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THEORETICAL MOTIVATIONS FOR EXTRA DIMENSIONS

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- Standard model of particle physics and beyond.
  Need for new physics at higher energy scales.
- Theoretical prejudices suggesting extra dimensions.
  Number and size of new dimensions.
- Possible scenarios for an ultimate fundamental theory.
  String theories and their peculiarities.

STANDARD MODEL AND BEYOND

Gauge interactions among elementary particles are described by a quantum field theory: the SM. This theory has a typical scale

$$M_{\rm ew} = G_F^{-\frac{1}{2}} \sim 300 \text{ GeV} \;\Leftrightarrow\; L_{\rm ew} \sim 10^{-16} \text{ mm}$$

It is well tested for $L > 10^{-15}$ mm $\Leftrightarrow$ $E < 100$ GeV, and the strength of classical interactions is

- Electromagnetic: $g \sim 1$.
- Strong: $g \sim 1$.
- Weak: $g_{\rm eff} \sim E/M_{\rm ew}$ at low E; $g \sim 1$ at high E.

Gravitational interactions among macroscopic bodies are instead described by a classical field theory: Einstein's GR. This theory has a fundamental scale

$$M_{\rm pl} = G_N^{-\frac{1}{2}} \sim 10^{19} \text{ GeV} \;\Leftrightarrow\; L_{\rm pl} \sim 10^{-32} \text{ mm}$$

It is tested only in the region $L > 1$ mm $\Leftrightarrow$ $E < 10^{-13}$ GeV, and the strength of interaction is:

- Gravity: $g_{\rm eff} \sim E/M_{\rm pl}$ at low E; $g \sim 1$ at high E ?

There are strong doubts that this description of gravitational interactions can hold true at the quantum level.

When all interactions become equally important, a yet more fundamental quantum theory describing them in a unified way is supposed to take over.

This suggests that the SM and GR are effective field theories, giving a satisfactory description of physics only at low enough energies, $E \ll M_{\rm eff}$. The UV cut-off Λ needed to regulate quantum corrections acquires a physical meaning: $\Lambda \sim M_{\rm eff}$.

Hierarchy problem

The classical symmetry breaking scale $M_{\rm ew}$ is destabilized by large quantum corrections, which naturally drive it to $M_{\rm eff}$, unless some parameter is fine-tuned:

$$M_{\rm eff} \gg M_{\rm ew} \text{ unnatural}$$

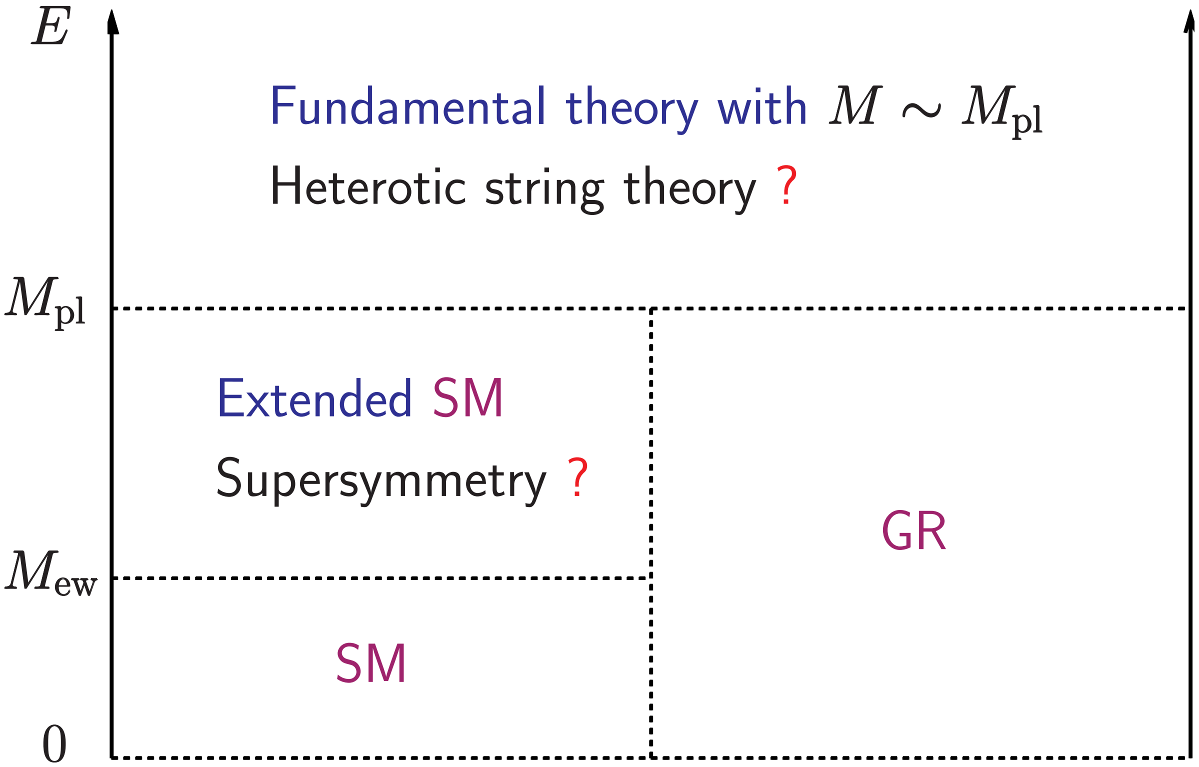
In order to stabilize $M_{\rm ew}$, its seems unavoidable to have new physics already not much beyond $M_{\rm ew}$ and much before $M_{\rm pl}$, although the SM is renormalizable.

The fundamental scale $M_{\rm pl}$ is instead related to physics of the fundamental theory.

Unification

What is the fundamental scale M at which the strength of all interactions becomes comparable ?

HIGH SCALE SCENARIO



Nice features:

- Effective couplings unify quite precisely around $M_{\rm pl}$.
- Nice properties of the SM natural up to $M_{\rm pl}$.
- Neutrino masses of order $M_{\rm ew}^2/M_{\rm pl}$ are natural.

Bad features:

- The huge hierarchy $M_{\rm ew}/M_{\rm pl}$ is difficult to explain.

Experimental implications

- Heavy superpartners.
- Gravity holds true at short distances.

EXTRA DIMENSIONS

Presently available candidates for a fundamental theory of particle physics including gravity all require more dimensions than what we apparently see.

A possible way out is that our spacetime of the form:

$$X_{4+n} = X_4 \times X_n \;, \;\; V_{X_n} = R^n$$

In this setting, long range interactions behave as:

$$V(r) \sim \begin{cases} \tilde{G} \dfrac{1}{r^{n+1}} \;, & r \ll R \\ \dfrac{\tilde{G}}{R^n} \dfrac{1}{r} \;, & r \gg R \end{cases}$$

At $L \gg R$, one finds thus effectively a standard Coulomb interaction on X_4 with strength:

$$G \sim \frac{\tilde{G}}{R^n}$$

New compact dimensions affect thus physics only for $E > R^{-1}$, and are compatible with experiments if R is small enough:

- Gauge interactions: $R < 10^{-15}$ mm
- Gravity: $R < 1$ mm

In a fundamental theory with characteristic scale M in which gauge and gravitational interactions feel different numbers n_1 and n_2 of extra dimensions:

$$\tilde{G}_{\rm gauge} \sim q^2 M^{-n_1} \quad \Rightarrow \; g \sim \prod_{i=1}^{n_1} (MR_i)^{-\frac{1}{2}}$$
$$\tilde{G}_{\rm grav} \sim m^2 M^{-2-n_2} \; \Rightarrow \; M_{\rm pl} \sim M \prod_{i=1}^{n_2} (MR_i)^{\frac{1}{2}}$$

Generically, $R_1, ..., R_{n_1}$ must be around M^{-1} to keep $g \sim 1$, but $R_{n_1+1}, ..., R_{n_2}$ can be tuned to adjust $M_{\rm pl}$:

$$\frac{M_{\rm pl}}{M} \sim g^{-1} \prod_{i=n_1}^{n_2} (MR_i)^{\frac{1}{2}}$$

Transverse dimensions felt by gravity but not gauge interactions control therefore their relative effective strength. Gravity could be weaker than gauge interactions due to flux spreading through large dimensions with $R_i > M^{-1}$.

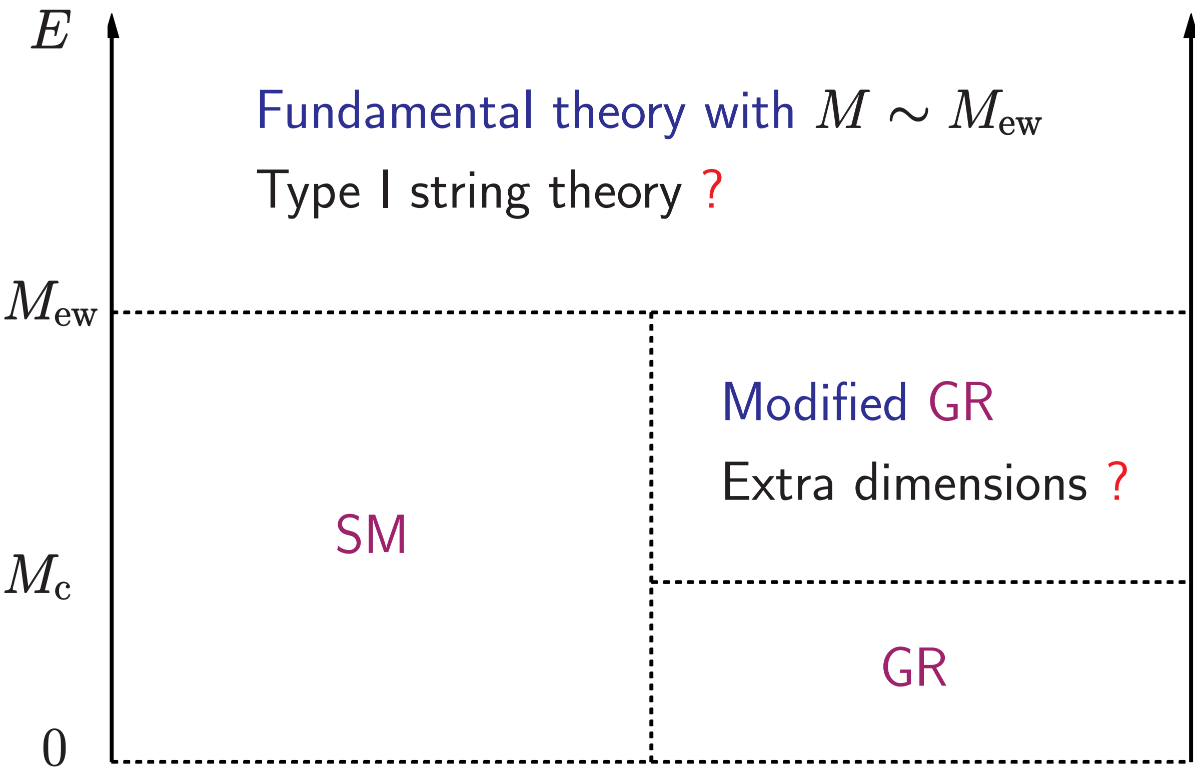
In the presence of n transverse directions of radius R, corresponding to $M_{\rm c} = R^{-1}$, one finds:

$$M \sim M_{\rm pl} \Big(\frac{M_{\rm c}}{M_{\rm pl}}\Big)^{\frac{n}{n+2}} \rightarrow \begin{cases} M_{\rm pl} \; , & n \ll 2 \\ M_{\rm c} \; , & n \gg 2 \end{cases}$$

For $n \geq 2$, one can make M as low as $M_{\rm ew}$ with $M_{\rm c}$ safely big and approaching M:

$$M_c \sim M \Big(\frac{M}{M_{\rm pl}}\Big)^{\frac{2}{n}} \xrightarrow[n \gg 2]{} M$$

LOW SCALE SCENARIO



Nice features:

- No stabilization problem and smaller hierarchy.

Bad features:

- Effective couplings unify only approx. at $M_{\rm ew}$.
- Specific symmetries must be imposed on the fundamental theory.

Experimental implications:

- Exotic physical processes and new heavy particles.
- Gravity modified at short distances.

STRING MODELS

String theories are characterized only by their fundamental scale $M_s = T_s^{\frac{1}{2}} \Leftrightarrow L_s = T_s^{-\frac{1}{2}}$, and require 10 dimensions.

One can postulate that spacetime has 4 non-compact and 6 compact dimensions at a scale M_c. However, this geometry should arise dynamically $\Rightarrow$ $M_{\rm c}$ naturally close to $M_{\rm s}$.

The modes occurring in the effective theory are:

- Vibration modes: $m \sim \sqrt{n}\, M_{\rm s}$.
- KK modes: $m \sim p\, M_{\rm c}$.
- Winding modes: $m \sim q\, M_{\rm s}^2/M_{\rm c}$.

Closed strings

Non-localized.

Open strings

Localized end-points.

Unification scenarios

- High scale if neutral $\Leftrightarrow$ closed, charged $\Leftrightarrow$ closed.
- Any scale if neutral $\Leftrightarrow$ closed, charged $\Leftrightarrow$ open.

OUTLOOK

- Extra dimensions should be taken seriously !
- Forthcoming experiments could be exciting ...
- Time to merge bottom-up and top-down views.