TARGET-SPACE SYMMETRY AND ANOMALIES IN HETEROTIC AND UNORIENTED STRINGS

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Scrucca, Serone : **hep-th**/0006201, **hep-th**/0012124

- DUALITY BETWEEN HETEROTIC ORBIFOLDS
  AND TYPE IIB ORIENTIFOLDS WITH $D = 4$ $N = 1$.
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- ANOMALIES IN HETEROTIC ORBIFOLDS.
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TYPE IIB - HETEROTIC DUALITY

It has been proposed that certain $D=4$ $N=1$ Type IIB orientifolds and heterotic orbifolds could be dual.

Angelantonj, Bianchi, Pradisi, Sagnotti, Stanev
Kakushadze, Shiu

The duality can be weak - weak $\Rightarrow$ the effective low-energy dynamics and symmetries of the two theories must match.

Simplest case: $\mathbf{Z}_N$, with N odd $\Rightarrow$ no threshold corrections nor D5-branes.

Gauge symmetry and diffeomorphisms

Mixed gauge/gravitational anomalies cancel through a GS mechanism, and the anomalous $U(1)$'s get spontaneously broken.

In heterotic models, the GS mechanism is mediated by the universal NSNS axion, and there is at most one anomalous $U(1)$.

Dine, Seiberg, Witten

In orientifold models, the GS mechanism is mediated by the twisted RR axions, and there can be several anomalous $U(1)$'s.

Ibáñez, Rabadán, Uranga
Scrucca, Serone

Target-space symmetry

In heterotic models, there is an exact $SL(2,Z)_i$ target-space symmetry for each internal T_i^2. It corresponds to T-duality.

Mixed target-space/gauge and target-space/gravitational anomalies cancel through a GS mechanism mediated by the NSNS axion.

Derendinger, Ferrara, Kounnas, Zwirner
Cardoso, Ovrut
Ibáñez, Lüst
Antoniadis, Gava, Narain, Taylor

In orientifold models, these $SL(2,Z)_i$ symmetries do not correspond to T-duality or any other string symmetry, and constitute a test of the proposed duality.

This symmetry exists at the classical level, and it was conjectured that mixed target-space/gauge and target-space/gravitational anomalies cancel through a GS mechanism involving all the axions.

Ibáñez, Rabadán, Uranga

A more detailed analysis indicates that this apparently fails to work, although not by much.

Lalak, Lavignac, Nilles

ANOMALIES IN $D=4$ $N=1$ MODELS

The anomaly $\mathcal{A}$ has to satisfy the WZ consistency condition. This implies that it is the WZ descent of some closed form I. Defining $I = dI^{(0)}$ and $\delta I^{(0)} = dI^{(1)}$, one has:

$$\mathcal{A} = 2\pi i \int I^{(1)}$$

Field theory computation

The Kähler potential of the effective SUGRA is:

$$\kappa^2 K = -\ln(S+\bar{S}) - \sum_{i=1}^{3} \ln(T_i + \overline{T}_i) + \kappa^2 K_{\text{mat.}}(\Phi_\alpha, \bar{\Phi}_\alpha, T_i, \overline{T}_i)$$

with

$$\kappa^2 K_{\text{mat.}}(\Phi_\alpha, \bar{\Phi}_\alpha, T_i, \overline{T}_i) \sim \sum_\alpha \prod_{i=1}^{3} (T_i + \overline{T}_i)^{n_i^\alpha} \Phi_\alpha \bar{\Phi}_\alpha$$

The target-space transformation is a σ-model reparametrisation:

$$T_i \longrightarrow \frac{a_i T_i - i b_i}{i c_i T_i + d_i}$$

$$\Phi_\alpha \longrightarrow \exp\left\{ -\sum_{i=1}^{3} n_i^\alpha \ln(i c_i T_i + d_i) \right\} \Phi_\alpha$$

and induces the Kähler transformation

$$\kappa^2 K \longrightarrow \kappa^2 K + 2 \sum_{i=1}^{3} \text{Re} \ln(i c_i T_i + d_i)$$

Each of these transformations is a continuous symmetry of the action, with an associated composite connection:

$$Z^{(\sigma)}_{\mu\alpha\beta} = i \sum_{i=1}^{3} \Big(\Gamma_{\alpha i \beta}\, \partial_\mu T_i - \Gamma_{\alpha \bar{\imath} \beta}\, \partial_\mu \overline{T}_i \Big) \sim \sum_{i=1}^{3} 2\, n_i^\alpha\, \delta_{\alpha\beta}\, Z^i_\mu$$

$$Z^{(K)}_{\mu} = -\frac{i}{2}\, \kappa^2 \sum_{i=1}^{3} \Big(K_i\, \partial_\mu T_i - K_{\bar{\imath}}\, \partial_\mu \overline{T}_i \Big) \sim \sum_{i=1}^{3} Z^i_\mu$$

The basic composite connections are:

$$Z^i_\mu = \frac{i}{2} \frac{\partial_\mu (T_i - \overline{T}_i)}{(T_i + \overline{T}_i)}$$

and transform as

$$Z^i_\mu \rightarrow Z^i_\mu + \partial_\mu \,\mathsf{Im}\, \ln(i c_i T_i + d_i)$$

The covariant derivatives of chiral fermions involve the composite connections Z^i_μ, the gauge connections A^a_μ and the spin connection ω_μ, and there are one-loop anomalies. We will focus on the CP-odd part.

Standard approach: treat the composite connections as elementary $U(1)$ connections. In this way, one finds:

$$I = \mathrm{ch}_{\mathbf{1}_{-1}}(G)\, \widehat{G}(R) + \mathrm{ch}_{\mathrm{Adj}}(F)\, \mathrm{ch}_{\mathbf{1}_{-1}}(G)\, \widehat{A}(R) + \sum_{\alpha} \mathrm{ch}_{\mathrm{R}^\alpha}(F)\, \mathrm{ch}_{\mathbf{1}_{1+2n_i^\alpha}}(G)\, \widehat{A}(R)$$

It not clear whether this result is correct beyond the leading order in the composite connection.

Safer approach: compute one-loop diagrams with external gluons, gravitons and T_i moduli. The latter should pair and reconstruct composite Z_i connections. Technically difficult.

The cancellation of these anomalies must happen through the GS mechanism, and the appropriate GS coupling should be provided by string theory.

String theory computation

Possible anomalies arise from non-vanishing amplitudes involving an unphysical particle. We focus on CP-odd amplitudes in odd spin-structures on world-sheets with $\chi = 0$:

$$\mathcal{A}_{12\cdots n} = \int_{\mathcal{F}} d\tau \left\langle T_F \, V_1^{unphy.} \, V_2^{phy.} \cdots V_n^{phy.} \right\rangle$$

The insertion of T_F is due to the gravitino zero-mode, and the vertices must have total superghost charge -1.

A non-trivial amplitude is obtained by choosing:

$$V_1^{unphy.} : -1 \text{ picture}, \quad \xi_1 \sim p_1 \;\Rightarrow\; V_1^{unphy.} = Q \cdot \hat{V}_1^{unphy.}$$

$$V_i^{phy.} : 0 \text{ picture}, \quad p_i \cdot \xi_i = 0 \;\Rightarrow\; Q \cdot V_i^{phy.} = 0$$

Move Q on the other operators: $Q \cdot V_i^{phy.} = 0$ but $Q \cdot T_F = T_B$. The net effect of T_B is to take ∂_τ of the remaining correlation:

$$\mathcal{A}_{12\cdots n} = \int_{\partial\mathcal{F}} d\tau \left\langle \hat{V}_1^{unphy.} \, V_2^{phy.} \ldots V_n^{phy.} \right\rangle$$

This is the WZ descent of

$$I_{12\cdots n} = \int_{\partial\mathcal{F}} d\tau \left\langle V_1^{phy.} \, V_2^{phy.} \ldots V_n^{phy.} \right\rangle$$

The total anomaly polynomial is the generating functional:

$$\boxed{I = \int_{\partial\mathcal{F}} d\tau \, Z(\tau; F, R, G)}$$

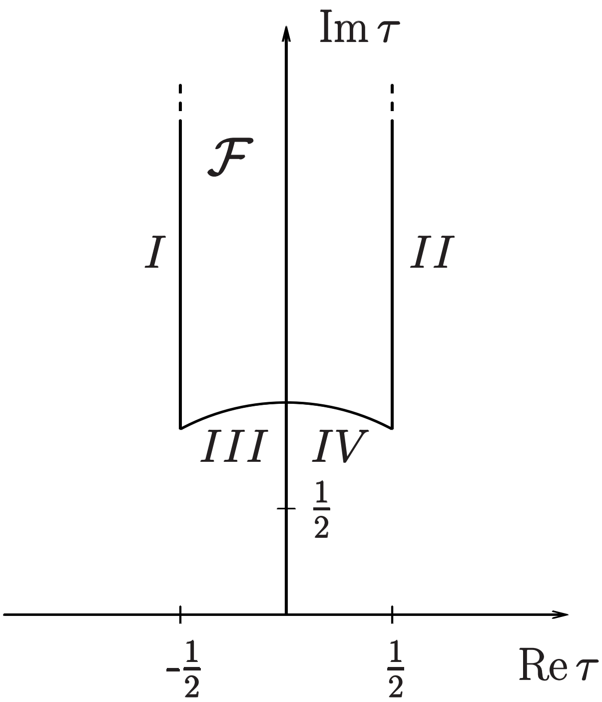
This is the string analogue of Fujikawa's method.

$Z(\tau; F, R, G)$ encodes the anomaly, and $\int_{\partial\mathcal{F}} d\tau$ its cancellation.

Schwinger parameter: $\alpha'\tau$. IR for $\alpha'\tau \to \infty$, UV for $\alpha'\tau \to 0$.

For α' = finite, $\tau \to \infty$ is in the IR and does not contribute.

For the Torus (T):



$$\Rightarrow \quad I = I^{I} + I^{II} + I^{III} + I^{IV}$$

Notice that:

$\tau \to \tau + 1$: $I \to II$

$\tau \to -1/\tau$: $III \to IV$

$\Rightarrow$ $I_T = 0$ by modular invariance

Suzuki, Sugamoto

Kutasov

For the Annulus (A), Möbius strip (M) and Klein bottle (K):

$$\mathcal{F} = i\,[0, \infty[\qquad \Rightarrow \quad I = I^0$$

$\Rightarrow$ $I_A + I_M + I_K = 0$ by tadpole cancellation

Inami, Kanno, Kubota

Polchinski, Cai

General result:

No divergences $\Rightarrow$ No anomalies

For $\alpha' \to 0$, $\tau \to \infty$ can be in the UV region. Interpretation:

One-loop anomaly: $\tau \to \infty$

Tree-level inflow: $\tau =$ finite

In this limit, one can restrict to leading order in the momenta, and use quadratic effective vertices depending on:

$$F = \frac{1}{2} F_{\mu\nu}(x_0)\, \psi_0^\mu\, \psi_0^\nu$$

$$R_{\mu\nu} = \frac{1}{2} R_{\mu\nu\rho\sigma}(x_0)\, \psi_0^\rho\, \psi_0^\sigma$$

$$G_i = \frac{1}{2} G^i_{\mu\nu}(x_0)\, \psi_0^\mu\, \psi_0^\nu$$

$Z(\tau; F, R, G)$ can be computed exactly, since the effective action becomes quadratic:

$$S^{\text{eff.}}(F,R,G) = S_0 + V_\gamma^{\text{eff.}}(F) + V_g^{\text{eff.}}(R) + \sum_{i=1}^{3} V_{Z_i}^{\text{eff.}}(G_i)$$

The $\int dx_0^\mu \int d\psi_0^\mu$ select the form of appropriate degree and integrate it over space-time. The result is a topological index expressed in terms of characteristic classes.

In this way, one recovers a well-known framework.

Alvarez-Gaumé, Witten

Characteristic classes for orbifold models

Twisted characteristic classes naturally appear, since the generator g of $\mathbf{Z}_N = \{g^k, k = 0, 1, \cdots, N-1\}$ acts as a twist v in the internal space and as a twist w in the gauge bundle.

For the gauge bundle:

$$\mathrm{ch}_\rho^k(F) = \mathrm{tr}_\rho \, \exp \, i(\tfrac{F}{2\pi} + 2\pi k w)$$

For the tangent bundle:

$$\widehat{A}(R) = \prod_{a=1}^{D/2} \frac{\frac{R_a}{4\pi}}{\sinh \frac{R_a}{4\pi}} \,, \quad \widehat{L}(R) = \prod_{a=1}^{D/2} \frac{\frac{R_a}{2\pi}}{\tanh \frac{R_a}{2\pi}}$$

$$\widehat{G}(R) = \prod_{a=1}^{D/2} \frac{\frac{R_a}{4\pi}}{\sinh \frac{R_a}{4\pi}} \Big(2 \sum_{b=1}^{D/2} \cosh \frac{R_b}{2\pi} \, - \, 1\Big)$$

For the internal target-space bundle:

$$\mathrm{ch}_{\mathbf{1}_{\mathbf{q_i}}}(G) = \prod_{i=1}^{3} \exp \, i \, q_i \, \frac{G_i}{2\pi}$$

$$\widehat{A}_k(G) = \prod_{i=1}^{3} \frac{\sin(\pi k v_i)}{\sin(\frac{G_i}{4\pi} + \pi k v_i)} \,, \quad \widehat{L}_k(G) = \prod_{i=1}^{3} \frac{\tan(\pi k v_i)}{\tan(\frac{G_i}{2\pi} + \pi k v_i)}$$

$$\widehat{G}_k(G) = \prod_{i=1}^{3} \frac{\sin(\pi k v_i)}{\sin(\frac{G_i}{4\pi} + \pi k v_i)} \Big(2 \sum_{j=1}^{3} \cos(\tfrac{G_j}{2\pi} + 2\pi k v_j) \, - \, 1\Big)$$

The number of g^k-fixed points is $N_k = C_k^2$, where:

$$C_k = \prod_{i=1}^{3} 2 \sin(\pi k v_i)$$

Field theory versus string theory

By averaging over all the twists, twisted characteristic classes get projected onto standard characteristic classes, but products of twisted characteristic classes remain entangled.

There are two types of states: the untwisted states (g^0 sector) living in $D = 4 + 6$ and the twisted states (g^l sector, $l \in S_l$) living in $D = 4$.

In field theory, the computation is in the Einstein frame, and the target-space dependence is:

$$I \sim \mathrm{ch}(G) \;, \quad \text{for all states}$$

In string theory, the computation is in the string frame, and the target-space dependence is:

$$I \sim \begin{cases} \widehat{A}_k(G),\; \widehat{L}_k(G)\;, \widehat{G}_k(G)\;, & \text{for untwisted states} \\ \mathrm{ch}(G)\;, & \text{for twisted states} \end{cases}$$

The total anomaly should be the same in the two approaches. However, the string and field theory results differ beyond the leading order in $G \Rightarrow$ subtleties due to the compositeness of Z.

HETEROTIC ORBIFOLDS

The vertex operators for gluons and gravitons are:

$$V_\gamma = \xi^a_\mu \int d^2z\, J_a \left(\bar\partial X^\mu + ip{\cdot}\psi\psi^\mu\right) e^{ip\cdot X}$$

$$V_g = \xi_{\mu\nu} \int d^2z\, \partial X^\mu \left(\bar\partial X^\nu + ip{\cdot}\psi\psi^\nu\right) e^{ip\cdot X}$$

For the T_i moduli:

$$V_{T_i} = T_i \int d^2z\, \partial \overline{X}^i \left(\bar\partial X^i + ip{\cdot}\psi\psi^i\right) e^{ip\cdot X}$$

$$V_{\overline{T}_i} = \overline{T}_i \int d^2z\, \partial X^i \left(\bar\partial \overline{X}^i + i\bar p{\cdot}\psi\overline{\psi}^i\right) e^{i\bar p\cdot X}$$

T amplitude (R)

In the low-energy limit, one can use:

$$V^{\text{eff.}}_\gamma = F^a \int d^2z\, Q_a$$

$$V^{\text{eff.}}_g = R_{\mu\nu} \int d^2z\, X^\mu \partial X^\nu$$

with

$$F^a = \frac{1}{2} F^a_{\mu\nu} \psi^\mu_0 \psi^\nu_0 \,, \quad R_{\mu\nu} = \frac{1}{2} R_{\mu\nu\rho\sigma} \psi^\rho_0 \psi^\sigma_0$$

Similarly:

$$V^{\text{eff.}}_{T_i} = dT_i \int d^2z\, \psi^i \partial \overline{X}^i \,, \quad V^{\text{eff.}}_{\overline{T}_i} = d\overline{T}_i \int d^2z\, \overline{\psi}^i \partial X^i$$

with

$$dT_i = ip_\mu T_i \psi^\mu_0 \,, \quad d\overline{T}_i = i\bar p_\mu \overline{T}_i \psi^\mu_0$$

A generic correlation is non-vanishing only for equal number of T_i and $\overline{T}_i$ vertices. As expected, these pair to reconstruct composite connections Z_i.

This can be understood also in the path-integral representation of the generating functional. The relevant part of the action is

$$S_i = \int d^2z \left(g_{i\bar{\imath}}\, \partial X^i\, \bar{\partial} \overline{X}^i + g_{i\bar{\imath}}\, \psi^i\, \partial \overline{\psi}^i + dT_i\, \psi^i\, \partial \overline{X}^i + d\overline{T}_i\, \overline{\psi}^i\, \partial X^i \right)$$

Redefining $\psi^i \to \psi^i - g^{i\bar{\imath}}\, dT_i\, X^i$ and rescaling by $\sqrt{g_{i\bar{\imath}}}$ to normalise, one finds:

$$V^{\text{eff.}}_{Z_i} = G_i \int d^2z\, \overline{X}^i\, \partial X^i$$

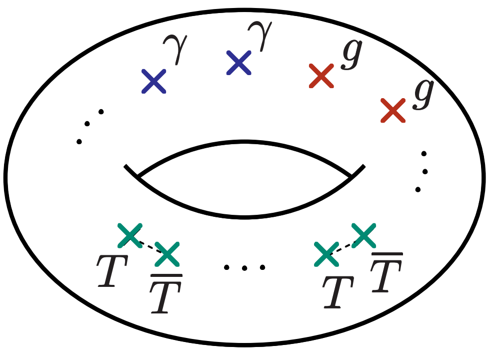
with

$$G_i = \frac{i}{2} \frac{p_\mu T_i\, \bar{p}_\nu \overline{T}_i}{(T_i + \overline{T}_i)^2}\, \psi^\mu_0\, \psi^\nu_0$$

This is very similar to an internal graviton.

The path-integral for $Z(\tau; F, R, G_i)$ is easily computed:

$$Z_T =$$



Typical chiral determinant to compute:

$$\det_{\alpha,\beta}(\lambda) = \prod_{m,n} \frac{2\pi}{\operatorname{Im}\tau}\left[(m+\tfrac{1}{2}-\beta)+(n+\tfrac{1}{2}-\alpha)\,\tau+\lambda\right]$$

One finds:

$$\det_{\alpha,\beta}(\lambda) = \frac{\theta\left[{\alpha \atop \beta}\right](\lambda|\tau)}{\eta(\tau)}\, e^{a\,\frac{\pi\lambda^2}{2\operatorname{Im}\tau}}$$

with

$$a = \begin{cases} 0\,, & \text{Holomorphic reg.} \\ 1\,, & \text{Modular invariant reg.} \end{cases}$$

The structure of the partition function is

$$Z_T^a(\tau; F, R, G) = \exp\left\{-\,a\,\frac{X_4(F,R,G)}{64\,\pi^3\operatorname{Im}\tau}\right\} A(\tau; F, R, G)$$

Notice that

$$\lim_{\tau\to i\infty} Z_T^a(\tau; F, R, G) = \lim_{\tau\to i\infty} A(\tau; F, R, G)$$

One finds:

$$A = \frac{1}{8N}\sum_{k,l=0}^{N-1} N_{k,l}\,\eta^2(\tau)\prod_{a=1}^{2}\frac{\frac{iR_a}{2\pi}\,\eta(\tau)}{\theta\left[{\frac{1}{2} \atop \frac{1}{2}}\right]\left(\frac{iR_a}{4\pi^2}|\tau\right)}\prod_{i=1}^{3}\frac{\eta(\tau)}{\theta\left[{\frac{1}{2}+lv_i \atop \frac{1}{2}+kv_i}\right]\left(\frac{-G_i}{4\pi^2}|\tau\right)}$$

$$\sum_{a,b}\prod_{p=1}^{8}\frac{\theta\left[{a+lw_p \atop b+kw_p}\right]\left(\frac{-F_p}{4\pi^2}|\tau\right)}{\eta(\tau)}\sum_{a',b'}\prod_{q=1}^{8}\frac{\theta\left[{a' \atop b'}\right]\left(\frac{-F'_q}{4\pi^2}|\tau\right)}{\eta(\tau)}$$

and

$$X_4 = 2\left(\sum_{a=1}^{2} R_a^2 + \sum_{i=1}^{3} G_i^2 - \sum_{p=1}^{8} F_p^2 - \sum_{q=1}^{8} {F'_q}^2\right)$$

Holomorphic reg. ($a = 0$):

$$\partial_{\bar{\tau}} Z(\tau; F, R, G) = 0$$

Anomaly cancellation remains manifest if we include the boundary at infinity:

$$I = \oint_{\partial \mathcal{F}} d\tau\, Z(\tau) = 0$$

Then:

$$I_{\text{quantum}} = \int_{\partial \mathcal{F}_\infty} d\tau\, Z(\tau) = \lim_{\tau \to i\infty} A(\tau)$$

$$I_{\text{inflow}} = \int_{\partial \mathcal{F}_{\text{finite}}} d\tau\, Z(\tau) = - \lim_{\tau \to i\infty} A(\tau)$$

Modular invariant reg. ($a = 1$):

$$Z(\tau + 1; F, R, G) = Z(\tau; F, R, G)$$

$$Z(-1/\tau; F, R, G) = \tau^2 Z(\tau; F, R, G)$$

In this case, the boundary at infinity should not be included:

$$I = \int_{\partial \mathcal{F}_{\text{finite}}} d\tau\, Z(\tau) = 0$$

The low-energy cancellation can be checked by computing separately the quantum anomaly and the classical inflow.

By definition:

$$I_{\text{quantum}} = \int_{\partial\mathcal{F}_\infty} d\tau\, Z(\tau) = \lim_{\tau\to i\infty} A(\tau)$$

Schellenkens, Warner

The couplings relevant for the inflow are:

$$L_{GS} = \frac{1}{12}\left|dB - Y_4^{(0)}\right| - 2\pi\, B\wedge Y_2$$

The modified kinetic term is a SUGRA effect and implies $dH = Y_4$.

The GS term is a string effect. They induce $I_{\text{inflow}} = -Y_2\wedge Y_4$.

One computes:

$$Y_2 = -\frac{1}{128\pi^3}\int_{\mathcal{F}}\frac{d^2\tau}{(\text{Im}\tau)^2}Z(\tau) = \frac{1}{X_4}\oint_{\partial\mathcal{F}} d\tau\, Z(\tau)$$
$$= \frac{1}{X_4}\int_{\partial\mathcal{F}_\infty} d\tau\, Z(\tau)$$

Lerche, Nilsson, Schellenkens, Warner

Assuming then that:

$$Y_4 = X_4$$

one finds as required

$$I_{\text{inflow}} = -\int_{\partial\mathcal{F}_\infty} d\tau\, Z(\tau) = -\lim_{\tau\to i\infty} A(\tau)$$

Additional information:

$$A_6 = X_2\wedge X_4$$

$$dH = X_4$$

One finds:

$$A \rightarrow \frac{1}{2N}\sum_{k=1}^{N-1} C_k \Big[\widehat{A}_k(G)\,\widehat{G}(R) + \widehat{G}_k(G)\,\widehat{A}(R) + \Big(\mathrm{ch}^k_{\mathbf{248}}(F) + \mathrm{ch}_{\mathbf{248}}(F)\Big)\widehat{A}_k(G)\,\widehat{A}(R)\Big] - i\sum_{\alpha\in\mathrm{tw.}} \mathrm{ch}_{\mathbf{R}^\alpha}(F)\,\mathrm{ch}_{\mathbf{1}_{\mathbf{1+2n}_\mathbf{i}^\alpha}}(G)\,\widehat{A}(R)$$

All the modular weights come out automatically and in agreement with explicit computations.

Low-energy interpretation

All the anomalies are cancelled by a universal GS mechanism mediated by the NSNS two-form.

One finds:

$$I_{\mathrm{quantum}} = -I_{\mathrm{inflow}} = X_2 \wedge X_4$$

with

$$X_2 = \frac{15}{2(2\pi)^3}\sum_{i=1}^{3} G_i$$

$$X_4 = \mathrm{tr}\,R^2 - \mathrm{tr}\,F^2 + 2\sum_{i=1}^{3} G_i^2$$

In the chiral basis:

$$K_S = -\ln(S + \bar{S} + 2\pi\, X_2^{(0)}) \;\Rightarrow\; \delta S = -2\pi\, X_2^{(1)}$$

$$L_S = S X_4 \;\Rightarrow\; \delta L_S = -2\pi X_2^{(1)} \wedge X_4$$

Geometric interpretation

The form of X_2 and X_4 can be understood by compactification.

In $D = 10$:

$$Y_8 = \frac{1}{8}\,\mathrm{tr}\, R^4 + \frac{1}{32}(\mathrm{tr}\, R^2)^2 + \frac{1}{4}(\mathrm{tr}\, F^2)^2 - \frac{1}{8}\,\mathrm{tr}\, F^2\,\mathrm{tr}\, R^2$$

$$Y_4 = \mathrm{tr}\, R^2 - \mathrm{tr}\, F^2$$

In the presence of torsion, R is the curvature of ω, the sum of the spin connection and the torsion connection constructed from H.

Hull

Compactifying to $D = 4$ on $\mathcal{M}$:

$$\omega_{(10)} \longrightarrow \omega_{(4)} + \sum_{i=1}^{3} Z_i$$

$$\mathrm{tr}\, R^2_{(10)} \longrightarrow \mathrm{tr}\, R^2_{(4)} + 2\sum_{i=1}^{3} G_i^2$$

and

$$X_2 = \int_{\mathcal{M}} Y_8$$

$$X_4 = Y_4\Big|_{(10)\to(4)}$$

TYPE IIB ORIENTIFOLDS

The vertices for gluons and gravitons are:

$$V_\gamma = \xi^a_\mu \oint dz\, \lambda_a \left(\dot{X}^\mu + ip{\cdot}\psi\psi^\mu\right) e^{ip\cdot X}$$

$$V_g = \xi_{\mu\nu} \int d^2z \left(\partial X^\mu + ip{\cdot}\psi\psi^\mu\right)\left(\bar{\partial} X^\nu + ip{\cdot}\tilde{\psi}\tilde{\psi}^\nu\right) e^{ip\cdot X}$$

For the T_i moduli:

$$V_{T_i+\overline{T}_i} = (T_i + \overline{T}_i) \int d^2z \left(\partial \overline{X}^i + ip{\cdot}\overline{\psi}\overline{\psi}^i\right)\left(\bar{\partial} X^i + ip{\cdot}\tilde{\psi}\tilde{\psi}^i\right) e^{ip\cdot X} + \text{h.c.}$$

$$V_{T_i-\overline{T}_i} = \cdots$$

In principle, we need both $V_{T_i\pm\overline{T}_i}$, but on general grounds:

$$\langle V_{T_i\pm\overline{T}_i} V_{T_i\pm\overline{T}_i}\rangle = \alpha_i\, d(T_i \pm \overline{T}_i) \wedge d(T_i \pm \overline{T}_i) = 0$$

$$\langle V_{T_i\pm\overline{T}_i} V_{T_i\mp\overline{T}_i}\rangle = \alpha_i\, d(T_i \pm \overline{T}_i) \wedge d(T_i \mp \overline{T}_i) \neq 0$$

Trick: compute the first and deduce the second by

$$d(T_i + \overline{T}_i) \wedge d(T_i + \overline{T}_i) \;\rightarrow\; d(T_i - \overline{T}_i) \wedge d(T_i + \overline{T}_i)$$

T amplitude (RR,RNS,RNS)

In the low-energy limit, one can use:

$$V_g^{\text{eff.}} = R_{\mu\nu} \int d^2z \left[X^\mu\, \partial X^\nu + \psi^\mu\psi^\nu\right]$$

with

$$R_{\mu\nu} = \frac{1}{2} R_{\mu\nu\rho\sigma}\, \tilde{\psi}_0^\rho\, \tilde{\psi}_0^\sigma$$

Similarly:

$$V^{\text{eff.}}_{T_i+\overline{T}_i} = d(T_i+\overline{T}_i)\int d^2z\left[\psi^i\bar{\partial}\overline{X}^i+\cdots\right] + \text{h.c.}$$

with

$$d(T_i+\overline{T}_i) = ip_\mu(T_i+\overline{T}_i)\,\tilde{\psi}^\mu_0$$

Performing a shift on the internal fermions, one finds:

$$V^{\text{eff.}}_{(T_i+\overline{T}_i)^2} = \frac{d(T_i+\overline{T}_i)\wedge d(T_i+\overline{T}_{\bar{i}})}{(T_i+\overline{T}_i)^2}\int d^2z\left[\overline{X}^i\,\partial X^i+\cdots\right]$$

Using the trick and fixing the fermionic terms by world-sheet SUSY, one finds:

$$V^{\text{eff.}}_{Z_i} = G_i\int d^2z\left[\overline{X}^i\,\partial X^i+\bar{\psi}^i\psi^i\right]$$

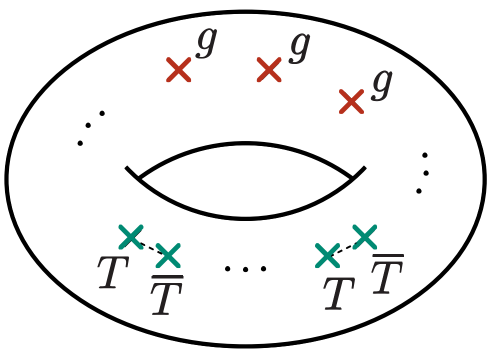
with

$$G_i = \frac{i}{2}\,\frac{p_\mu T_i\,\bar{p}_\nu\overline{T}_i}{(T_i+\overline{T}_i)^2}\,\tilde{\psi}^\mu_0\,\tilde{\psi}^\nu_0$$

This is very similar to an internal graviton.

The partition function is then easily evaluated:

$$Z_T =$$



The typical combination of determinants is:

$$\frac{\det_{a+\delta,b+\gamma}(\lambda)}{\det_{\frac{1}{2}+\delta,\frac{1}{2}+\gamma}(\lambda)} = \frac{\theta\begin{bmatrix}a+\delta\\b+\gamma\end{bmatrix}(\lambda|\tau)}{\theta\begin{bmatrix}\frac{1}{2}+\delta\\\frac{1}{2}+\gamma\end{bmatrix}(\lambda|\tau)} \quad , \quad \forall\, a$$

In this case, there is no regularisation dependence and the result is both holomorphic and modular invariant.

The partition functions is

$$Z_T = \frac{1}{8N}\sum_{k,l=0}^{N-1} N_{k,l} \sum_{a,b} \eta_{ab} \frac{\eta^3(\tau)}{\theta\begin{bmatrix}a\\b\end{bmatrix}(0|\tau)} \prod_{a=1}^{2} \frac{\frac{iR_a}{2\pi}\,\theta\begin{bmatrix}a\\b\end{bmatrix}(\frac{iR_a}{2\pi^2}|\tau)}{\theta\begin{bmatrix}\frac{1}{2}\\\frac{1}{2}\end{bmatrix}(\frac{iR_a}{2\pi^2}|\tau)}$$

$$\prod_{i=1}^{3} \frac{\theta\begin{bmatrix}a+lv_i\\b+kv_i\end{bmatrix}(\frac{-G_i}{2\pi^2}|\tau)}{\theta\begin{bmatrix}\frac{1}{2}+lv_i\\\frac{1}{2}+kv_i\end{bmatrix}(\frac{-G_i}{2\pi^2}|\tau)}$$

Holomorphicity:

$$\partial_{\bar\tau}\, Z(\tau; R, G) = 0$$

This implies:

$$\oint_{\partial\mathcal{F}} d\tau\, Z(\tau) = 0$$

Modular invariance:

$$Z(\tau+1; R, G) = Z(\tau; R, G)$$

$$Z(-1/\tau; R, G) = \tau^2 Z(\tau; R, G)$$

This implies:

$$\int_{\partial\mathcal{F}_{\rm finite}} d\tau\, Z(\tau) = 0$$

Putting these together:

$$I_{\text{quantum}} = \int_{\partial\mathcal{F}_\infty} d\tau\, Z(\tau) = 0$$
$$I_{\text{inflow}} = -\int_{\partial\mathcal{F}_\infty} d\tau\, Z(\tau) = 0$$

Indeed, one finds:

$$\begin{aligned} Z_T \to & -\frac{1}{16N}\sum_{k=1}^{N-1} C_{2k}\,\widehat{L}_k(G)\,\widehat{L}(R) \\ & +\frac{1}{2N}\sum_{k=1}^{N-1} C_k\left[\widehat{A}_k(G)\,\widehat{G}(R)+\widehat{G}_k(G)\,\widehat{A}(R)\right] \\ & + i\sum_{\alpha\in\text{tw.}} \text{ch}_{\mathbf{1}_1}(G)\,\widehat{A}(R) \\ = 0 & \end{aligned}$$

All the known modular weights for untwisted fields are correctly reproduced.

For the twisted moduli, one finds $n_i = 0$, but the opposite sign compared to the SUGRA computation. This is equivalent to having $n_i = -1$ with the expected sign.

Remark: the relative sign between untwisted and twisted sectors is fixed by modular invariance.

A, M and K amplitudes (RR)

In the low-energy limit, one can use:

$$V_\gamma^{\text{eff.}} = F^a \oint dz\, \lambda_a$$

$$V_g^{\text{eff.}} = R_{\mu\nu} \int d^2z \left[X^\mu (\partial + \bar\partial) X^\nu + (\psi - \tilde\psi)^\mu (\psi - \tilde\psi)^\nu\right]$$

with

$$F^a = \frac{1}{2} F^a_{\mu\nu}\, \psi_0^\mu \psi_0^\nu\,, \quad R_{\mu\nu} = \frac{1}{2} R_{\mu\nu\rho\sigma}\, \psi_0^\rho \psi_0^\sigma$$

Similarly:

$$V^{\text{eff.}}_{T_i+\overline{T}_i} = d(T_i + \overline{T}_i) \int d^2z \left[\psi^i \bar\partial \overline{X}^i + \tilde\psi^i \partial \overline{X}^i + \cdots\right] + \text{h.c.}$$

with

$$d(T_i + \overline{T}_i) = i p_\mu (T_i + \overline{T}_i)\, \psi_0^\mu$$

Performing a shift on the internal fermions, one finds:

$$V^{\text{eff.}}_{(T_i+\overline{T}_i)^2} = \frac{d(T_i + \overline{T}_i) \wedge d(T_i + \overline{T}_i)}{(T_i + \overline{T}_i)^2} \int d^2z \left[\overline{X}^i (\partial + \bar\partial) X^i + \cdots\right]$$

Using the trick and fixing the fermionic terms by world-sheet SUSY, one finds:

$$V^{\text{eff.}}_{Z_i} = G_i \int d^2z \left[\overline{X}^i (\partial + \bar\partial) X^i + (\bar\psi - \tilde{\bar\psi})^i (\psi - \tilde\psi)^i\right]$$

with

$$G_i = \frac{i}{2} \frac{p_\mu T_i\, \bar p_\nu \overline{T}_i}{(T_i + \overline{T}_i)^2}\, \psi_0^\mu\, \psi_0^\nu$$

This is again very similar to an internal graviton.

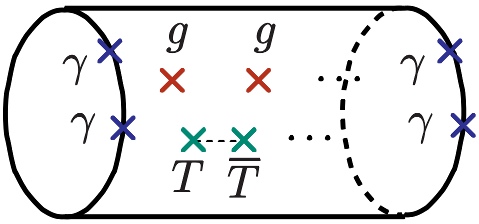
In this case, only massless states contribute, and:

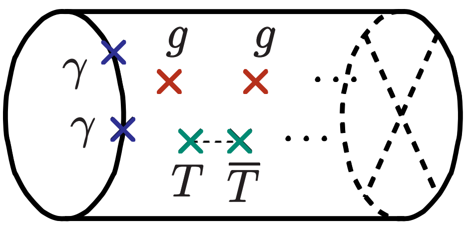
$$Z(\tau; F, R, G) = Z(F, R, G)$$

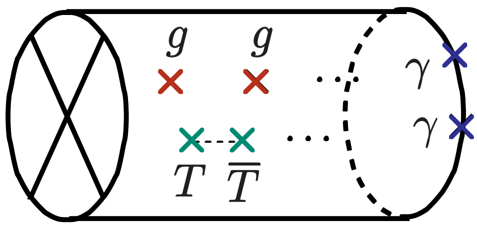
Anomaly cancellation is manifest:

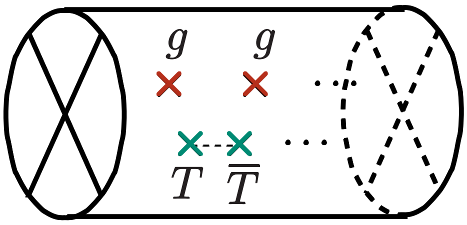
$$I_{\text{quantum}} = \lim_{\tau \to i\infty} Z(\tau) = Z$$
$$I_{\text{inflow}} = -\lim_{\tau \to 0} Z(\tau) = -Z$$

The partition functions correspond to:

$Z_A =$ 

$Z_M =$ 

$+$ 

$Z_K =$ 

One finds:

$$Z_A = \frac{1}{4N} \sum_{k=1}^{N-1} C_k \, \mathrm{ch}^k_{\mathbf{32^2}}(F) \, \hat{A}_k(G) \, \hat{A}(R)$$

$$Z_M = -\frac{1}{4N} \sum_{k=1}^{N-1} C_k \, \mathrm{ch}^{2k}_{\mathbf{32}}(2F) \, \hat{A}_k(G) \, \hat{A}(R)$$

$$Z_K = \frac{1}{16N} \sum_{k=1}^{N-1} C_{2k} \, \hat{L}_k(G) \, \hat{L}(R)$$

Anomaly cancellation requires that these factorise as:

$$Z_A = \frac{1}{2N} \sum_{k \in S_k} N_k \, Q_k^B \wedge Q_k^B$$

$$Z_M = \frac{1}{2N} \sum_{k \in S_k} N_k \left(Q_{2k}^B \wedge Q_{2k}^O + Q_{2k}^O \wedge Q_{2k}^B \right)$$

$$Z_K = \frac{1}{2N} \sum_{k \in S_k} N_k \, Q_{2k}^O \wedge Q_{2k}^O$$

These are interpreted as the inflows induced by the RR anomalous couplings:

$$S_B = \sqrt{\frac{2\pi}{N}} \sum_{k \in S_k} \sum_{i_k=1}^{N_k} \int C_k^{i_k} \wedge Q_k^B$$

$$S_O = \sqrt{\frac{2\pi}{N}} \sum_{k \in S_k} \sum_{i_k=1}^{N_k} \int C_{2k}^{i_k} \wedge Q_{2k}^O$$

Factorisation is possible thanks to the properties

$$\sqrt{\widehat{A}(R)}\,\sqrt{\widehat{L}(R/4)} = \widehat{A}(R/2)$$
$$\sqrt{\widehat{A}_{2k}(G)}\,\sqrt{\widehat{L}_k(G/4)} = \widehat{A}_k(G/2)$$

one finds

$$Q_k^B = \sqrt{\left|\frac{1}{C_k}\right|}\,\mathrm{ch}_{\mathbf{32}}^k(F)\,\sqrt{\widehat{A}_k(G)}\,\sqrt{\widehat{A}(R)}$$
$$Q_{2k}^O = -4\sqrt{\left|\frac{C_{2k}}{C_k^2}\right|}\,\sqrt{\widehat{L}_k(G/4)}\,\sqrt{\widehat{L}(R/4)}$$

These are the generalisation of the usual RR charges of D-branes and O-planes, which are also related to anomalies.

Green, Harvey, Moore

Dasgupta, Jatkar, Mukhi

Morales, Scrucca, Serone

This kind of factorisation is natural also for models with a rational internal CFT.

Bianchi, Morales

Low-energy interpretation

All the anomalies are cancelled by a generalised GS mechanism involving the twisted RR two-forms. The GS couplings are given by the anomalous couplings for D-branes and O-planes.

One finds:

$$I_{\text{quantum}} = -I_{\text{inflow}} = \sum_{k \in S_k} N_k \, X_2^k \wedge X_4^k$$

with

$$X_2^{2k} = \frac{N_k^{-1/4}}{\sqrt{N}(2\pi)} \left[i \, \mathrm{tr}^{2k} F + 2 \sum_{i=1}^{3} \tan(\pi k v_i) \, G_i \right]$$

$$X_4^{2k} = \frac{N_k^{-1/4}}{\sqrt{N}(2\pi)^2} \left[\frac{1}{2} \mathrm{tr}^{2k} F^2 + \frac{i}{4} \sum_{i=1}^{3} \cot(2\pi k v_i) \, \mathrm{tr}^{2k} F \, G_i \right.$$
$$- \frac{1}{16} \mathrm{tr} \, R^2 - \frac{1}{8} \sum_{i=1}^{3} \tan^2(\pi k v_i) \, G_i^2$$
$$\left. - \frac{1}{4} \sum_{i \neq j = 1}^{3} \frac{\cos(2\pi k v_i) \cos(2\pi k v_j) - 1}{\sin(2\pi k v_i) \sin(2\pi k v_j)} G_i \, G_j \right]$$

In the chiral basis:

$$K_M = F\left(M_{i_k}^k + \overline{M}_{i_k}^k + 2\pi \, X_2^{k(0)} \right) \;\Rightarrow\; \delta M_{i_k}^k = -2\pi \, X_2^{k(1)}$$

$$L_M = \sum_{k \in S_k} \sum_{i_k = 1}^{N_k} M_{i_k}^k \, X_4^k \;\Rightarrow\; \delta L_M = -2\pi \sum_{k \in S_k} N_k \, X_2^{k(1)} \wedge X_4^k$$

Puzzle: in this way, the M's would have $n_i = 0$ and not $n_i = -1$ $\Rightarrow$ wrong sign for their contribution to the anomaly.

Possible solutions:

- Subtleties in the linear-chiral duality.
- The M's have $n_i = 0$, but for some reason they contribute to the anomaly with the unexpected sign.
- The M's have $n_i = -1$, due to some T_i-dependence in K_M. Perhaps this is provided by the GS shift, since in SUGRA it looks tree-level due to a g_s hidden in the definition of T_i.

CONCLUSIONS

The mechanism of anomaly cancellation in string theory is quite universal, and works in the same way for gauge, gravitational and target-space anomalies.

T amplitude

Z_T must be modular invariant, and one gets a non-vanishing anomaly only if it is not holomorphic. The GS term for NSNS axion is determined by the non-holomorphicity.

A, M and K amplitude

Z_A, Z_M and Z_K satisfy tadpole cancellation and factorise. The GS terms for the twisted RR axions are determined by the anomalous couplings of D-branes and O-planes.

This applies to all the anomalies in the simplest $D=4$ $N=1$ heterotic orbifolds and Type IIB orientifolds.

OPEN PROBLEM

What are the implications of the unexpected sign for the contribution of the twisted moduli in orientifold models ?