton sector domination

The dilaton belongs to a fixed and factorized manifold $SU(1,1)/U(1)$ with constant curvature -2. One then finds:

$$R(f) = -2 \qquad R(v,f) = 0$$

This unavoidably leads to a negative m^2_{sgold}, but automatically yields a positive universal m^2_{sferm}:

$$m^2_{\mathrm{sgold}} = -4 m^2_{3/2} \qquad m^2_{\mathrm{sferm}} = m^2_{3/2}$$

This means that it is impossible to realize this scenario in a controllable weak coupling situation.

No-scale sector domination

Cremmer, Ferrara, Kounnas, Nanopoulos 1983
Ellis, Lahanas, Nanopoulos, Tamvakis 1984
Covi, Gomez-Reino, Gross, Louis, Palma, Scrucca 2008

The moduli and matter fields span a no-scale manifold. For 1 modulus and m matter fields, one gets $SU(1,1+m)/(U(1)\times SU(1+m))$ with constant curvature $-\frac{2}{3}$. One then finds

$$R(f) = -\frac{2}{3} \qquad R(v,f) = -\frac{1}{3}$$

This implies vanishing $m^2_{\rm sgold}$ and vanishing $m^2_{\rm sferm}$, which can be a good starting point:

$$m^2_{\rm sgold} = 0 \qquad m^2_{\rm sferm} = 0$$

For $1+n$ moduli and m matter fields, one gets a more general $\mathcal{M}_{\rm ns}$ with a curvature that is a priori not constant but must behave as in the previous case along some special direction.

This shows that it may be possible to realize this scenario in a controllable weak coupling situation, at least in models with several moduli.

GEOMETRY OF NO-SCALE MANIFOLDS

General no-scale manifolds

A general no-scale manifold spanned by moduli and matter fields fields $Z^i = T^A, \Phi^\alpha$ is described by a Kähler potential of the form

$$K = -\log Y(J^A) \qquad J^A = T^A + \bar{T}^A + N^A(\Phi^\alpha \bar{\Phi}^\beta)$$

The real functions N^A are arbitrary, while the real function Y must be homogeneous of degree three in the variables J^A. This implies that:

$$K^i = -\delta^i_A J^A \qquad K^i K_i = 3$$

As a consequence, the geometry of such spaces has a restricted form along the special direction $k^i = -\frac{1}{\sqrt{3}} K^i$ in the hidden sector and any direction v^i in the visible sector.

Geometry

Covi, Gomez-Reino, Gross, Louis, Palma, Scrucca 2008
Farquet, Scrucca 2012

The metric, Chistoffel symbol and Riemann tensor are found to be:

$$g_{i\bar{\jmath}} = -Y^{-1} Y_{i\bar{\jmath}} + Y^{-2} Y_i Y_{\bar{\jmath}}$$

$$\Gamma_{ij\bar{k}} = -Y^{-1} Y_{ij\bar{k}} + Y^{-2} Y_{ij} Y_{\bar{k}} - Y^{-1} \big(g_{i\bar{k}} Y_j + g_{j\bar{k}} Y_i \big)$$

$$R_{i\bar{\jmath}p\bar{q}} = g_{i\bar{\jmath}}\, g_{p\bar{q}} + g_{i\bar{q}}\, g_{p\bar{\jmath}} - Y^{-1} Y_{i\bar{\jmath}p\bar{q}} - Y^{-2} Y_{ip\bar{s}} Y^{\bar{s}}{}_{\bar{\jmath}\bar{q}}$$

Along the special directions k^i and v^i one then finds:

$$g_{i\bar{\jmath}}\, k^i \bar{k}^{\bar{\jmath}} = 1 \qquad g_{i\bar{\jmath}}\, v^i \bar{k}^{\bar{\jmath}} = 0 \qquad g_{i\bar{\jmath}}\, v^i \bar{v}^{\bar{\jmath}} = 1$$

$$\Gamma_{ij\bar{k}}\, k^i k^j \bar{k}^{\bar{k}} = -\frac{2}{\sqrt{3}} \qquad \Gamma_{ij\bar{k}}\, v^i k^j \bar{k}^{\bar{k}} = 0 \qquad \Gamma_{ij\bar{k}}\, v^i v^j \bar{k}^{\bar{k}} = 0$$

$$R_{i\bar{\jmath}p\bar{q}}\, k^i \bar{k}^{\bar{\jmath}} k^p \bar{k}^{\bar{q}} = \frac{2}{3} \qquad R_{i\bar{\jmath}p\bar{q}}\, v^i \bar{k}^{\bar{\jmath}} k^p \bar{k}^{\bar{q}} = 0 \qquad R_{i\bar{\jmath}p\bar{q}}\, v^i v^{\bar{\jmath}} k^p \bar{k}^{\bar{q}} = \frac{1}{3}$$

This implies that

$$R(k) = -\frac{2}{3} \qquad R(v,k) = -\frac{1}{3}$$

HETEROTIC MODELS

Cecotti, Ferrara, Girardello 1988
Candelas, de la Ossa 1990
Buchbinder, Ovrut 2003
Paccetti Correia, Schmidt 2008
Andrey, Scrucca 2011

Geometry of the no-scale sector

One finds:

$$Y = \frac{1}{6} d_{ABC} t^A t^B t^C$$

where

$$t^A = J^A$$

$$J^A = T^A + \bar{T}^A - c^A_{\alpha\beta} \Phi^\alpha \bar{\Phi}^\beta$$

The function $Y(J^A)$ is homogeneous of degree 3 and also polynomial. The quantities d_{ABC} and $c^A_{\alpha\beta}$ are defined by integrals of harmonic forms:

$$d_{ABC} = \int_X \omega_A \wedge \omega_B \wedge \omega_C$$

$$c^A_{\alpha\beta} = \int_X \omega^A \wedge \mathrm{tr}(u_\alpha \wedge \bar{u}_\beta)$$

ORIENTIFOLD MODELS

Grimm, Louis 2004
Graña, Grimm, Jockers, Louis 2005
Jockers, Louis 2005

Geometry of the no-scale sector

One finds:

$$Y = \left(\frac{1}{6} d^{ABC} t_A t_B t_C\right)^2$$

where

$$t_A = t_A(J^B) \quad \text{such that} \quad d^{ABC} t_B t_C = 2 J^A$$

$$J^A = T^A + \bar{T}^A - c^A_{\alpha\beta} \Phi^\alpha \bar{\Phi}^\beta$$

The function $Y(J^A)$ is homogeneous of degree 3 but not polynomial. The quantities d_{ABC} and $c^A_{\alpha\beta}$ are defined by integrals of harmonic forms:

$$d^{ABC} = \int_X \omega^A \wedge \omega^B \wedge \omega^C$$

$$c^A_{\alpha\beta} = \int_C i^* \omega^A \wedge \mathrm{tr}(u_\alpha \wedge \bar{u}_\beta)$$

CANONICAL PARAMETRIZATION

Gunaydin, Sierra, Townsend 1984
Cremmer, Kounnas, Van Proeyen, Derendinger, Ferrara, de Wit, Girardello 1985
Farquet, Scrucca 2012

Canonical frame

At any reference point corresponding to $T^A \neq 0$ and $\Phi^\alpha = 0$, one may switch to a canonical parametrization where

$$T^0 = \frac{\sqrt{3}}{2} \qquad T^a = 0 \qquad \Phi^\alpha = 0$$

One may moreover require that $g_{i\bar{\jmath}} = \delta_{ij}$ and $Y = 1$, by a further linear field redefinition and a Kähler transformation.

In this new frame, T^0, T^a and Φ^α correspond to the volume modulus, cycle moduli and suitably rotated matter fields, and one finds:

$$d_{000} = \frac{2}{\sqrt{3}} \qquad d_{00a} = 0 \qquad d_{0ab} = -\frac{1}{\sqrt{3}}\,\delta_{ab} \qquad d_{abc} = \text{generic}$$

$$c^0_{\alpha\beta} = \frac{1}{\sqrt{3}}\,\delta_{\alpha\beta}\,, \quad c^a_{\alpha\beta} = \text{generic}$$

GEOMETRY IN THE CANONICAL FRAME

Metric

The metric is by construction trivial:

$$g_{0\bar{0}} = 1 \qquad g_{a\bar{b}} = \delta_{ab}$$
$$g_{\alpha\bar{\beta}} = \delta_{\alpha\beta}$$

Christoffel symbol

The Christoffel symbol is found to be identical in heterotic and orientifold models and reads:

$$\Gamma_{00\bar{0}} = -\frac{2}{\sqrt{3}} \qquad \Gamma_{0a\bar{b}} = -\frac{2}{\sqrt{3}}\delta_{ab} \qquad \Gamma_{ab\bar{c}} = -d_{abc}$$
$$\Gamma_{0\alpha\bar{\beta}} = -\frac{1}{\sqrt{3}}\delta_{\alpha\beta} \qquad \Gamma_{a\alpha\bar{\beta}} = -c^{a}_{\alpha\beta}$$

Riemann tensor

Andrey, Scrucca 2011
Farquet, Scrucca 2012

The Riemann tensor for heterotic and orientifold models is instead:

$$R_{0\bar{0}0\bar{0}} = \frac{2}{3} \quad R_{0\bar{0}a\bar{b}} = \frac{2}{3}\delta_{ab} \quad R_{a\bar{b}c\bar{0}} = \frac{1}{\sqrt{3}}d_{abc} \quad R_{a\bar{b}c\bar{d}} = (x \mp a)_{abcd}$$

$$R_{\alpha\bar{\beta}0\bar{0}} = \frac{1}{3}\delta_{\alpha\beta} \quad R_{\alpha\bar{\beta}a\bar{b}} = -(y+b)_{\alpha\beta ab} \quad R_{\alpha\bar{\beta}0\bar{b}} = \frac{1}{\sqrt{3}}c^{b}_{\alpha\beta}$$

$$R_{\alpha\bar{\beta}\gamma\bar{\delta}} = \frac{1}{3}\big(\delta_{\alpha\beta}\delta_{\gamma\delta} + \delta_{\alpha\delta}\delta_{\gamma\beta}\big) + c^{r}_{\alpha\beta}c^{r}_{\gamma\delta} + c^{r}_{\alpha\delta}c^{r}_{\gamma\beta}$$

in terms of the following combinations of parameters:

$$a_{abcd} = \frac{1}{2}\big(d_{abr}d_{rcd} + d_{adr}d_{rbc} + d_{acr}d_{rbd}\big) - \frac{1}{3}\big(\delta_{ab}\delta_{cd} + \delta_{ad}\delta_{bc} + \delta_{ac}\delta_{bd}\big)$$

$$x_{abcd} = \frac{1}{2}\big(d_{abr}d_{rcd} + d_{adr}d_{rbc} - d_{acr}d_{rbd}\big) + \frac{2}{3}\big(\delta_{ab}\delta_{cd} + \delta_{ad}\delta_{bc} - \delta_{ac}\delta_{bd}\big)$$

$$b_{\alpha\beta ab} = \frac{1}{2}\{c^{a}, c^{b}\}_{\alpha\beta} - \frac{1}{3}\delta_{ab}\delta_{\alpha\beta} - \frac{1}{2}d_{abr}c^{r}_{\alpha\beta}$$

$$y_{\alpha\beta ab} = \frac{1}{2}\big[c^{a}, c^{b}\big]_{\alpha\beta} - \frac{1}{3}\delta_{ab}\delta_{\alpha\beta} - \frac{1}{2}d_{abr}c^{r}_{\alpha\beta}$$

Coset spaces

Cremmer, Kounnas, Van Proeyen, Derendinger, Ferrara, de Wit, Girardello 1985
Farquet, Scrucca 2012

The space is symmetric, with a covariantly constant Riemann tensor, whenever:

$$a_{abcd} = 0 \qquad b_{\alpha\beta ab} = 0$$

There is also another mild algebraic condition on the matrices $c^{a}_{\alpha\beta}$, but it is essentially automatically satisfied whenever these form an algebra.

Degeneracy of heterotic and orientifold models

D'Auria, Ferrara, Trigiante 2004
Farquet, Scrucca 2012

The manifolds arising in the heterotic and orientifold models based on the same Calabi-Yau space coincide if and only if:

$$a_{abcd} = 0 \qquad b_{\alpha\beta ab} = \text{arbitrary}$$

AVERAGE SGOLDSTINO MASS

Covi, Gomez-Reino, Gross, Louis, Palma, Scrucca 2008
Farquet, Scrucca 2012

Sectional curvature

The sectional curvature controlling $m^2_{\rm sgold}$ is, for real f^i:

$$R(f) = -\frac{2}{3} \pm a(f) + 4\,b(f) - 2\,\omega^a(f)\,\omega^a(f)$$

where

$$a(f) = a_{abcd}\, f^a f^b f^c f^d \qquad b(f) = b_{\alpha\beta ab}\, f^a f^b f^\alpha f^\beta$$

$$\omega^a(f) = \frac{2}{\sqrt{3}} f^a f^0 + \frac{1}{2} d_{abc}\, f^b f^c + c^a_{\alpha\beta}\, f^\alpha f^\beta$$

Metastability and the lightest scalar

The condition $R(f) > -\frac{2}{3}$ for metastability implies:

$$\pm\, a(f) > 0 \quad \text{or} \quad b(f) > 0$$

The lightest scalar then has $m^2_{\rm light} \le \max\left\{0, (\pm a)_{\rm up}, 2\,b_{\rm up}\right\} m^2_{3/2}$.

SOFT SFERMION MASSES

Bisectional curvature

Andrey, Scrucca 2011
Farquet, Scrucca 2012

The bisectional curvature controlling m^2_{sferm} is, for real f^i and v^i:

$$R(v,f) = -\frac{1}{3} + b(v,f) - c^a(v)\,\omega^a(f)$$

where

$$b(v,f) = b_{\alpha\beta ab}\, v^\alpha v^\beta f^a f^b$$

$$c^a(v) = c^a_{\alpha\beta}\, v^\alpha v^\beta \qquad \omega^a(f) = \frac{2}{\sqrt{3}} f^a f^0 + \frac{1}{2} d_{abc} f^b f^c + c^a_{\alpha\beta} f^\alpha f^\beta$$

Positivity, universality and global symmetries

The condition $R(\not{v}, f) > -\frac{1}{3}$ for positivity and universality calls for:

$$\omega^a(f) = 0 \quad \text{and} \quad b(\not{v}, f) > 0$$

A set of global symmetries might explain the first of these conditions.

CONCLUSIONS

- The scalar geometry in Calabi-Yau string models is controlled by two kinds of parameters a_{abcd} and $b_{\alpha\beta ab}$, related to the deviations from coset situations in the moduli and matter sectors.
- Heterotic and orientifold models lead to dual geometries, which coincide in symmetric situations with $a_{abcd} = 0$ and $b_{\alpha\beta ab} = 0$, but also in non-symmetric cases with $a_{abcd} = 0$ but $b_{\alpha\beta ab} \neq 0$.
- The properties of the sgoldstino and sfermion masses are directly linked to the parameters a_{abcd} but $b_{\alpha\beta ab}$, and this allows to study the possibilities of achieving metastability and universality.