SUSY BREAKING IN EXTRA DIMENSIONS

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- Mechanisms for the transmission of SUSY breaking.
  Generic problems and possible solutions.
- Geometrical sequestering through extra dimensions.
  Soft masses and relevance of quantum corrections.
- Brane worlds from orbifold models.
  Microscopic and low energy effective theories.
  Loop corrections and SUSY cancellations.
- Radius-dependent loop effects in sequestered models.
  Universal correction to soft masses.
- Prototype of viable model.

Rattazzi, Scrucca, Strumia (hep-th/0305184)

SUSY BREAKING

The standard scenario is that SUSY breaking occurs at a scale M in a hidden sector and is transmitted to the visible sector through some interactions, generating soft breaking terms.

There are two main delicate points for phenomenology:

- Sflavour: m_0^2 must be positive and nearly universal.
- Shierarchy: must have $\mu \sim m_0 \sim m_{1/2} \ll \Lambda_{\rm UV}$.

Gauge mediation

SUSY breaking at $M < M_{\rm M}$ mediated by gauge interactions:

- $m_{1/2} \sim \dfrac{g^2}{16\pi^2}\dfrac{M^2}{M_M}$, $m_0^2 \sim \Big(\dfrac{g^2}{16\pi^2}\Big)^2\Big(\dfrac{M^2}{M_M}\Big)^2$.
- m_0^2 universal $(M_{\rm F} \gg M_M)$ and positive.
- $\mu \sim M_{\rm M}$ from interactions.

Gravity mediation

SUSY breaking at $M \ll M_{\rm P}$ mediated by gravity interactions:

- $m_{1/2}, m_{3/2} \sim \dfrac{M^2}{M_{\rm P}}$, $m_0^2 \sim \Big(\dfrac{M^2}{M_{\rm P}}\Big)^2$.
- m_0^2 generic $(M_{\rm F} \sim M_{\rm P})$.
- $\mu \sim \dfrac{M^2}{M_{\rm P}}$ from interactions.

Symmetries

To naturally solve both problems, one can try to introduce new symmetries. Main options:

- Gauge mediation + complications for Shierarchy.
  No simple and compelling model so far.
- Gravity mediation + constraints for Sflavour.
  Difficult to forbid mixing of the two sectors at M_{P}.

Geometry

An interesting possibility, natural in string theory, is to separate the visible and the hidden sectors along an extra dimension. This framework has very peculiar characteristics going beyond symmetries:

- Geometric distinction between visible sector, hidden sector, and mediating interactions.
- New physical scale M_{C} acting as cut-off for the mixing between the two sectors.

GEOMETRICAL SEQUESTERING IN SUGRA

Consider a general SUGRA theory with:

Visible: $\Phi_0 = (\phi_0, \chi_0; F_0)$, $V_0 = (A_0^\mu, \lambda_0; D_0)$.

Hidden: $\Phi_\pi = (\phi_\pi, \chi_\pi; F_\pi)$, $V_\pi = (A_\pi^\mu, \lambda_\pi; D_\pi)$.

Interactions: $C = (e_\mu^a, \psi_\mu; a_\mu, b_\mu)$, $S = (\phi_S, \psi_S; F_S)$.

After superconformal gauge-fixing, $b_\mu = 0$, $\phi_S = 1$, $\psi_S = 0$, and the structure of the matter action reads:

$$\mathcal{L}_{\rm mat} = \Big[\Omega(\Phi,\Phi^\dagger) S S^\dagger\Big]_D + \Big[P(\Phi) S^3\Big]_F + \Big[P(\Phi) S^3\Big]_F^\dagger + \Big[\tau(\Phi)\mathcal{W}^2\Big]_F + \Big[\tau(\Phi)\mathcal{W}^2\Big]_F^\dagger$$

The functions Ω, $\tau\mathcal{W}^2$ and P have expansions of the type:

$$\Omega = -3M_{\rm P}^2 + \Phi_0\Phi_0^\dagger + \Phi_\pi\Phi_\pi^\dagger + \frac{h}{M_{\rm P}^2}\Phi_0\Phi_0^\dagger\Phi_\pi\Phi_\pi^\dagger + \dots$$

$$P = \Lambda^3 + M_\pi^2\Phi_\pi + \dots$$

$$\tau\mathcal{W}^2 = \frac{1}{g_0^2}\mathcal{W}_0^2 + \frac{1}{g_\pi^2}\mathcal{W}_\pi^2 + \frac{k}{M_{\rm P}}\Phi_\pi\mathcal{W}_0^2 + \dots$$

For a vanishing cosmological constant, we tune $\Lambda^3 \sim M_\pi^2 M_{\rm P}$.

The SUSY breaking VEVs are then:

$$|F_\pi| \sim M_\pi^2\,, \quad |F_S| \sim \frac{\Lambda^3}{M_{\rm P}^2} \sim \frac{M_\pi^2}{M_{\rm P}}$$

Classical theory

Leading soft masses at classical level:

$$m_{3/2} \sim |F_S| \sim \frac{M_\pi^2}{M_\mathrm{P}}$$

$$m_{1/2} \sim k\,\frac{|F_\pi|}{M_\mathrm{P}} \sim k\,\frac{M_\pi^2}{M_\mathrm{P}}$$

$$m_0^2 \sim h\,\frac{|F_\pi|^2}{M_\mathrm{P}^2} \sim h\,\frac{M_\pi^4}{M_\mathrm{P}^2}$$

Non-universal; separating visible and hidden sectors in an extra dimension, $h = k = 0 \Rightarrow$ quantum corrections important.

Quantum corrections

Corrections from gauge loops due to superconformal anomaly:

$$\delta m_{1/2} \sim \frac{g^2}{16\pi^2}\,|F_S| \sim \frac{g^2}{16\pi^2}\,\frac{M_\pi^2}{M_\mathrm{P}}$$

$$\delta m_0^2 \sim \left(\frac{g^2}{16\pi^2}\right)^2 |F_S|^2 \sim \left(\frac{g^2}{16\pi^2}\right)^2 \frac{M_\pi^4}{M_\mathrm{P}^2}$$

Universal; positive for squarks and negative for sleptons !

Randall, Sundrum;

Giudice, Luty, Murayama, Rattazzi

With an extra dimension, corrections from gravity loops are cut off at $M_\mathrm{C} = (\pi R)^{-1}$ and computable:

$$\delta m_0^2 \sim \frac{M_\mathrm{C}^2}{16\pi^2 M_\mathrm{P}^2}\,\frac{|F_\pi|^2}{M_\mathrm{P}^2} \sim \frac{M_\mathrm{C}^2}{16\pi^2 M_\mathrm{P}^2}\,\frac{M_\pi^4}{M_\mathrm{P}^2}$$

Universal; positive or negative ?

Gauge and gravitational quantum corrections can compete if (gravity loop at $M_{\rm C}$) $\sim$ (gauge loop)^2, that is:

$$\frac{M_{\rm C}^2}{16\pi^2 M_{\rm P}^2} \sim \left(\frac{g^2}{16\pi^2}\right)^2 \Rightarrow \frac{M_{\rm C}}{M_{\rm P}} \sim \frac{g^2}{4\pi}$$

This is reasonable $\Rightarrow$ possible very interesting hybrid models of SUSY breaking.

Dynamics of extra dimensions

In the 4D effective theory for $E \ll M_{\rm C}$, the dynamics of an extra dimension is described by a chiral multiplet:

Radion: $T = (T, \psi_T; F_T)$.

The VEV of T controls the radius $({\rm Re}\, T = \pi R)$, whereas a VEV for F_T gives additional SUSY-breaking effects.

There are various ways to get a satisfactory radion dynamics.
F-terms: e.g. strong coupling condensation of bulk gaugino.
D-terms: e.g. Casimir energy with localized kinetic terms.

Luty, Sundrum;
Ponton, Poppitz

To compute radiative effects involving the radion multiplet, one needs a full 5D supergravity description. All these effects are non-local and therefore finite and insensitive to UV physics.

$S^1/\mathbf{Z_2}$ ORBIFOLD MODELS

The extra dimension is a circle with coordinate $x^5 \in [0, 2\pi]$ and gauged parity $\mathbf{Z_2} : \mathbf{x^5} \to -\mathbf{x^5}$. The radius is $e_{\dot 5}^{\dot 5} = R$.

The visible and hidden sectors are located at the fixed-points at 0 and π, and have $N = 1$ SUSY with $U(1)$ R-symmetry (bosons: q, fermions: $q-1$, aux: $q-2$):

Visible: $\Phi_0 = (\phi_0, \chi_0; F_0)$, $V_0 = (A_0^\mu, \lambda_0; D_0)$.

Hidden: $\Phi_\pi = (\phi_\pi, \chi_\pi; F_\pi)$, $V_\pi = (A_\pi^\mu, \lambda_\pi; D_\pi)$.

The interactions are in the bulk, and have $N = 2$ SUSY with $SU(2)$ R-symmetry (bosons: $\mathbf{1}$, fermions: $\mathbf{2}$, aux: $\mathbf{1}$ or $\mathbf{3}$):

Gauge: $\mathcal{V} = (A_M, \lambda, \Sigma; \vec{X})$.

Gravity: $\mathcal{M} = (e_M^A, \psi_M, A_M; \vec{V}_M, \vec{t}, v_{AB}, \lambda, C)$,
$\mathcal{T} = (\vec{Y}, B_{MNP}, \rho; N)$.

Bulk and boundary theories: fixed by $N = 2$ and $N = 1$ SUSY. Bulk-boundary couplings: fixed by $N = 1$ SUSY with $N = 2$ bulk multiplets decomposed into $N = 1$ boundary multiplets.

The Lagrangian (with e factored out) has the form:

$$\mathcal{L} = \mathcal{L}_5 + e_{\dot 5}^{\dot 5}\,\delta(x^5 - 0)\mathcal{L}_{4,0} + e_{\dot 5}^{\dot 5}\,\delta(x^5 - \pi)\mathcal{L}_{4,\pi}$$

Singularities

Auxiliary fields have a dimensionless propagator and could give divergences in the sums over KK modes with $m_n = n/R$.

In the natural formulation, auxiliary and odd fields mix through $\partial_5 \Rightarrow$ propagators $\Box_4/\Box_5$ and $1/\Box_5$. Matter couples to auxiliary fields $\Rightarrow$ no singularities.

Making a shift, auxiliary and odd fields can be decoupled $\Rightarrow$ propagators 1 and $1/\Box_5$. Matter couples to odd fields through $\partial_5 \Rightarrow$ singularities cancelled by contact terms proportional to

$$\delta(0) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} 1 = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \frac{p^2 - m_n^2}{p^2 - m_n^2}$$

Gauge interactions

In this case, the off-shell formulation of the bulk theory is simple. The bulk-to-boundary couplings are well understood.

Mirabelli, Peskin

Gravity interactions

In this case, the off-shell formulation is rather involved and has been formulated only recently.

Zucker

The bulk-to-boundary couplings have been only partly studied.

Gherghetta, Riotto

GAUGE INTERACTIONS

The Lagrangian for the $N=2$ bulk vector mult. $\mathcal{V}$ is $(g_5 \to 1)$:

$$\mathcal{L}_5 = -\frac{1}{4}F_{MN}^2 + \frac{i}{2}\bar{\lambda}\not{\partial}\lambda + \frac{1}{2}|\partial_M\Sigma|^2 + \frac{1}{2}\vec{X}^2$$

The $\mathbf{Z_2}$ parities of $\mathcal{V}$ are:

$\mathcal{V}$	A_M	λ	Σ	$\vec{X}$
$+$	A_μ	λ^1		X^3
$-$	A_5	λ^2	Σ	$X^{1,2}$

At the fixed-points, the even components of $\mathcal{V}$ form an $N=1$ vector multiplet $V=(A_\mu, \lambda^1; D)$ with

$$D = X^3 - \partial_{\dot{5}}\Sigma$$

The interaction with an $N=1$ boundary chiral multiplet Φ is:

$$\mathcal{L}_4^\Phi = |D_\mu\phi|^2 + i\bar{\chi}\not{D}\chi + |F|^2 + |\phi|^2 D + \ldots$$

with

$$D_\mu = \partial_\mu - iA_\mu$$

After integrating out $X^{1,2}$ and F, the total Lagrangian reads:

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F_{MN}^2 + \frac{i}{2}\bar{\lambda}\not{\partial}\lambda + \frac{1}{2}|\partial_\mu\Sigma|^2 + \frac{1}{2}D^2 \\ &\quad + e_{\dot{5}}^5\,\delta(x^5)\Big[|D_\mu\phi|^2 + i\bar{\chi}\not{D}\chi\Big] + \Big(\partial_{\dot{5}}\Sigma + \rho_{\dot{5}}(x^5)\Big)D + \ldots\end{aligned}$$

The density which couples to D is given by:

$$\rho_{\dot{5}}(x^5) = e_{\dot{5}}^5\,\delta(x^5)\,|\phi|^2$$

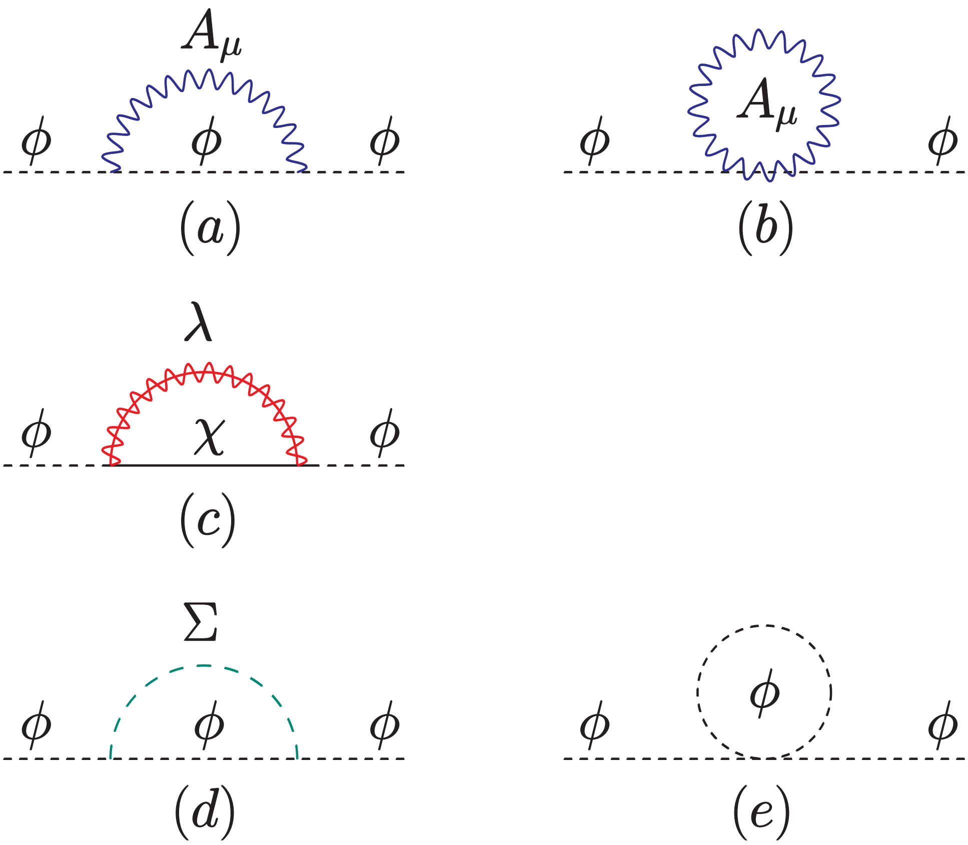
Redefining D through a shift to complete squares, one gets:

$$\begin{aligned}\mathcal{L} \;=\; & -\frac{1}{4}F_{MN}^2 + \frac{i}{2}\bar{\lambda}\partial\!\!\!/\lambda + \frac{1}{2}|\partial_\mu\Sigma|^2 + \frac{1}{2}\tilde{D}^2 \\ & + e_{\dot{5}}^5\,\delta(x^5)\Big[|D_\mu\phi|^2 + i\bar{\chi}D\!\!\!\!/\chi\Big] \\ & -\frac{1}{2}\Big(\partial_{\dot{5}}\Sigma + \rho_{\dot{5}}(x^5)\Big)^2 + \ldots\end{aligned}$$

Loop corrections

Consider for example the 1-loop correction to the mass of ϕ.

This must vanish by SUSY non-renormalization theorem.



The result is:

$$\Delta m^2 = \frac{i}{2\pi R} \sum_{\alpha} \sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \frac{N_{\alpha,n}}{p^2 - m_n^2}$$

with

$$\begin{aligned} N_{a,n} &= -p^2 & N_{b,n} &= 4\,p^2 \\ N_{c,n} &= -4\,p^2 & & \\ N_{d,n} &= m_n^2 & N_{e,n} &= p^2 - m_n^2 \end{aligned}$$

Low-energy theory

The low energy theory for $E \ll M_{\rm C}$ is obtained by integrating out Σ. Neglecting $\partial_\mu \sim E$ with respect to $\partial_5 \sim M_{\rm C}$, its equation of motion is:

$$\partial_{\dot 5}\Big(\partial_{\dot 5}\Sigma + \rho_{\dot 5}(x^5)\Big) = 0$$

The solution is

$$\partial_{\dot 5}\Sigma = -\Big(\rho_{\dot 5}(x^5) - \frac{1}{2\pi R}\,\rho\Big)$$

with

$$\rho = \int_0^{2\pi} dx^5 e_5^{\dot 5}\, \rho_{\dot 5}(x^5) = |\phi|^2$$

Substituting back in the Lagrangian and integrating over x^5, one finds:

$$\begin{aligned} \mathcal{L}^{\rm eff} \;=\; & -\frac{1}{4}F_{\mu\nu}^2 + \frac{i}{2}\bar\lambda^1 \not\partial \lambda^1 \\ & + |D_\mu\phi|^2 + i\bar\chi \not D \chi - \frac{1}{2}\frac{\rho^2}{2\pi R} + \dots \end{aligned}$$

GRAVITY INTERACTIONS

The Lagrangians for the $N=2$ bulk minimal multiplet $\mathcal{M}$ and tensor multiplet $\mathcal{T}$ are $(M_5 \to 1)$:

$$\begin{aligned}
\mathcal{L}_5^{\mathcal{M}} = {} & -32\,\vec{t}^{\,2} - \frac{1}{\sqrt{3}} F_{AB} v^{AB} + \bar{\psi}_M \vec{\tau} \gamma^{MN} \psi_N \vec{t} \\
& - \frac{1}{6\sqrt{3}} \varepsilon^{MNPQR} (A_M F_{NP} - \frac{3}{2} \bar{\psi}_M \gamma_N \psi_P) F_{QR} \\
& - 4C - 2i\bar{\lambda}\gamma^M \psi_M
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_5^{\mathcal{T}} = {} & Y^{-1}\Big(-\frac{1}{4}|\mathcal{D}_M \vec{Y}|^2 + W_A^2 - \frac{i}{2}\bar{\rho}\not{\mathcal{D}}\rho - (N + 6\,\vec{t}\vec{Y})^2 \\
& - \frac{1}{24}\varepsilon^{MNPQR}\vec{Y}(\vec{H}_{MN} - Y^{-2}\mathcal{D}_M \vec{Y} \times \mathcal{D}_N \vec{Y}) B_{PQR} \\
& - \frac{1}{4}\bar{\psi}_M \vec{\tau}\gamma^{MNP}\psi_N(\vec{Y} \times \mathcal{D}_P \vec{Y}) + 4\,\bar{\rho}\,\vec{\tau}\lambda\,\vec{Y} Y \Big) \\
& + Y\Big(-\frac{1}{4}\mathcal{R}(\widehat{\omega}) - \frac{i}{2}\bar{\psi}_M \gamma^{MNP}\mathcal{D}_N \psi_P - \frac{1}{6}\widehat{F}_{MN}^2 \\
& + 20\,\vec{t}^{\,2} + v_{AB}^2 - \frac{i}{2}\bar{\psi}_A \psi_B v^{AB} - \bar{\psi}_M \vec{\tau}\,\gamma^{MN}\psi_N\,\vec{t} \\
& - \frac{i}{4\sqrt{3}}\bar{\psi}_P \gamma^{MNPQ}\psi_Q \widehat{F}_{MN} + 4C + 2i\bar{\lambda}\gamma^M \psi_M \Big) \\
& + \rho\text{-dep. } \lambda\text{-indep.}
\end{aligned}$$

Notation:

$$W^M = \frac{1}{12}\epsilon^{MNPQR}\partial_N B_{PQR} + \frac{1}{4}\bar\psi_P \vec\tau \gamma^{PMQ}\psi_Q \vec Y$$

$$\widehat F_{MN} = \partial_M A_N - \partial_N A_M + i(\sqrt{3}/2)\,\bar\psi_M \psi_N$$

$$\vec H_{MN} = \mathcal{D}_M \vec V_N - \mathcal{D}_N \vec V_M$$

The derivatives $\mathcal{D}_M$ are $SU(2)_R$ and super-Lorentz covariant.

In particular:

$$\mathcal{D}_M \vec Y = \partial_M \vec Y + \vec V_M \times \vec Y$$

$$\mathcal{D}_M \vec V_N = D_M(\hat\omega)\vec V_N + \vec V_M \times \vec V_N$$

$$\mathcal{D}_M \psi_N = D_M(\hat\omega)\psi_N - \frac{i}{2}\vec V_M \vec\tau \psi_N$$

The $\mathbf{Z_2}$ parities of $\mathcal{M}$ and $\mathcal{T}$ are:

$\mathcal{M}$	e_M^A	ψ_M	A_M	$\vec t$	v_{AB}	$\vec V_M$	λ	C
$+$	$e_\mu^a, e_5^{\dot 5}$	ψ_μ^1, ψ_5^2	A_5	$t^{1,2}$	$v_{a\dot 5}$	$V_\mu^3, V_5^{1,2}$	λ^1	C
$-$	$e_\mu^{\dot 5}, e_5^a$	ψ_μ^2, ψ_5^1	A_μ	t^3	v_{ab}	$V_\mu^{1,2}, V_5^3$	λ^2	

$\mathcal{T}$	$\vec Y$	B_{MNP}	ρ	N
$+$	$Y^{1,2}$	$B_{\mu\nu\rho}$	ρ^1	N
$-$	Y^3	$B_{\mu\nu 5}$	ρ^2	

At the fixed-points, the even components of $\mathcal{M}$ form an $N=1$ intermediate multiplet $I=(e^a_\mu, \psi^1_\mu; a_\mu, b_a, t^2+it^1, \lambda^1, S)$ with

$$S = C+\frac{1}{2}e^{\dot 5}_5\bar\lambda^1\psi^2_5-\frac{1}{2}\mathcal{D}_{\dot 5}\,t^3$$

$$a_\mu = -\frac{1}{2}\Big(V^3_\mu+4\,v_{\mu\dot 5}\Big)-\frac{2}{\sqrt{3}}\,e^{\dot 5}_5\,\widehat{F}_{\mu 5}$$

$$b_a = v_{a\dot 5}$$

plus an $N=1$ chiral mult. $T=(\pi e^{\dot 5}_5+i(2\pi/\sqrt{3})A_5, \pi\psi^2_5; F_T)$ with $q_T=0$ and

$$F_T=\pi\Big[V^1_5-4\,e^{\dot 5}_5\,t^2\Big]+i\pi\Big[V^2_5+4\,e^{\dot 5}_5\,t^1\Big]$$

Similarly, the even components of $\mathcal{T}$ form an $N=1$ chiral multiplet $S=(Y^2+iY^1, \rho; F_S)$ with $q_S=2$ and

$$F_S=\Big[-2N+\mathcal{D}_{\dot 5}Y^3\Big]+i\Big[-2W_{\dot 5}+12(Y^2t^1-Y^1t^2)\Big]$$

After gauge-fixing: I conformal gravity multiplet, $S^{1/3}$ chiral compensator multiplet, T radion chiral multiplet.

The Lagrangians for an $N=1$ boundary chiral multiplet Φ and vector multiplet V with $q_\Phi=2/3$ and $q_V=0$ are:

$$\mathcal{L}^\Phi_4 = |\mathcal{D}_\mu\phi|^2+i\bar\chi\not{\mathcal{D}}\chi+|F-4\,\phi\,(t^2-it^1)|^2$$
$$+\frac{1}{6}\,|\phi|^2\Big(\mathcal{R}+2i\bar\psi^1_\mu\gamma^{\mu\nu\rho}D_\nu\psi^1_\rho\Big)+\cdots$$
$$\mathcal{L}^V_4 = -\frac{1}{4}G^2_{\mu\nu}+i\bar\lambda\not{\mathcal{D}}\lambda+\frac{1}{2}D^2+\cdots$$

The chiral $U(1)_R$-covariant derivatives are given by

$$\mathcal{D}_\mu = D_\mu + i\,q\left(a_\mu + 3\,b_\mu\right)(i\gamma^{\dot 5})^F$$

with

$$q_\phi \;=\; 2/3 \qquad q_\chi \;=\; -1/3$$
$$q_{A_\mu} = \; 0 \qquad q_\lambda \;=\; -1$$

The only non-trivial auxiliary field dependence in the boundary Lagrangian is through $a_\mu + 3\,b_\mu$. All the other auxiliary fields can be integrated out through their equations of motion.

The fields C and λ act as Lagrangian multipliers and enforce the contraints $Y = 1$ and $\rho = 0$. After gauge-fixing $\vec{Y} = (0,1,0)^T$, the remaining decoupled auxiliary fields can be integrated out, keeping only the coupled combination:

$$V_\mu = -2(a_\mu + 3\,b_\mu) = V^3_\mu - 2\,v_{\mu\dot 5} - \frac{2}{\sqrt{3}}\,e_{\dot 5}^5\,\widehat{F}_{\mu 5}$$

The resulting Lagrangian is:

$$\begin{aligned}\mathcal{L} \;=\; & \frac{1}{6}\Omega_{\dot 5}(x^5)\Big[\mathcal{R} + 2i\bar\psi_M\gamma^{MNP}D_N\psi_P + \frac{2}{3}V_\mu^2\Big] - \frac{1}{4}\widehat{F}^2_{\mu\nu} \\ & + e_{\dot 5}^5\,\delta(x^5)\Big[|\partial_\mu\phi|^2 + i\bar\chi\not{D}\chi - \frac{1}{4}G^2_{\mu\nu} + i\bar\lambda\not{D}\lambda\Big] \\ & + \frac{1}{\sqrt{3}}\Big(\partial_{\dot 5}A_\mu + \frac{1}{\sqrt{3}}J_{\mu\dot 5}(x^5)\Big)V^\mu + \cdots\end{aligned}$$

The Kähler kinetic function is defined as

$$\Omega_{\dot 5}(x^5) = -\frac{3}{2} + e_{\dot 5}^{5}\,\delta(x^5)|\phi|^2$$

The current which couples to V_μ is the sum of

$$\begin{aligned} J^{\Phi}_{\mu\dot 5}(x^5) &= e_{\dot 5}^{5}\,\delta(x^5)\Big[i(\phi^*\partial_\mu\phi - \text{c.c.}) - \frac{i}{2}\bar\chi\gamma_\mu\gamma^{\dot 5}\chi + \cdots\Big] \\ J^{V}_{\mu\dot 5}(x^5) &= e_{\dot 5}^{5}\,\delta(x^5)\Big[-\frac{3i}{2}\bar\psi\gamma_\mu\gamma^{\dot 5}\psi + \cdots\Big] \\ J^{T}_{\mu\dot 5}(x^5) &= -\sqrt{3}\,e_{\dot 5}^{5}\,\partial_\mu A_5 + \cdots \end{aligned}$$

Note that:

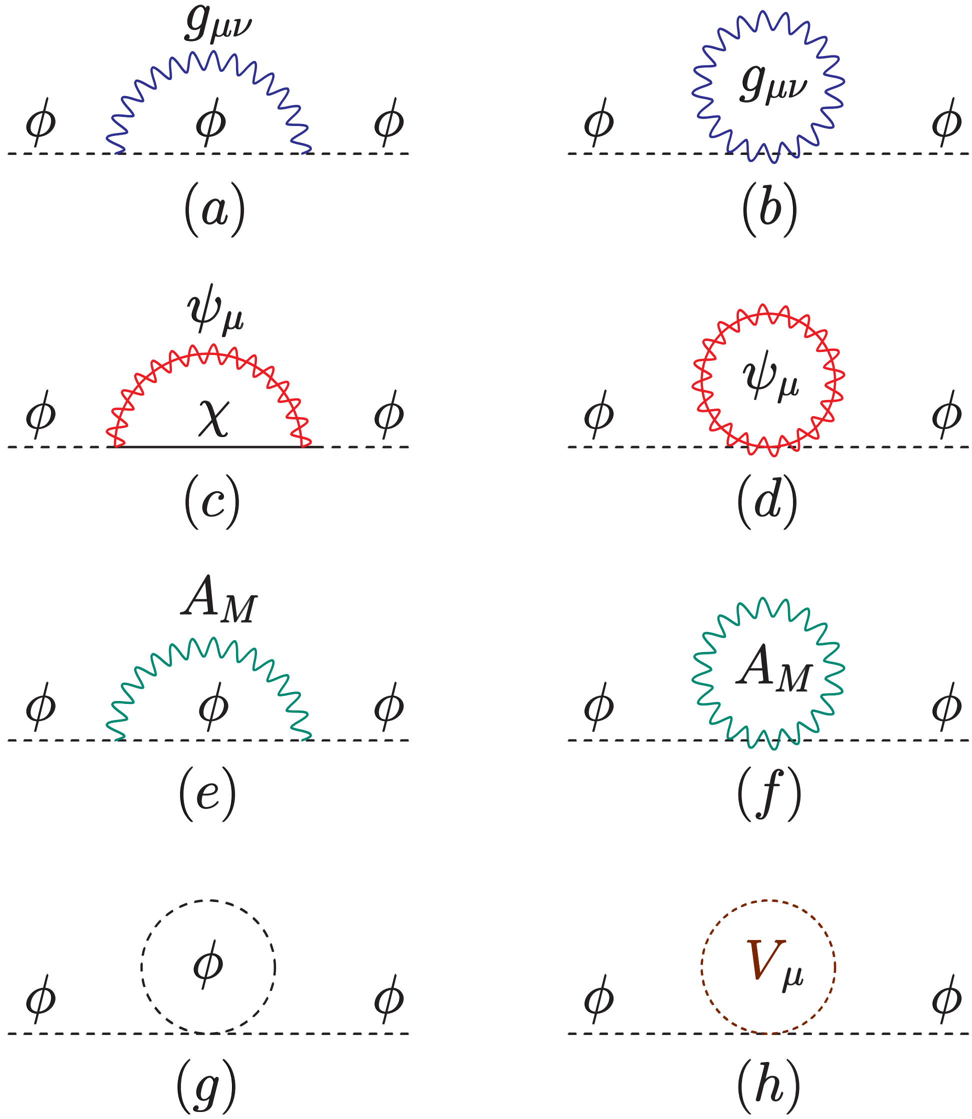
$$\partial_{\dot 5}A_\mu + \frac{1}{\sqrt{3}}J^{T}_{\mu\dot 5}(x^5) = -\,e_{\dot 5}^{5}\widehat{F}_{\mu 5}$$

Redefining V_μ through a shift to complete the squares, one finds finally:

$$\begin{aligned} \mathcal{L} &= \frac{1}{6}\Omega_{\dot 5}(x^5)\Big[\mathcal{R} + 2i\bar\psi_M\gamma^{MNP}D_N\psi_P + \frac{2}{3}\tilde V_\mu^2\Big] - \frac{1}{4}\widehat{F}^2_{\mu\nu} \\ &\quad + e_{\dot 5}^{5}\,\delta(x^5)\Big[|\partial_\mu\phi|^2 + i\bar\chi\not{D}\chi - \frac{1}{4}G^2_{\mu\nu} + i\bar\lambda\not{D}\lambda\Big] \\ &\quad - \frac{3}{4\,\Omega_{\dot 5}(x^5)}\Big(\partial_{\dot 5}A_\mu + \frac{1}{\sqrt{3}}J_{\mu\dot 5}(x^5)\Big)^2 + \cdots \end{aligned}$$

Loop corrections

Consider as before the 1-loop correction to the mass of ϕ, which must vanish by SUSY non-renormalization theorem.



The result is:

$$\Delta m^2 = \frac{i}{6\pi R} \sum_{\alpha} \sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \frac{N_{\alpha,n}}{p^2 - m_n^2}$$

with

$$N_{a,n} = 0 \qquad N_{b,n} = 5\,p^2$$

$$N_{c,n} = 0 \qquad N_{d,n} = -8\,p^2$$

$$N_{e,n} = p^2 - m_n^2 \qquad N_{f,n} = -p^2 + 4\,m_n^2$$

$$N_{g,n} = -p^2 + m_n^2 \qquad N_{h,n} = 4\,p^2 - 4\,m_n^2$$

Low-energy theory

The low energy theory for $E \ll M_{\rm C}$ is obtained by integrating out A_μ. Neglecting $\partial_\mu \sim E$ with respect to $\partial_5 \sim M_{\rm C}$, its equation of motion is:

$$\partial_{\dot 5}\Big[\frac{1}{\Omega_{\dot 5}(x^5)}\Big(\partial_{\dot 5}A_\mu + \frac{1}{\sqrt{3}}J_{\mu\dot 5}(x^5)\Big)\Big] = 0$$

The solution is

$$\partial_{\dot 5}A_\mu = -\frac{1}{\sqrt{3}}\Big(J_{\mu\dot 5}(x^5) - \frac{\Omega_{\dot 5}(x^5)}{\Omega}J_\mu\Big)$$

with

$$\Omega = \int_0^{2\pi} dx^5 e_5^{\dot 5}\,\Omega_{\dot 5}(x^5) = -\frac{3}{2}(T+T^*) + |\phi|^2$$

and

$$J_\mu^\Phi = \int_0^{2\pi} dx^5 e_5^{\dot 5} J_{\mu\dot 5}^\Phi(x^5) = i(\Omega_\phi\partial_\mu\phi - \text{c.c.}) - \frac{i}{2}\Omega_{\phi\phi^*}\bar\chi\gamma_\mu\gamma^{\dot 5}\chi + \cdots$$

$$J_\mu^V = \int_0^{2\pi} dx^5 e_5^{\dot 5} J_{\mu\dot 5}^V(x^5) = -\frac{3i}{2}\bar\lambda\gamma_\mu\gamma^{\dot 5}\lambda + \cdots$$

$$J_\mu^T = \int_0^{2\pi} dx^5 e_5^{\dot 5} J_{\mu\dot 5}^T(x^5) = i(\Omega_T\partial_\mu T - \text{c.c.}) + \cdots$$

Substituting back in the Lagrangian and integrating over x^5, one finds:

$$\begin{aligned}\mathcal{L}^{\rm eff} &= \frac{1}{6}\Omega\Big[\mathcal{R} + 2i\bar\psi_\mu^1\gamma^{\mu\nu\rho}D_\nu\psi_\rho^1\Big] - \frac{1}{4\,\Omega}J_\mu^2 \\ &\quad + \Omega_{\phi\phi^*}\Big[|\partial_\mu\phi|^2 + \bar\chi \not{D}\chi\Big] + \Big[-\frac{1}{4}G_{\mu\nu}^2 + i\bar\lambda\not{D}\lambda\Big] + \cdots\end{aligned}$$

LOOP EFFECTS IN SEQUESTERED MODELS

A generic sequestered model is defined by:

$$\Omega_{\dot{5}}(x^5) = -\frac{3}{2}M_5^3 + \Omega_0\, e_{\dot{5}}^5\, \delta(x^5 - 0) + \Omega_\pi\, e_{\dot{5}}^5\, \delta(x^5 - \pi)$$

We take:

$$\Omega_{0,\pi} = -3L_{0,\pi}M_5^3 + \Phi_{0,\pi}\Phi_{0,\pi}^\dagger$$

The kinetic function of the effective theory is then

$$\Omega(\Omega_{0,\pi}, T + T^\dagger) = -\frac{3}{2}(T + T^\dagger)M_5^3 + \Omega_0 + \Omega_\pi$$

and

$$M_{\rm P}^2 = \Big(\mathrm{Re}\, T + L_0 + L_\pi\Big)M_5^3$$

The 1-loop correction to this has a divergent T-indep. (local) plus a finite T-dep. (non-local) parts. The relevant part is:

$$\Delta\Omega(\Omega_{0,\pi}, T + T^\dagger) = \sum_{m,n=0}^{\infty} \frac{c_{m,n}\,\Omega_0^m\,\Omega_\pi^n}{M_5^{3(m+n)}(T + T^\dagger)^{2+m+n}}$$

The corresponding component effective action is $\Delta\Gamma = \Big[\Delta\Omega\Big]_D$.

In particular, when $F_\pi \neq 0$ and/or $F_T \neq 0$:

Vac. energy: $c_{m,n}L_0^m L_\pi^n |F_T|^2$, $c_{m,n}L_0^m L_\pi^{n-1}|F_\pi|^2$

Soft masses: $c_{m,n}L_0^{m-1} L_\pi^n |F_T|^2$, $c_{m,n}L_0^{m-1} L_\pi^{n-1}|F_\pi|^2$

To derive the $c_{m,n}$s, one chooses one operator for each superspace term in $\Delta\Omega$, and computes its induced coefficient.

Strategy

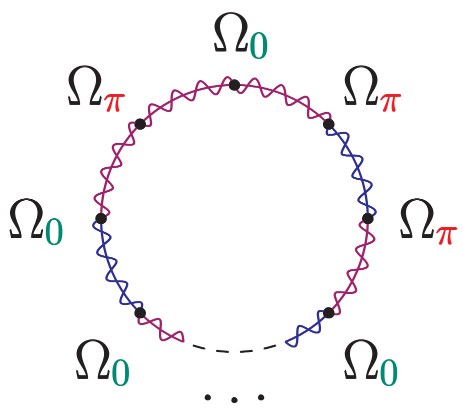
Crucial trick: use non-SUSY background with $F_T = 2\pi\epsilon \neq 0$. This corresponds to a $SU(2)_R$ Wilson line, and can be achieved in two ways: SS twist or constant boundary superpotentials. Only the gravitino KK modes are affected: $m_n = (n+\epsilon)/R$.

von Gersdorff, Quiros, Riotto;
Bagger, Feruglio, Zwirner

This leads to huge simplifications:

- One can use operators with scalars and no derivatives
  $\Rightarrow$ Few diagrams, mostly with quartic couplings.
- The amplitudes must vanish in the SUSY limit $\epsilon \to 0$
  $\Rightarrow$ All the information is in the gravitino diagrams.

In the end, there is a single type of diagram for each $c_{m,n}$:



The set of operator that we want to compute is given by the effective potential $\Delta V = -\partial_T \partial_{T^*} \Delta\Omega |F_T|^2$, function of R and

$$\alpha_{0,\pi} = \frac{\Omega_{0,\pi}}{6\pi R M_5^3} = -\frac{L_{0,\pi}}{2\pi R} + \frac{|\phi_{0,\pi}|^2}{6\pi R M_5^3} = -r_{0,\pi} + |\varphi_{0,\pi}|^2$$

The precise expression of the operators to be matched is:

$$\Delta V(\alpha_{0,\pi}, R, \epsilon) = \frac{-\epsilon^2}{4\pi^2 R^4} \sum_{m,n=0}^{\infty} c_{m,n}(2+m+n)(3+m+n)\alpha_0^m \alpha_\pi^n$$

Computation

The gravitino contribution to the full effective potential is:

$$\Delta W_\psi(\alpha_{0,\pi}, R, \epsilon) = -\frac{1}{2} \ln\det\left[\Box_5 + \left(\alpha_0\, \delta_0 + \alpha_\pi\, \delta_\pi\right)\Box_4\right]$$

The $\alpha_{0,\pi}$-independent part is

$$\ln\det\left[\Box_5\right] = 8\,\mathrm{Re}\int \frac{d^4p}{(2\pi)^4} \ln\left[F(pR, \epsilon)\right]$$

with

$$F(pR, \epsilon) = \prod_{n=-\infty}^{+\infty}\left(p + i\, m_n\right) = (\mathrm{Div.})\sinh \pi(pR + i\epsilon)$$

The $\alpha_{0,\pi}$-dependent part is

$$\ln\det\left[1 + \left(\alpha_0\, \delta_0 + \alpha_\pi\, \delta_\pi\right)\frac{\Box_4}{\Box_5}\right]$$

$$= 8\,\mathrm{Re}\int \frac{d^4p}{(2\pi)^4} \ln\begin{vmatrix} 1 - p\alpha_0 G_0(pR, \epsilon) & -p\alpha_\pi G_\pi(pR, \epsilon) \\ -p\alpha_0 G_\pi(pR, \epsilon) & 1 - p\alpha_\pi G_0(pR, \epsilon) \end{vmatrix}$$

with

$$G_0(pR, \epsilon) = \frac{1}{2\pi R} \sum_{n=-\infty}^{+\infty} \frac{e^{i0n}}{p + i\, m_n} = \frac{1}{2} \coth \pi(pR + i\epsilon)$$

$$G_\pi(pR, \epsilon) = \frac{1}{2\pi R} \sum_{n=-\infty}^{+\infty} \frac{e^{i\pi n}}{p + i\, m_n} = \frac{1}{2} \operatorname{csch} \pi(pR + i\epsilon)$$

Putting these two pieces together and simplifying one finds:

$$\Delta W_\psi(\alpha_{0,\pi}, R, \epsilon)$$
$$= \text{Div.} - \frac{1}{2\pi^6 R^4}\text{Re}\int_0^\infty dl\, l^3 \ln\left|1 - \frac{1+\alpha_0 l}{1-\alpha_0 l}\frac{1+\alpha_\pi l}{1-\alpha_\pi l}e^{-2(l+i\pi\epsilon)}\right|$$

The $\mathcal{O}(\epsilon^0)$ part cancels the contributions of other bulk fields.
The $\mathcal{O}(\epsilon^2)$ part yields the relevant potential ΔV that we need.
The $\mathcal{O}(\epsilon^{2n})$ terms map to D-terms with superderivatives.

Expanding $\Delta W_\psi|_{\epsilon^2}$ in powers of $\alpha_{0,\pi}$ and comparing with the general expression for ΔV, one extracts the coefficients $c_{m,n}$. The first few ones are:

$$c_{0,0} = \frac{\zeta(3)}{4\pi^2}\,, \quad c_{1,0} = c_{0,1} = \frac{\zeta(3)}{6\pi^2}\,, \quad c_{1,1} = \frac{\zeta(3)}{6\pi^2}\,, \quad \dots$$

An independent and direct computation exploiting supergraph techniques leads to the same results.

Buchbinder et al.

Since we know the exact expression $\Delta W_\psi|_{\epsilon^2}$ for ΔV, we can do better and find the exact expression for $\Delta\Omega$ by solving the differential equation $\Delta V = -\epsilon^2\,\partial_R^2\,\Delta\Omega$. The result is:

$$\Delta\Omega(\Omega_{0,\pi}, T+T^\dagger)$$
$$= -\frac{9}{\pi^2}M_5^2\int_0^\infty dx\, x \ln\left[1 - \frac{1+\frac{\Omega_0}{M_5^2}x}{1-\frac{\Omega_0}{M_5^2}x}\frac{1+\frac{\Omega_\pi}{M_5^2}x}{1-\frac{\Omega_\pi}{M_5^2}x}e^{-6(T+T^\dagger)M_5 x}\right]$$

This shows in particular that all the $c_{m,n}$s are positive.

Results

The results for the vacuum energy and soft masses are:

$$\delta\mathcal{E}^4 = -\frac{\zeta(3)}{16\pi^2}\left[\frac{1}{3}f_\pi\frac{|F_\pi|^2}{M_4^2}M_C^2 + \frac{3}{2}f_T|F_T|^2M_C^4\right]$$

$$\delta m_0^2 = -\frac{\zeta(3)M_C^2}{16\pi^2M_4^2}\left[\frac{1}{6}g_\pi\frac{|F_\pi|^2}{M_4^2} + g_T|F_T|^2M_C^2\right]$$

These depend on the parameters $r_{0,\pi}$ through

$$M_4^2 = \frac{1}{1+r_0+r_\pi}M_P^2$$

and the normalized functions

$$f_\pi = \frac{4}{3\zeta(3)}\int_0^\infty dl\, l^2 e^{-2l}\,\frac{(1-r_0 l)/(1+r_\pi l)}{[(1+r_0 l)(1+r_\pi l)-(1-r_0 l)(1-r_\pi l)e^{-2l}]}$$

$$f_T = \frac{2}{3\zeta(3)}\int_0^\infty dl\, l^3 e^{-2l}\,\frac{(1-r_0^2 l^2)(1-r_\pi^2 l^2)}{[(1+r_0 l)(1+r_\pi l)-(1-r_0 l)(1-r_\pi l)e^{-2l}]^2}$$

$$g_\pi = \frac{8}{3\zeta(3)}\int_0^\infty dl\, l^3 e^{-2l}\,\frac{1}{[(1+r_0 l)(1+r_\pi l)-(1-r_0 l)(1-r_\pi l)e^{-2l}]^2}$$

$$g_T = \frac{4}{3\zeta(3)}\int_0^\infty dl\, l^4 e^{-2l}\,\frac{(1-r_\pi^2 l^2)[(1+r_0 l)(1+r_\pi l)+(1-r_0 l)(1-r_\pi l)e^{-2l}]}{[(1+r_0 l)(1+r_\pi l)-(1-r_0 l)(1-r_\pi l)e^{-2l}]^3}$$

For $r_{0,\pi} = 0$, $\delta\mathcal{E}^4$ and δm_0^2 are negative $\Rightarrow$ not interesting.

For $r_{0,\pi} \neq 0$, $\delta\mathcal{E}^4$ and δm_0^2 can have any sign $\Rightarrow$ interesting.

Three main cases for the dependence on R at fixed $L_{0,\pi}$:

- $L_0 = 0$, $L_\pi = 0$: $\delta\mathcal{E}^4$ unstable, $\delta m_0^2 \sim -(\delta\mathcal{E}^4)'$.
- $L_0 = 0$, $L_\pi \neq 0$: $\delta\mathcal{E}^4$ stable, $\delta m_0^2 \sim -(\delta\mathcal{E}^4)'$.
- $L_0 \neq 0$, $L_\pi \neq 0$: $\delta\mathcal{E}^4$ metastable, $\delta m_0^2 \not\sim -(\delta\mathcal{E}^4)'$.

PROTOTYPE MODEL

The goal is to achieve values of T, F_T, F_π, F_S such that:

- $\mathcal{E}^4 \sim 0 \Rightarrow$ tuning of P.
- $\delta^{\rm grav} m_0^2 > 0 \Rightarrow$ needs $r_\pi \neq 0$.
- $\delta^{\rm grav} m_0^2 \sim \delta^{\rm gau} m_0^2 \Rightarrow$ indep. stab. mech.

One can try to combine localized kinetic terms with gaugino condensation, with:

$$\Omega = -\frac{3}{2}(T + T^\dagger) M_5^3 + \Phi_0 \Phi_0^\dagger - 3 L_\pi M_5^3 + \Phi_\pi \Phi_\pi^\dagger$$

$$P = \Lambda_\pi^3 + M_\pi^2 \Phi_\pi + \Lambda^3 e^{-\alpha \Lambda T}$$

To have $\mathcal{E}^4 \sim 0$ we need $\Lambda_\pi^3 \sim M_\pi^2 M_{\rm P}$. We then get:

$$M_{\rm C} \sim \alpha \Lambda\,, \quad F_T \sim \frac{M_\pi^2}{\Lambda M_{\rm P}}\,, \quad F_S \sim \frac{M_\pi^2}{M_{\rm P}}\,, \quad F_\pi \sim M_\pi^2$$

To have $r_\pi \gg 1$ we need $L_\pi \gg (\alpha\Lambda)^{-1}$. In this limit:

$$f_\pi, g_\pi \to \frac{2 \ln(2)}{3 \zeta(3)} \frac{1}{r_\pi^2}\,, \quad f_T, g_T \to -\frac{3}{4}$$

$\delta^{\rm grav} m_0^2$ becomes positive for $r_\pi \sim \alpha^{-1}$; OK with $\alpha \ll 1$.

$\delta^{\rm grav} m_0^2$ is of the same order of magnitude as $\delta^{\rm gau} m_0^2$ if:

$$\alpha^2 \frac{M_{\rm C}^2}{16 \pi^2 M_4^2} \sim \left(\frac{g^2}{16 \pi^2}\right)^2 \Rightarrow \frac{M_{\rm C}}{M_{\rm P}} \sim \frac{g^2}{4 \pi \sqrt{\alpha}}$$

OUTLOOK

- Bulk-to-boundary couplings now well understood and full theory under controll.
- Radius-dependent quantum corrections to sfermion squared masses generally negative, but can become positive with sizable localized kinetic terms.
- Sequestered models can work, but radion dynamics plays a crucial.