

Electroweak symmetry breaking and fermion masses from extra dimensions

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Abstract

We study higher-dimensional non-supersymmetric orbifold models where the Higgs field is identified with some internal component of a gauge field. We address two important and related issues that constitute severe obstacles towards model building within this type of constructions: the possibilities of achieving satisfactory Yukawa couplings and Higgs potentials. We consider models where matter fermions are localized at the orbifold fixed-points and couple to additional heavy fermions in the bulk. When integrated out, the latter induce tree-level non-local Yukawa interactions and a quantum contribution to the Higgs potential that we explicitly evaluate and analyse. The general features of these highly constrained models are illustrated through a minimal but potentially realistic five-dimensional example. Finally, we discuss possible cures for the persisting difficulties in achieving acceptable top and Higgs masses. In particular, we consider in some detail the effects induced in these models by adding localized kinetic terms for gauge fields.

1 Introduction

The Standard Model (SM) of electroweak and strong interactions is extremely successful in reproducing all the available experimental data up to currently accessible energies. However, the problem of stabilizing the electroweak scale against quadratically divergent radiative corrections to the Higgs mass suggests the presence of new physics at scales not much larger than the Z mass. While supersymmetric extensions of the SM technically solve the hierarchy problem and suggest the unification of gauge couplings, the ever-rising lower bounds on supersymmetric particles have recently stimulated studies of alternative methods of supersymmetry and electroweak breaking based on the presence of extra dimensions at the TeV scale [1].

Higher-dimensional models open up the interesting possibility that the Higgs boson arises from the internal component of a gauge field, with a dynamics protected by higher-dimensional gauge invariance and controlled in a predictive way by a higher-dimensional effective theory. Such a possibility, already advocated several years ago [2], has recently received renewed interest [3]–[8]. In the case of toroidal internal spaces, the effective potential turns out to be free of quadratic divergences, and even in phenomenologically more viable orbifold models [9], power divergences arising through operators localized at the fixed-points are strongly constrained [10]. Exploiting these facts, one can construct interesting orbifold models in which electroweak interactions are unified in a larger group, which is broken through the orbifold projection. Gauge bosons and Higgs fields arise respectively from the four-dimensional and internal components of the higher-dimensional gauge bosons. Matter fields can be introduced either as bulk fields in representations of the unified group G , or as boundary fields localized at the fixed-points where this is broken to a subgroup H . The construction of realistic models of this type is, however, a difficult task, because Higgs interactions are severely constrained by the higher-dimensional gauge invariance. In particular, achieving satisfactory flavour structures and electroweak symmetry breaking at the same time represents the main problem in this class of theories.

Flavour symmetry breaking can be achieved essentially in two ways, depending on whether standard matter fields are introduced in the bulk or at the fixed-points of the orbifold, i.e. the boundaries. In the case of bulk matter fields, standard Yukawa couplings can originate only from higher-dimensional gauge couplings, which are flavour-symmetric and completely determined by the group representation. A possible way of obtaining couplings with a non-trivial flavour structure consists in switching on mass terms with odd coefficients and introducing mixings between the bulk matter fields and additional boundary fields in such a way that unwanted light fields eventually decouple [8]. In the case of boundary matter fields, standard Yukawa couplings cannot be directly introduced, because they would violate the higher-dimensional symmetry that protects the potential and therefore induce quadratic divergences, just as in the

SM [10]. The only invariant interactions that can be used are non-local interactions involving Wilson lines [5]; an interesting possibility to generate this kind of interactions was suggested in [7] and consists in introducing mixings between the matter fields and additional heavy fermions in the bulk, which are then integrated out. These two flavour symmetry breaking mechanisms are very similar from the microscopic point of view, and rely basically on a mixing of bulk and boundary fields. Interestingly, they also provide a natural explanation for the large hierarchy of fermion masses, since they are exponentially sensitive on some parameters of the microscopic theory.

Electroweak symmetry breaking occurs radiatively in this class of models, and is equivalent to a Wilson line symmetry breaking [11, 12] (or Scherk–Schwarz twist [13]), which reduces the rank of H (as required for electroweak symmetry breaking) if the Wilson line and the embedding of the orbifold twist do not commute. Since the tree-level Higgs potential is strongly constrained by gauge invariance, any theory of this type is potentially very predictive and constrained, even though it is only an effective theory. The generic predictive power of the theory depends on the parameter M_c/Λ , which controls the effect of non-renormalizable operators at the compactification scale M_c . Λ is the physical cut-off scale, whose actual value can be roughly estimated by using Naïve Dimensional Analysis (NDA) [14, 15]. There are, however, special quantities, such as the ratio of Higgs and gauge boson masses, that turn out to be quite insensitive to Λ , because of the non-local nature of the electroweak symmetry breaking in the internal dimensions.

The construction of phenomenologically viable models requires a detailed and combined analysis of the flavour and electroweak symmetry breaking mechanisms, since the two are closely connected. Such an analysis has never been performed so far, and the aim of this work is to fill this gap with an investigation of the possibility of achieving realistic effective Yukawa couplings and at the same time a satisfactory Higgs potential. We focus for simplicity on five-dimensional (5D) models, where the tree-level Higgs potential is actually vanishing and electroweak symmetry breaking is thus entirely governed, at sufficiently weak coupling, by the one-loop Higgs effective potential. The contribution to the latter arising from gauge fields or bulk fermions in fundamental or adjoint representations is easily computed and well-known [12, 16, 17]. In the presence of a flavour symmetry breaking mechanism of the types described above, however, the computation of the matter contribution to the effective potential is more involved, since in both cases there are simultaneously bulk fermions and boundary fermions mixing among each other. Putting matter fields in the bulk as suggested in [8], the situation is further worsened by the need of odd mass terms that distort the wave functions of the bulk fields on their own. For this reason, we consider here the case suggested in [7], with matter fields located at the fixed-points.

The basic construction that we study consists of a pair of left-handed and right-handed matter fermions located at possibly different fixed-points and a heavy bulk

fermion with quantum numbers allowing couplings to both boundary fermions, in such a way that non-local Wilson line effective interactions between the two can be generated. We explicitly compute the tree-level effective action that is induced for the matter fermions by integrating out the massive bulk fermions. The result consists of a mass term, a wave-function correction, and an infinite series of higher-derivative interactions. All these terms are directly proportional to the bulk-to-boundary couplings and exponentially sensitive to the bulk masses. At energies much below the bulk mass, higher-derivative terms can be neglected and after rescaling the wave-function one finds a physical mass that is bounded in size. At energies much above the bulk mass, on the contrary, the induced interactions get exponentially suppressed by their derivative dependence. We also analyse the full one-loop effective potential for the Higgs field induced by charged bulk fields. Its dependence on the bulk-to-boundary couplings can be reinterpreted in terms of diagrams involving boundary fields and the momentum-dependent effective vertex between Higgs and boundary fields that is induced at tree level by the bulk fermion. The soft behaviour of this effective vertex at high momenta ensures that the loop integral is free of any divergence. The full one-loop effective potential for the Higgs fields is therefore finite, as expected on symmetry grounds¹. Moreover, it turns out to have non-trivial symmetry-breaking minima. This shows that the flavour and electroweak symmetry breaking mechanisms that we consider are indeed compatible.

The above building blocks can be easily used to construct models that incorporate all SM fermions and Yukawa couplings and offer therefore a realistic arena for more quantitative investigations. The only universal quantity to play with is the radius R , which can be considered as a parameter. In phenomenologically viable models, it must be small enough to satisfy the current experimental bounds (see e.g. [18] and references therein), and the Vacuum Expectation Value (VEV) α of the Wilson line phase must therefore be small as well in order to match the mass of the W bosons, given by $m_W = \alpha/R$. For the minimal version of the model, with the simplest possible choice of gauge group and representations, we find that the induced effective potential $V(\alpha)$ has minima at $\alpha \sim 0.2$, leading to too-low values for $1/R$. The Higgs mass $m_H = (g_4 R/2)\sqrt{V''(\alpha)}$, which is further suppressed by a gauge loop factor, tends consequently to be too low as well. Finally, the top mass m_{top} cannot be adjusted to a high-enough value. A possible way of improving the situation is to add to the model new heavy bulk fermions that do not couple to the SM matter fields but transform

¹This should be contrasted to [7], where a Wilson line Yukawa coupling was introduced as a fundamental coupling in the tree-level Lagrangian and found to induce divergences in the one-loop Higgs potential. The distinction between our situation and that of [7] lies in the locality of the fundamental Lagrangian. If the latter is non-local, then quantum divergences are allowed to appear even in non-local operators (such as the Higgs mass in these models), since all of them must now be introduced in the theory from the beginning as independent couplings and can be used as counterterms.

in large representations of the gauge group, in such a way as to modify $V(\alpha)$ and obtain minima for much lower α . For representations of rank $r \gg 1$, one can obtain minima at values as low as $\alpha \sim 1/r$, with a $V''(\alpha)$ at the minimum that rapidly grows with r . This helps considerably in increasing $1/R$ and m_H , and to some extent m_{top} . However, fields with high rank r induce electroweak quantum corrections that are enhanced by large group-theoretical factors, thereby lowering the cut-off and the predictive power of the theory.

A simple and natural generalization of our minimal set-up consists in introducing additional localized kinetic terms for gauge fields [19]². These have the effect of increasing the bulk gauge coupling and to distort in a non-trivial way the wave functions and mass spectrum of the Kaluza–Klein (KK) modes of the gauge bosons. They also modify the gauge contribution to the effective potential, which we explicitly compute for arbitrary values of the localized couplings. It turns out that for sufficiently large values of the latter, where the analysis simplifies, it is possible to get values of α leading to acceptable values of $1/R$; at the same time, m_H is significantly increased and m_{top} can be reproduced. We show, however, that the simultaneous presence of localized gauge kinetic terms and a Wilson-line symmetry breaking leads in general to an unwanted deformation of the electroweak sector of the theory. The main point is that the gauge interactions are no longer universal and the masses of the lowest KK gauge bosons are deformed. This implies, among other things, that the tree-level value of the ρ parameter departs from 1 and hence imposes generically very severe bounds on the size of localized gauge kinetic terms. These could strongly constrain model building even if they are only radiatively generated, although it is not excluded that large values can be tolerated in special models with new symmetries. Finally, the scale at which electroweak interactions become strong is again lowered, so that the theory tends to become less predictive.

The paper is organized as follows. In section 2 we present a basic model that captures all the essential features we want to exploit. In section 3, the non-local Yukawa couplings between boundary fermions and the Higgs field are computed. In section 4 we compute the one-loop effective potential for the Higgs field. In section 5 we construct a prototype 5D model and discuss its main properties. In section 6, the effects of localized gauge kinetic terms are studied. Finally, section 7 is devoted to some general conclusions, and some technical details are collected in the appendix.

2 The basic construction

The simplest framework allowing an implementation of the Higgs field as the internal component of a gauge field is a 5D gauge theory with gauge group $G = SU(3)$ on

²We thank R. Rattazzi for suggesting this possibility to us.

an $S^1/\mathbf{Z}_2$ orbifold [20]. The $\mathbf{Z}_2$ orbifold projection is embedded in the gauge group through the matrix

$$P = e^{i\pi\lambda_3} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (1)$$

where λ_a are the standard $SU(3)$ Gell-Mann matrices, normalized as $\text{Tr } \lambda_a \lambda_b = 2\delta_{ab}$, so that $A_M = A_M^a \lambda_a/2$. The group G is broken in 4D to the commutant $H = SU(2) \times U(1)$ of the projection P . The massless 4D fields are the gauge bosons A_μ^a in the adjoint of H and a charged scalar doublet, arising from the internal components A_5^a of the gauge field. A VEV for A_5 induces an additional spontaneous symmetry breaking to $E = \hat{U}(1)$, generated by $\hat{A}_\mu = (A_\mu^8 + \sqrt{3}A_\mu^3)/2$. Using the residual H symmetry, it is always possible to align $\langle A_5 \rangle$ along the λ_7 component, corresponding to the down component of the doublet, and take

$$\langle A_5^a \rangle = \frac{2\alpha}{g_5 R} \delta^{a7}. \quad (2)$$

Identifying H with the electroweak gauge group, E with the electromagnetic group, and the zero-mode of A_5 with the Higgs field H , the above construction gives a description of electroweak symmetry breaking. In this minimal version, the weak mixing angle θ_W turns out to be too large, $\theta_W = \pi/3$, but this problem can be solved by starting with a different unified group G , as we will see. Taking into account the normalization of the zero-mode (see the appendix), (2) corresponds to a VEV for the neutral component of the Higgs doublet H equal to $2\alpha/(g_4 R)$, with

$$g_4 = \frac{g_5}{\sqrt{2\pi R}}. \quad (3)$$

It is well known from [12, 17] that a VEV for A_5 induces a Wilson line which is equivalent to a Scherk-Schwarz twist, the two situations being related through a non-periodic gauge transformation. The twist matrix $T(\alpha)$ satisfies the consistency condition $TPT = P$ [21]; for the choice (2), it is given by

$$T(\alpha) = e^{2i\pi\alpha\lambda_7} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 2\pi\alpha & \sin 2\pi\alpha \\ 0 & -\sin 2\pi\alpha & \cos 2\pi\alpha \end{pmatrix}. \quad (4)$$

The orbifold projection represents an explicit symmetry breaking of G to H , the masses of the fields in G/H being of order $1/R$, whereas the Scherk-Schwarz twist corresponds to a spontaneous symmetry breaking of H to E , the masses of the fields in H/E being of order α/R . This situation is equivalent to an $S^1/(\mathbf{Z}_2 \times \mathbf{Z}_2')$ orbifold with two non-commuting projections P and $P'(\alpha) = T(\alpha)P = T(\alpha/2)PT(-\alpha/2)$, and radius $2R$. From this point of view, there are two perfectly symmetric projections, the first breaking G to H with fixed-points $y = 0, 2\pi R$, and the second G to H' with

fixed-points $y = \pi R, 3\pi R$. Together, they break G to $E = H \cap H'$ in a non-local way: the two subgroups H and H' are isomorphic, but their embeddings in G form an angle α . The most general field content consists of bulk fields in representations of G , and boundary fields in representations of the subgroup H or H' surviving at the fixed-point where they are localized (see e.g. [22]).

In the following, we will adopt the perspective of the $S^1/\mathbf{Z}_2$ orbifold with a Wilson line breaking, and work on the fundamental interval $y \in [0, \pi R]$. The brane fields of our basic construction consist of a left-handed fermion doublet $Q_L = (u_L, d_L)^T$ and two right-handed fermion singlets u_R and d_R of $H = SU(2) \times U(1)$; these fields can also be described by their charge conjugates $Q_R^c = (d_R^c, -u_R^c)^T$, d_L^c and $-u_L^c$. We will assume that the doublet and the two singlet fields are located respectively at positions y_1 and y_2 , equal to either 0 or πR . The bulk fields are, in addition to the gauge fields A_M , one pair of fermions Ψ_a and $\tilde{\Psi}_a$ with opposite $\mathbf{Z}_2$ parities for each type of matter field $a = u, d$; we take these in the symmetric and the fundamental representations of $G = SU(3)$ for $a = u$ and $a = d$ respectively.

The parity assignments for the bulk fermions allow for bulk mass terms M_a mixing Ψ_a and $\tilde{\Psi}_a$, as well as boundary couplings $e_{1,2}^a$ with mass dimension 1/2 mixing the bulk fermion Ψ_a to the boundary fermion a . Denoting the doublet and singlet components arising from the decomposition of the bulk fermion Ψ_a under $G \rightarrow H$ (see the appendix) by ψ_a and χ_a respectively, the complete Lagrangian for matter fields is then given by the following expression³:

$$\begin{aligned} \mathcal{L}_{\text{mat}} = & \sum_a \left[\bar{\Psi}_a i \not{D}_5 \Psi_a + \tilde{\bar{\Psi}}_a i \not{D}_5 \tilde{\Psi}_a + \left(\bar{\Psi}_a M_a \tilde{\Psi}_a + \text{h.c.} \right) \right] \\ & + \delta(y - y_1) \left[\bar{Q}_L i \not{D}_4 Q_L + \left(e_1^d \bar{Q}_L \psi_d + e_1^u \bar{Q}_L^c \psi_u + \text{h.c.} \right) \right] \\ & + \delta(y - y_2) \left[\bar{d}_R i \not{D}_4 d_R + \bar{u}_R i \not{D}_4 u_R + \left(e_2^d \bar{d}_R \chi_d + e_2^u \bar{u}_R^c \chi_u + \text{h.c.} \right) \right]. \end{aligned}$$

All bulk fermion modes are massive and, neglecting the bulk-to-boundary couplings, their mass spectrum is given by $M_{a,n} = \pm \sqrt{m_n^2 + M_a^2}$, where $m_n = n/R$. After the spontaneous symmetry breaking induced by (2), a new basis has to be defined for the bulk fermion modes in which they have diagonal mass terms, with a shift in the KK masses $m_n \rightarrow m_n(\alpha)$. The procedure is outlined in the appendix. The new fields $\Psi^{(i)}$ and $\tilde{\Psi}^{(i)}$ defining this basis are given by eqs. (56) and (61), and have KK masses $m_n^{(i)} = (n + q^{(i)}\alpha)/R$, with $q^{(i)}$ being an integer charge, defined by eqs. (59) and (64); $i = 1, 2$ for the fundamental representation and $i = 1, 2, 3, 4$ for the symmetric one. Similarly, a new basis has to be defined for the gauge-field modes to diagonalize their mass terms. The new fields $A_M^{(i)}$ and their KK masses are defined as in eqs. (66) and (69), where the field $\Psi^\pm$ are respectively identified with the gauge field components A_μ and A_5 . More precisely, the complex gauge field $A_\mu^{(1)}$ with charge $q^{(1)} = 1$ ($\Psi_0^{+(1)}$)

³ $\not{D}_4$ and $\not{D}_5$ denote the 4D and 5D covariant derivatives. Defining the Hermitian matrix $\gamma_5 = i\gamma_4$, they are related by $\not{D}_5 = \not{D}_4 + i\gamma_5 D_5$.

is identified with the W boson, the real field $A_\mu^{(2)}$ with $q^{(2)} = 2$ ($\Psi_0^{+(2)}$) with the Z boson and the neutral field $A_\mu^{(0)}$ ($\Psi_0^{+(3)}$) with the photon. Similarly, the scalar field $A_5^{(0)}$ ($\Psi_0^{-(4)}$) is identified with the component of the Higgs field H that is left over after the spontaneous symmetry breaking. Using the fact that $\sec \theta_W = 2$, the masses of the W and Z fields can be written as

$$m_W = \frac{\alpha}{R}, \quad m_Z = \frac{\alpha}{R} \sec \theta_W. \quad (5)$$

The Higgs mass is radiatively induced after the electroweak symmetry breaking and depends on the second derivative of the potential evaluated at the minimum:

$$m_H = \frac{g_4 R}{2} \sqrt{V''(\alpha)}. \quad (6)$$

In the following, it will be convenient to take the size πR of the orbifold as reference length scale and use it to define dimensionless quantities. In particular, it will be useful to introduce the parameters $\lambda^a = \pi R M_a$ and $\epsilon_i^a = \sqrt{\pi R / 2} e_i^a$, and the integer $\delta = (\pi R)^{-1} |y_1 - y_2|$ parametrizing the distance between the location of left- and right-handed fields.

3 Induced couplings for the boundary fermions

The heavy bulk fields couple to boundary fermions, and can therefore induce gauge-invariant non-local couplings mixing matter fermions localized at the fixed-points and the field A_5 through Wilson lines [7]. Since the VEV (2) for A_5 mixes the modes of different components of the bulk fermions, it is convenient to use the fields $\Psi^{(i)}$ and $\tilde{\Psi}^{(i)}$ defined before and to group them into a two-vector $(\Psi^{(i)}, \tilde{\Psi}^{(i)})^T$. In this notation, the kinetic term of the n -th KK mode of each component of the bulk fermions is encoded in the following two-by-two matrix in momentum space:

$$K^\Psi = \begin{pmatrix} \not{p} - m_n & M \\ M & \not{p} + m_n \end{pmatrix}. \quad (7)$$

The tree-level propagator is obtained by inverting this matrix, and is given by

$$\Delta^\Psi = \frac{i}{p^2 - m_n^2 - M^2} \begin{pmatrix} \not{p} + m_n & -M \\ -M & \not{p} - m_n \end{pmatrix}. \quad (8)$$

The couplings between bulk and boundary fields can be easily rewritten in terms of the new fields as well; the new doublet and singlet components under the decomposition $G \rightarrow H$ can be easily read off from eqs. (57) and (62). The only bulk field that couples both to the left-handed and right-handed boundary fields, and can therefore induce mass terms, is always $\Psi^{(2)}$. The other components of the bulk fermions

couple only separately to left-handed and right-handed fields; they can therefore induce only wave-function corrections. For each bulk-to-boundary coupling, there is a wave-function factor $\xi_{i,n}$ for the n -th mode, which is equal to 1 if $y_i = 0$ and $(-1)^n$ if $y_i = \pi R$. The interaction Lagrangian reads

$$\begin{aligned} \mathcal{L}_{\text{int}} = \frac{1}{\pi R} \sum_{n=-\infty}^{\infty} & \left[\epsilon_1^u \xi_{1,n} \left(\bar{d}_R^c \Psi_{u,n}^{(1)} - \bar{u}_R^c \Psi_{u,n}^{(2)} \right) - \frac{\epsilon_2^u}{\sqrt{2}} \xi_{2,n} \bar{u}_L^c \left(\Psi_{u,n}^{(2)} + \eta_n \Psi_{u,n}^{(4)} \right) \right. \\ & \left. + \epsilon_1^d \xi_{1,n} \left(\eta_n \bar{u}_L \Psi_{d,n}^{(1)} + \bar{d}_L \Psi_{d,n}^{(2)} \right) + \epsilon_2^d \xi_{2,n} \bar{d}_R \Psi_{d,n}^{(2)} + \text{h.c.} \right], \end{aligned} \quad (9)$$

where the factor η_n is defined to be 1 for $n = 0$ and $1/\sqrt{2}$ for $n \neq 0$. The bulk fermions can be disentangled from the boundary fermions by completing the squares in the kinetic operator and performing an appropriate redefinition of the bulk fields. This generates corrections to the kinetic operators K^u and K^d for the boundary fields $u = u_L + u_R$ and $d = d_L + d_R$, originally given just by $\not{p}$, which correspond diagrammatically to the exchange of bulk fermions. These contributions have the structure $\sum_n (e \xi_n P_{L,R}) \Delta_n^\Psi (e \xi_n P_{R,L})$, where $P_{L,R}$ are chiral projectors and Δ_n^Ψ is given by the first entry of (8) with m_n as in eqs. (59) and (64). To compute the infinite sums, it is convenient to go to Euclidean space, and define the dimensionless momentum variables $x = \pi R p$ and $x^a = \pi R \sqrt{p^2 + M_a^2}$, as well as the basic functions f_δ given by

$$f_0(x, \alpha) = \sum_{n=-\infty}^{\infty} \frac{1}{x + i\pi(n + \alpha)} = \coth(x + i\pi\alpha), \quad (10)$$

$$f_1(x, \alpha) = \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{x + i\pi(n + \alpha)} = \sinh^{-1}(x + i\pi\alpha). \quad (11)$$

The functions $f_\delta(x, \alpha)$ are related to the propagation of bulk fields between two fixed-points separated by a distance $\delta \pi R$. This is particularly clear from their Taylor expansion, which takes the simple form

$$f_\delta(x, \alpha) = \sum_{k=-\infty}^{\infty} e^{-|2k+\delta|(x+i\pi\alpha)}. \quad (12)$$

After a straightforward computation, the total effective actions for the boundary fields are found to be given by the following expressions:

$$\begin{aligned} K^u = \not{p} \text{Re} & \left[1 + \frac{\epsilon_1^{d2}}{x^d} P_L f_0(x^d, 0) + \frac{\epsilon_2^{u2}}{2x^u} P_R f_0(x^u, 0) + \left(\frac{\epsilon_1^{u2}}{x^u} P_L + \frac{\epsilon_2^{u2}}{2x^u} P_R \right) f_0(x^u, 2\alpha) \right] \\ & + \frac{i}{\pi R} \text{Im} \left[\frac{\epsilon_1^u \epsilon_2^u}{\sqrt{2}} f_\delta(x^u, 2\alpha) \right], \end{aligned} \quad (13)$$

$$\begin{aligned} K^d = \not{p} \text{Re} & \left[1 + \left(\frac{\epsilon_1^{d2}}{x^d} P_L + \frac{\epsilon_2^{d2}}{x^d} P_R \right) f_0(x^d, \alpha) + \frac{\epsilon_1^{u2}}{x^u} P_L f_0(x^u, \alpha) \right] \\ & + \frac{i}{\pi R} \text{Im} \left[\epsilon_1^d \epsilon_2^d f_\delta(x^d, \alpha) \right]. \end{aligned} \quad (14)$$

From the above expressions, we see that an infinite set of non-renormalizable interactions involving the Higgs field are generated together with the standard Yukawas. The new interactions have small effects in physical processes as long as the radius R is small enough, but they all contribute comparably to spontaneous symmetry breaking, whose effect is not only to induce masses m^u and m^d mixing d_L with d_R and u_L with u_R , but also to generate different wave-function corrections $Z_{1,2}^u$ and $Z_{1,2}^d$ for $u_{L,R}$ and $d_{L,R}$. Moreover, all these quantities are momentum-dependent.

To be precise, the boundary fields have an effect on the spectrum of the bulk fields as well, which in general cannot be neglected, and the mass eigenstates are mixtures of bulk and boundary states. To find the exact spectrum of fermions, one would have to diagonalize the full kinetic operator for the entangled bulk and boundary fermions. Assuming however that the physical mass induced for the boundary fields is much smaller than the masses of the bulk fields, one can neglect the distortion on the spectrum of the latter and use the free kinetic operator (7) for bulk fermions and the results (13) and (14) for the boundary fields. In this approximation, the spectrum of bulk fields is unchanged and the mass of the boundary fields is obtained by looking at the zeros (13) and (14). Notice that the momentum dependence in the latter can be safely neglected within the adopted approximation. The masses m^a and the wave-functions $Z_{1,2}^a$ for left and right components then reduce to α -dependent parameters given by

$$m^u = \frac{\epsilon_1^u \epsilon_2^u}{\sqrt{2}\pi R} \text{Im} f_\delta(\lambda^u, 2\alpha) , \quad (15)$$

$$m^d = \frac{\epsilon_1^d \epsilon_2^d}{\pi R} \text{Im} f_\delta(\lambda^d, \alpha) , \quad (16)$$

$$Z_i^u = 1 + \delta_{i1} \frac{\epsilon_1^{d2}}{\lambda^d} \text{Re} f_0(\lambda^d, 0) + \delta_{i2} \frac{\epsilon_2^{u2}}{2\lambda^u} \text{Re} f_0(\lambda^u, 0) + \frac{\epsilon_i^{u2}}{2\delta_{i2}\lambda^u} \text{Re} f_0(\lambda^u, 2\alpha) , \quad (17)$$

$$Z_i^d = 1 + \frac{\epsilon_i^{d2}}{\lambda^d} \text{Re} f_0(\lambda^d, \alpha) + \delta_{i1} \frac{\epsilon_1^{u2}}{\lambda^u} \text{Re} f_0(\lambda^u, \alpha) . \quad (18)$$

Finally, the physical masses after symmetry breaking are obtained by rescaling the fields to canonically normalize them; they are given by

$$m_{\text{phys}}^a = \frac{m^a}{\sqrt{Z_1^a Z_2^a}} . \quad (19)$$

The arguments of [23] suggest that the effective actions induced for the boundary fields should be non-local from the $5D$ point of view and generated when $\langle A_5 \rangle$ acquires the VEV (2) from operators involving Wilson lines

$$W_n(A_5) = \mathcal{P} \exp \left\{ ig_5 \int_0^{n\pi R} dy A_5 \right\} . \quad (20)$$

Indeed, the k -th term in the series expansion (12) corresponds to the components of $W_{2|n|}(\langle A_5 \rangle) = T(|n|\alpha)$ with a winding number $n = 2k + \delta$; this connects the two

boundaries through a path that winds k times around the internal circle, with total length $|n|\pi R$. Since the boundary fields break explicitly G to H , the effective action will be only H -invariant and involve the various components of the decomposition of (20). However, it is nevertheless convenient to embed the boundary fields Q and q into G representations of the same type as those of the bulk fields, completing them with vanishing components. This description of the boundary fields is similar to the one that would emerge for the boundary values of bulk fields with $+$ and $-$ parities. More precisely, the boundary fields Q and q can be embedded into fundamental representations Q_F and q_F or symmetric representations Q_S and q_S of $SU(3)$ as follows:

$$Q_F = \begin{pmatrix} u_L \\ d_L \\ 0 \end{pmatrix}, \quad d_F = \begin{pmatrix} 0 \\ 0 \\ d_R \end{pmatrix}, \quad Q_S^c = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & d_R^c \\ 0 & 0 & -u_R^c \\ d_R^c & -u_R^c & 0 \end{pmatrix}, \quad u_S^c = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -u_L^c \end{pmatrix}. \quad (21)$$

In this notation, the couplings between boundary and bulk fermions can be obtained simply by taking traces of products of eqs. (21) with eqs. (57) and (62).

Using the embeddings (21), one can easily verify that the leading part of the effective action in a derivative expansion is given by the sum of the original action $\mathcal{L}_0 = \bar{Q}i\!\not{\partial}Q + \bar{u}i\!\not{\partial}u + \bar{d}i\!\not{\partial}d$ and the following non-local interactions $\mathcal{L}_F$ and $\mathcal{L}_S$ induced by the bulk fermions in the fundamental and symmetric representations respectively (going back to Minkowski space):

$$\begin{aligned} \mathcal{L}_F = & \sum_k e^{-|2k|\lambda^d} \left[\frac{\epsilon_1^{d2}}{\lambda^d} \bar{Q}_F W_{|2k|} i\!\not{\partial} Q_F + \frac{\epsilon_2^{d2}}{\lambda^d} \bar{d}_F W_{|2k|} i\!\not{\partial} d_F \right] \\ & + \sum_k e^{-|2k+\delta|\lambda^d} \frac{\epsilon_1^d \epsilon_2^d}{\pi R} \left[\bar{Q}_F W_{|2k+\delta|} d_F + \text{h.c.} \right], \end{aligned} \quad (22)$$

$$\begin{aligned} \mathcal{L}_S = & \sum_k e^{-|2k|\lambda^u} \left[\frac{\epsilon_1^{u2}}{\lambda^u} \text{Tr} W_{|2k|}^T \bar{Q}_S^c W_{|2k|} i\!\not{\partial} Q_S^c + \frac{\epsilon_2^{u2}}{\lambda^u} \text{Tr} W_{|2k|}^T \bar{u}_S^c W_{|2k|} i\!\not{\partial} u_S^c \right] \\ & + \sum_k e^{-|2k+\delta|\lambda^u} \frac{\epsilon_1^u \epsilon_2^u}{\pi R} \text{Tr} \left[W_{|2k+\delta|}^T \bar{Q}_S^c W_{|2k+\delta|} u_S^c + \text{h.c.} \right]. \end{aligned} \quad (23)$$

We conclude that it is possible to induce generic Yukawa couplings for boundary fields through non-local operators involving Wilson lines that connect the two boundaries and wind around the orbifold an arbitrary number of times. The resulting physical mass m_{phys}^a of the boundary fields is exponentially sensitive to the parameter λ^a governing the bulk masses. Notice also that, because of the wave-function rescaling, the value of m_{phys}^a given by (19) cannot be made arbitrarily large by increasing the values of the boundary couplings ϵ_i^a . Indeed, for $\epsilon_i^a \gg 1$, m_{phys}^a quickly saturates to a value depending only on ratios of these parameters but not on their overall size.

4 One-loop effective potential for the Higgs

The field A_5 couples only to the gauge fields and to the two bulk fermions. Its radiatively induced potential thus depends only indirectly on the boundary couplings through diagrams in which the virtual bulk fermions temporarily switch to a virtual boundary fermion. The total potential is therefore the sum of a universal gauge contribution and a parameter-dependent contribution coming from the fermions.

The contribution of the fermions is obtained by summing up all possible one-loop diagrams of bulk fermions dressed by an arbitrary number of external A_5 lines and insertions of boundary couplings. Since bulk interactions conserve the KK momentum, whereas boundary interactions do not, it is convenient to separately resum diagrams with no insertion of boundary interactions and diagrams with an arbitrary but non-zero number of these. The first piece corresponds to the contribution of bulk fields in the absence of boundary couplings, with kinetic operator (7), whereas the second can be reinterpreted as the contribution of boundary fermions propagating with the effective kinetic operators (13) and (14) induced by the insertions of mixings with the bulk field. This decomposition corresponds precisely to the shift performed in the last section to disentangle bulk and boundary fermions. In this case, however, this leads to an exact result, because both bulk and boundary fields are integrated out.

The bulk fields $\Psi_a^{(i)}$ couple to the gauge field A_5 in a diagonal way, through the shift induced in their KK mass by the minimal coupling. For each mode one has $m^{(i)} = (n + q^{(i)}\alpha)/R$, where $q^{(i)}$ is the charge of the mode as specified by eqs. (59) and (64). In total, there are only three kinds of modes with non-vanishing charges, two with $q = 1$ and one with $q = 2$. The contribution to the effective action from a pair of such modes for the $\Psi^{(i)}$ and $\tilde{\Psi}^{(i)}$ fields with given charge q reads $\Gamma^\Psi(q\alpha) = -\ln \det[K^\Psi(q\alpha)]$, where K^Ψ is the Euclidean continuation of (7). The determinant of K^Ψ as a two-by-two matrix yields $p^2 + m_n^2 + M^2$. The determinant in the space of KK modes then reduces to an infinite product over these factors, which yields an irrelevant α -independent divergence plus a finite α -dependent function. Finally, the determinant over spinor indices yields a trivial factor of 4. For a pair of modes with charge q , one then finds

$$\begin{aligned} V_{\Psi_a}(q\alpha) &= \frac{-1}{2\pi^6 R^4} \int_0^\infty dx x^3 \ln |f_1(x^a, q\alpha)|^{-2} \\ &= \frac{3}{8\pi^6 R^4} \sum_{k=1}^\infty \frac{1 + 2k\lambda^a + 4/3k^2\lambda^{a^2}}{k^5} e^{-2k\lambda^a} \cos(2qk\pi\alpha). \end{aligned} \quad (24)$$

The boundary fields $a = u, d$ couple to A_5 only through the non-local Wilson line effective interactions induced by the bulk fermions, and their kinetic operators $K^a(\alpha)$ in Euclidean space are given by (13) and (14). Their contributions to the effective action read $\Gamma^a(\alpha) = -\ln \det[K^a(\alpha)]$. To evaluate this expression, we start by using the standard trick of rewriting it in terms of the scalar quantity $K^a K^{aT}$. The

determinant over spinor indices then yields just a factor of 4, and dropping irrelevant constant terms one finds:

$$V_u(\alpha) = \frac{-1}{4\pi^6 R^4} \int_0^\infty dx x^3 \ln \left[\prod_{i=1}^2 \text{Re} \left[1 + \delta_{i1} \frac{\epsilon_1^{d2}}{x^d} f_0(x^d, 0) + \delta_{i2} \frac{\epsilon_2^{u2}}{2x^u} f_0(x^u, 0) + \frac{\epsilon_i^{u2}}{2^{\delta_{i2}} x^u} f_0(x^u, 2\alpha) \right] + \prod_{i=1}^2 \text{Im} \left[\frac{\epsilon_i^{u2}}{2^{\delta_{i2}} x} f_\delta(x^u, \alpha) \right] \right], \quad (25)$$

$$V_d(\alpha) = \frac{-1}{4\pi^6 R^4} \int_0^\infty dx x^3 \ln \left[\prod_{i=1}^2 \text{Re} \left[1 + \frac{\epsilon_i^{d2}}{x^d} f_0(x^d, \alpha) + \delta_{i1} \frac{\epsilon_1^{u2}}{x^u} f_0(x^u, \alpha) \right] + \prod_{i=1}^2 \text{Im} \left[\frac{\epsilon_i^{d2}}{x} f_\delta(x^d, \alpha) \right] \right]. \quad (26)$$

The full contribution of bulk and boundary fermions to the one-loop effective potential is finally given by

$$V_f(\alpha) = V_{\Psi_u}(\alpha) + V_{\Psi_u}(2\alpha) + V_{\Psi_d}(\alpha) + V_u(\alpha) + V_d(\alpha). \quad (27)$$

As expected, the result is finite, thanks to the exponentially soft UV behaviour of the functions f_δ . Notice also that the argument of the logarithm in the boundary contributions can be rewritten as the determinant of a two-by-two matrix given by the identity δ_{ij} , plus a matrix Δ_{ij} encoding interactions between the fixed-points located at y_i and y_j and involving the function $f_{\delta_{ij}}$ with $\delta_{ij} = (\pi R)^{-1} |y_i - y_j|$.

It is worth noting that the result (27) contains indirect information on the exact spectrum of the bulk and boundary fermions, which allows in fact to probe the accuracy of (19). The information about the spectrum of eigenvalues $\xi_n = \pi R m_n$ can be extracted by comparing our result (27) with the definition of the effective action as a trace over all the mass eigenstates of the logarithm of their free kinetic operators. Turning the sum into a product inside the logarithm, one would then obtain in this approach a logarithm argument proportional to $\prod_n (x^2 + \xi_n^2)$, which has zeros at $x = i\xi_n$. This means that the exact spectrum of eigenvalues can be obtained by setting the total logarithm argument in (27) to zero. One can verify that the boundary contribution has poles exactly where the bulk contribution has double zeros corresponding to the original tower of degenerate bulk modes. Half of the original zeros remain and correspond to the combination of bulk fields whose modes are not perturbed. The other orthogonal combination has a deformed mass spectrum, which is determined, together with the masses of the boundary fields, by solving the transcendental equation arising from the argument of the boundary contribution alone. The latter can be solved numerically, or analytically for the lightest modes ξ_0^a , under the assumption that $\xi_0^a \ll 1$. In that limit, the momentum dependence can be completely neglected and the logarithm argument reduces to $Z_1(\alpha)Z_2(\alpha)x^2 + (\pi R m^a)^2$, which leads to (19). This expression is thus valid as long as $\xi_0^a \ll 1$. If the latter condition is not satisfied,

the mixing between bulk and boundary modes can have a significant effect on the mass of the lowest-lying state, which has to be computed by numerically solving the exact transcendental equation defining the spectrum.

The contributions to the effective action from gauge and ghost fields are easily computed [12, 16]. Going again to a diagonal basis, two modes with $q = 1$ and one with $q = 2$ are found. Each contributes

$$\begin{aligned} V_g^A(q\alpha) &= \frac{3}{16\pi^6 R^4} \int_0^\infty dx x^3 \ln |f_1(x, q\alpha)|^{-2} \\ &= -\frac{9}{64\pi^6 R^4} \sum_{k=1}^\infty \frac{1}{k^5} \cos(2qk\pi\alpha) , \end{aligned} \quad (28)$$

which gives, in total:

$$V_g(\alpha) = 2V_g^A(\alpha) + V_g^A(2\alpha) . \quad (29)$$

The total one-loop effective potential is given by $V = V_f + V_g$. It satisfies the symmetry property $V(1 \pm \alpha) = V(\alpha)$. As expected, the bulk-to-boundary couplings ϵ_i^a deform the potential in a non-trivial way; it is thus interesting to analyse the possible minima one can get in this case. Notice first that standard 5D bulk fermions in fundamental or symmetric representations, combined with gauge bosons, can lead to non-trivial minima for the values $\alpha = 0.5$ and $\alpha \sim 0.3$ respectively. In our situation, however, lower values can be obtained, thanks to the effect of the boundary interactions. This is easily understood by noticing that for $\alpha \ll 1$ both V_a and V_g increase with α , whereas V_{Ψ_a} decreases as α increases; the boundary contribution therefore tends to shift the minimum of V to lower values of α . Furthermore, both fermion contributions are very sensitive to λ^a and decrease exponentially with λ^a , whereas the dependence on the brane-to-bulk couplings ϵ_i^a of the boundary fermion potential is mild. At fixed ϵ_i^a , the dependence of V on the λ^a 's, which we assume to be equal to some common value λ for simplicity, is as follows. For $\lambda = 0$, V_{Ψ_a} dominates and we get $\alpha \sim 0.3$, roughly the same value as in the case of decoupled 5D massless fermions. As λ is increased, V_{Ψ_a} and V_a decrease and the minimum moves to lower values of α , down to $\alpha \sim 0.2$, the precise value depending on the ϵ_i^a couplings. When λ further increases, V_g eventually dominates and the only minimum that is left is the trivial one at $\alpha = 0$. We have performed a numerical study of V to determine the lowest values of α that can be achieved in this setting. We were able to find minima for $\alpha \sim 0.1 - 0.2$ for a wide range of the parameters λ^a and ϵ_i^a (see fig. 1).

Let us conclude with a comment on divergences. The above computation proves that no divergences are induced for the Higgs mass at the one-loop level, neither on the bulk nor on the boundary. There is actually a residual gauge invariance at the fixed-points, $A_5^a \rightarrow A_5^a + \partial_5 \xi^a$, with $a \in G/H$ and ξ^a the corresponding gauge parameters, which forbids any local boundary mass term for $A_{5,0}^7$ [10], and this shift symmetry is not broken by the localized couplings in (5). This implies that no direct

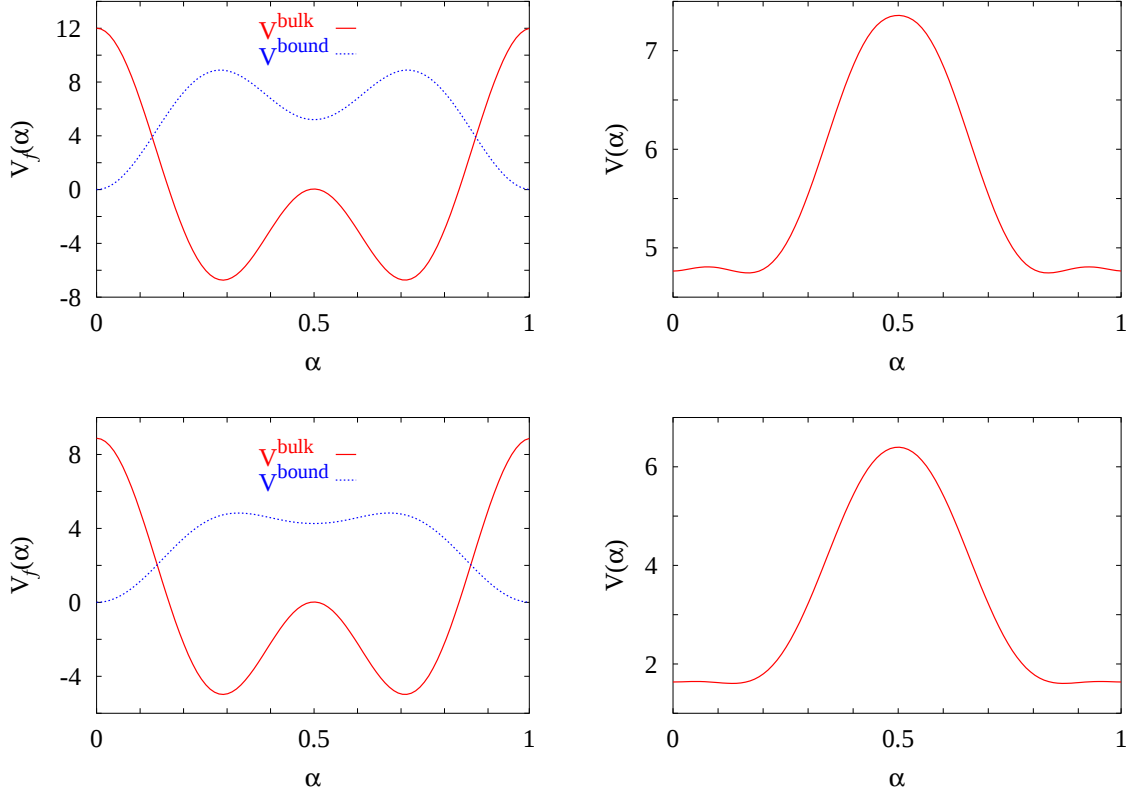


Figure 1: Different contributions to the effective potential (in arbitrary units): the bulk and boundary fermion contributions (upper left) and the full potential (upper right) for $\lambda = 1.57$, $\epsilon_1 = 3.1$, $\epsilon_2 = 0.7$ and $\delta = 0$; the bulk and boundary fermion contributions (lower left) and the full potential (lower right) for $\lambda = 1.83$, $\epsilon_1 = 6.4$, $\epsilon_2 = 6.1$ and $\delta = 1$.

divergence can appear in the potential at any order in perturbation theory. There will however be linearly divergent wave-function corrections for the gauge and Higgs fields, which can substantially influence the physical potential. It should be noticed, however, that these effects are G -symmetric, and G -violating quantities that are only H -symmetric are therefore completely insensitive to them and *finite*. Particularly important examples of such quantities are the ratios m_W/m_H or m_Z/m_H , since both the gauge fields W, Z and the Higgs field H are gauge fields from the higher-dimensional point of view, and hence receive a common wave-function correction. At leading order, these ratios depend only on α and therefore represent predictable quantities for the model. They can in principle be influenced by non-renormalizable operators, so that the accuracy of their leading-order values are controlled by the effective theory expansion parameter $(\Lambda R)^{-1}$. However, the very peculiar symmetries constraining A_5 do not allow for any higher-dimensional operator that could have a relevant effect at tree- or one loop-level.

5 A prototype 5D model

Having shown how a satisfactory mechanism for electroweak and flavour symmetry breaking can be achieved, we now turn to the construction of a simple prototype model in five dimensions. The first concern in model building is to introduce heavy bulk fermions for each pair of left- and right-handed SM fermions, so as to obtain all the required Yukawa couplings for the SM matter (including neutrinos) through the mechanism explained in section 3. The strong-interaction sector is completely factorized and $SU(3)_c$ therefore does not matter. The true constraint comes from the electroweak-interaction sector and is fixed by the $SU(2)_L \times U(1)_Y$ quantum numbers of the SM fields; the Higgs scalar H is a $\mathbf{2}_{1/2}$, the quarks Q_L , u_R , d_R are in the $\mathbf{2}_{1/6}$, $\mathbf{1}_{2/3}$, $\mathbf{1}_{-1/3}$, and the leptons L_L , l_R , ν_R in the $\mathbf{2}_{-1/2}$, $\mathbf{1}_{-1}$, $\mathbf{1}_0$.

There are many possibilities for gauge groups unifying electroweak and Higgs interactions, but we will stick to the basic structure in which a $SU(3)$ group is broken to $SU(2) \times U(1)$ through a $\mathbf{Z}_2$ orbifold projection. The decomposition of the simplest $SU(3)$ representations under this projection is as follows: the adjoint representation decomposes as $\mathbf{8} = \mathbf{3}_0 \oplus \mathbf{2}_{1/2} \oplus \bar{\mathbf{2}}_{-1/2} \oplus \mathbf{1}_0$, the fundamental as $\mathbf{3} = \mathbf{2}_{1/6} \oplus \mathbf{1}_{-1/3}$, the symmetric as $\mathbf{6} = \mathbf{3}_{1/3} \oplus \mathbf{2}_{-1/6} \oplus \mathbf{1}_{-2/3}$, and finally the rank-three symmetric as $\mathbf{10} = \mathbf{4}_{1/2} \oplus \mathbf{3}_0 \oplus \mathbf{2}_{-1/2} \oplus \mathbf{1}_{-1}$.

The simplest possibility is to take $G = SU(3)_w$ and $H = SU(2)_L \times U(1)_Y$. In this case, bulk fermions in the fundamental, symmetric, rank-three symmetric and adjoint representations would have the right charges to couple respectively to the down quarks, conjugate up quarks, charged leptons and conjugate neutrinos. However, this minimal choice of the gauge group would lead to too high a weak mixing angle: $\sin^2 \theta_W = 3/4$. A possible cure to this problem consists in adjusting θ_W by introducing different localized kinetic terms for $SU(2)_L$ and $U(1)_Y$ gauge bosons at the fixed-points of the orbifold. The distortion they cause introduces however other problems, as we will discuss in the next section. In addition, the computation of the Higgs potential is complicated by the presence of non-universal localized gauge couplings.

An alternative way of tuning θ_W to a reasonable value is to add an extra overall $U(1)'$ factor, with coupling constant g' , which remains unaffected by the orbifold projection, and identify the $U(1)_Y$ hypercharge as the sum of the $U(1)$ and $U(1)'$ charges after the orbifold projection $SU(3)_w \times U(1)' \rightarrow SU(2)_L \times U(1) \times U(1)'$. In this way the gauge field A_Y associated to the hypercharge and its orthonormal combination A_X are

$$A_Y = \frac{g' A_8 + \sqrt{3} g A'}{\sqrt{3g^2 + g'^2}}, \quad A_X = \frac{\sqrt{3} g A_8 - g' A'}{\sqrt{3g^2 + g'^2}}, \quad (30)$$

implying that $g_Y = \sqrt{3}gg'/\sqrt{3g^2 + g'^2}$ and thus

$$\sin^2 \theta_W = \frac{g_Y^2}{g^2 + g_Y^2} = \frac{3}{4 + 3g^2/g'^2} . \quad (31)$$

By appropriately choosing the ratio g/g' we can then restore the correct value of $\sin^2 \theta_W$. After electroweak symmetry breaking, the following mass term for the gauge fields is induced from their gauge kinetic term:

$$\mathcal{L}_m = \left(\frac{\alpha}{R}\right)^2 \left[|W|^2 + \frac{1}{2 \cos^2 \theta_W} \left(Z - \sqrt{3 - 4 \sin^2 \theta_W} A_X \right)^2 \right] . \quad (32)$$

In order for the model to be realistic, the gauge boson A_X must of course acquire a large mass by some mechanism. As we shall discuss below, its associated $U(1)_X$ symmetry is actually anomalous, and is therefore naturally expected to be spontaneously broken at the cut-off scale, with a mass term

$$\mathcal{L}'_m \simeq \frac{1}{2} \Lambda^2 (A_X)^2 . \quad (33)$$

An important consequence of (33) is that the mixing mass term arising in (32) between Z and A_X has a negligible effect. The corresponding distortion of the ρ parameter can be quantified by integrating out the heavy A_X gauge boson: this leaves a correction of order $(m_Z/\Lambda)^2$, which is safely small since Λ is experimentally constrained to be above a few TeV.

Bulk fermions in fundamental and symmetric representations allow couplings to all the matter fermions, since their $U(1)'$ charge can be tuned to achieve the required hypercharge; the way in which the hypercharge of the SM field is distributed as sum of $U(1)$ and $U(1)'$ charges is then completely fixed. In this model, one can thus implement the construction described in the section 3, without any additional complication. It turns out that four bulk fermions Ψ_a with $a = d, u, l, \nu$ (plus their mirrors $\tilde{\Psi}_a$ with opposite parities) do the job, the quantum numbers of bulk and brane fields with respect to $SU(3)_c \times SU(3)_w \times U(1)'$ and $SU(3)_c \times SU(2)_L \times U(1) \times U(1)'$ being as follows:

$$\begin{aligned} \Psi_d : (\mathbf{3}, \mathbf{3})_0^+ , & \quad \text{couples to } Q_L : (\mathbf{3}, \mathbf{2})_{1/6,0} \text{ and } d_R : (\mathbf{3}, \mathbf{1})_{-1/3,0} , \\ \Psi_u : (\bar{\mathbf{3}}, \mathbf{6})_0^- , & \quad \text{couples to } Q_R^c : (\bar{\mathbf{3}}, \mathbf{2})_{-1/6,0} \text{ and } u_L^c : (\bar{\mathbf{3}}, \mathbf{1})_{-2/3,0} , \\ \Psi_l : (\mathbf{1}, \mathbf{3})_{-2/3}^+ , & \quad \text{couples to } L_L : (\mathbf{1}, \mathbf{2})_{1/6,-2/3} \text{ and } l_R : (\mathbf{1}, \mathbf{1})_{-1/3,-2/3} , \\ \Psi_\nu : (\mathbf{1}, \mathbf{6})_{2/3}^- , & \quad \text{couples to } L_R^c : (\mathbf{1}, \mathbf{2})_{-1/6,2/3} \text{ and } \nu_L^c : (\mathbf{1}, \mathbf{1})_{-2/3,2/3} . \end{aligned} \quad (34)$$

To achieve the most general flavour structure, we introduce three generations $I = 1, 2, 3$ of the above bulk fields. These can have arbitrary kinetic matrices $(M_a)_{IJ}$ in flavour space, possibly different for $a = d, u, l, \nu$. The couplings of the bulk fermions

to the three generations of SM boundary fields can involve generic matrices $(\epsilon_i^a)_{IJ}$ in flavour space. However, these can be made proportional to the identity through a rotation of the bulk fields, whose only additional consequence will be to change the kinetic matrices $(M_a)_{IJ}$. Without loss of generality, we can therefore set the couplings to flavour-blind constants, parametrized by dimensionless coefficients ϵ_1^a and ϵ_2^a for left-handed and right-handed fields.

5.1 Mass matrices

The computation of the induced masses for the SM matter fields proceeds exactly as in section 3, the only novelty being the non-trivial flavour structure of the bulk masses. The latter can be written as $M_a = E_a^\dagger M_a^D F_a$, where E_a, F_a are unitary matrices and M_a^D is diagonal, and the kinetic term of the bulk fields can thus be diagonalized in flavour space by redefining $\Psi'_a = E_a \Psi_a$ and $\tilde{\Psi}'_a = F_a \tilde{\Psi}_a$. By doing so, one gets a diagonal parameter $\lambda^a = \pi R M_a^D$ for bulk fields, but the couplings between boundary fields and the bulk fields Ψ'_a become non-diagonal and involve the matrices E_a . The right-handed part of the boundary fields a , which couple only to the corresponding bulk fermion Ψ'_a , can be diagonalized by redefining $a'_R = U_a a_R$ with $U_{d,l} = E_{d,l}$ and $U_{u,\nu} = E_{u,\nu}^*$. For the left-handed fields, instead, this cannot be done, since each of them couples to two different bulk fields; this will be a first source of non-trivial mixing in the mass matrices. At this point, all the couplings are diagonal, except those mixing the boundary fields Q_L, L_L, Q_R^c and L_R^c to the bulk fields $\Psi'_d, \Psi'_l, \Psi'_u$ and Ψ'_ν , which involve the matrices U_d, U_l, U_u and U_ν .

The presence of the matrices U_a affects the results of section 3 in the following way. The contribution to the wave function Z_1^a of the left-handed field a_L from the bulk field Ψ'_b , call it $Z_1^a(\Psi'_b)$, is changed to $\tilde{Z}_1^a(\Psi'_b) = U_b^\dagger Z_1^a(\Psi'_b) U_b$, and the new total wave function $\tilde{Z}_1^a = \sum_b \tilde{Z}_1^a(\Psi'_b)$ is not diagonal. The wave-function corrections Z_2^a for the right-handed fields a_R are instead unchanged and diagonal: $\tilde{Z}_2^a = Z_2^a$. Finally, the mass m^a induced for the boundary fields is changed to $\tilde{m}^a = m^a U_a$. In order to determine the new physical mass, one has to diagonalize the kinetic term of the left-handed fields. This is achieved through a unitary transformation; writing $\tilde{Z}_1^a = V_a Z_1^{D^a} V_a^\dagger$, with $Z_1^{D^a}$ diagonal, one can redefine the left-handed fields as $a'_L = V_a^\dagger a_L$, to obtain a kinetic term for boundary fields which is diagonal in flavour space. The wave functions are now all diagonal and given by $Z_1^{D^a}$ and Z_2^a , whereas the mass matrices are all non-diagonal and given by $m^a U_a V_a$. Rescaling finally the wave-function factors, one finds the following physical mass matrices:

$$(m_{\text{phys}}^a)_{IJ} = \frac{m_{II}^a (V_a U_a)_{IJ}}{\sqrt{(Z_2^a)_{II} (Z_1^{D^a})_{JJ}}} \quad (\text{no sum on } I, J). \quad (35)$$

5.2 Anomalies

We now briefly comment on the issue of anomalies, paying attention to their distribution over the internal space. Since the bulk fermions are strictly vector-like, the only anomalies that can arise come from the SM fermions living at the fixed-points, and depend on how these are distributed among the two different fixed-points.

In the case where all SM fermions are located at the same fixed-point, all anomalies that do not involve the extra $U(1)_X$ gauge field vanish, thanks to the usual cancellations arising for the SM spectrum of fermions. We are then left with localized mixed anomalies involving the $U(1)_X$ gauge field, which can be cured by means of a 4D version of the Green–Schwarz mechanism (GS) [24]. One introduces a neutral 4D axion at $y = 0$, transforming non-homogeneously under the $U(1)_X$ symmetry, with non-invariant 4D Wess–Zumino couplings compensating for the one-loop anomaly. In this way all mixed $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$ gauge and gravitational anomalies are cancelled and the axion is eaten by the $U(1)_X$ gauge boson, which becomes massive and decouples.

For other distributions of the SM fermions such that all the SM anomalies are still cancelled locally, a GS mechanism is again sufficient to cancel all remaining anomalies involving the $U(1)_X$ gauge field. However, two neutral axions are now needed, one at $y = 0$ and one at $y = \pi R$, with non-invariant 4D Wess–Zumino couplings. One combination of axions is again eaten by the $U(1)_X$ gauge boson, but the other combination remains as a physical massless axion in the low-energy spectrum.

For a completely generic distribution of matter, for which not even the SM anomalies are locally cancelled, the situation is more complicated. In order to locally cancel the SM anomalies, one has to introduce a bulk Chern–Simons term with jumping coefficient [25]–[27], which can be naturally generated by integrating out certain massive states [28]. Since the hypercharge is embedded into a non-Abelian group in the bulk, this is however not sufficient to let all of the anomalies flow on a single fixed-point. One then has to introduce also two neutral axions, one at each fixed-point, to locally cancel all the remaining anomalies involving $U(1)_X$. As before, the $U(1)_X$ gauge boson gets a mass, but a combination of axions remains massless.

5.3 Quantitative analysis

This simple 5D orbifold model we constructed has all the qualitative features to represent a possible interesting extension of the SM, where the electroweak scale is stabilized without supersymmetry and the hierarchy of fermion masses is explained by the non-local origin of the Yukawa couplings. As mentioned at the end of section 4, the quantities m_H , m_W and m_Z , and especially their ratios, can be reliably computed,

having a mild dependence on the cut-off Λ . In order to get a better understanding of the range of validity of our model as an effective field theory, it is however necessary to know the magnitude of Λ . A reasonable estimate is obtained by defining Λ as the energy scale where the basic higher-dimensional gauge interactions become strong. NDA then yields $\Lambda \sim (l_D/g_D^2)^{1/(D-4)}$, where g_D is the higher-dimensional gauge coupling and $l_D = (4\pi)^{D/2}\Gamma(D/2)$ in D space-time dimensions. The predictive power of the effective theory at the compactification scale M_c is therefore governed by the 4D effective coupling $g_4 = g_D M_c^{(D-4)/2}$, since the small parameter controlling corrections due to non-renormalizable operators is given by $M_c/\Lambda \sim (g_4^2/l_D)^{1/(D-4)}$, and can thus be lowered by effects that tend to increase the effective gauge coupling⁴.

In our 5D model, the loop factor is given by $l_5 = 24\pi^3$ and the 5D and 4D gauge couplings are related by $g_4 = g_5/\sqrt{2\pi R}$, so that $\Lambda \sim (12\pi^2)/g_4^2$. Considering respectively the strong and the weak interactions, this would give roughly $\Lambda_c \sim 10/R$ and $\Lambda_w \sim 100/R$. This means that Λ can be identified with Λ_c and the theory is indeed reasonably predictive. In particular, the universal wave-function corrections for the H , Z and W fields are proportional to $(g_{5,w}^2/l_5)\Lambda$, that is Λ_c/Λ_w , and therefore represent small corrections that can be neglected. We can then go a step further and ask whether this minimal 5D model is also quantitatively a phenomenologically viable model. As we will see, this is not quite the case, because the values predicted by the model for $1/R$, m_H and m_{top} are too low.

The crucial parameter that sets the scale of the model is α , whose value is determined by minimizing the full effective potential $V(\alpha)$, as in section 4. Generically, the most relevant fermionic contribution to the potential is induced by the bulk fermion in the symmetric of $SU(3)_w$ that gives mass to the top quark, as expected. Neglecting the effect of the other bulk and boundary fermions, we have numerically analysed the form of $V(\alpha)$ as a function of λ^{top} and of the bulk-to-boundary couplings ϵ_i^{top} . As discussed in the previous section, the lowest non-trivial value for α that we get is $\alpha \simeq 0.16$ for $\delta = 0$ and $\alpha \simeq 0.12$ for $\delta = 1$ (see fig. 1), which by means of (5) implies $1/R \sim 500$ GeV. This is in conflict with experimental bounds for models such as ours, with localized interactions that do not conserve KK momentum, which require roughly $1/R \sim \text{few TeV}$ [18]. The Higgs mass computed using (6) is also too low, at most $m_H \sim 30$ GeV. Finally, the top Yukawa coupling arising from (15)–(18) turns out to be too small for any value of ϵ_i^{top} and λ^{top} , giving a bound $m_{\text{top}} \leq 65$ GeV. We actually have also evaluated the top mass by numerically solving the exact transcendental equation arising from the effective potential, as discussed in section 4, but the deviations from the approximate relations (15)–(18) turn out to be very small and negligible.

It is interesting to notice that all these problems could be alleviated if $V(\alpha)$ had

⁴Of course Λ is always larger than M_c , since at lower energies the theory returns 4D, and the above estimate for Λ is accurate only as long as $\Lambda \gg M_c$.

minima at lower α . For $\alpha \sim 0.01$, for instance, $1/R$ would be well above the current experimental bounds and m_H would increase up to more than 100 GeV. The top mass slightly increases, but still $m_{\text{top}} \leq 110$ GeV. This is not yet completely satisfactory, but it goes in the right direction. One should also remember that these predictions, in particular the top mass, could be affected by large corrections if the cut-off of the model is low. One should also remind that all our analysis is based on minimizing the one-loop effective potential $V(\alpha)$, and it is not easy to estimate how much higher-loop contributions to $V(\alpha)$ can alter the actual value of α . The latter, as we have seen, is the crucial parameter in these models, and it would be extremely interesting to find a mechanism that gives rise to a potential with such low values of α without spoiling the nice features of the model.

5.4 Possible extensions

As anticipated in the introduction, a possible way to lower α is to consider 5D massive bulk fermions in large $SU(3)_w$ representations. For example one can take a completely symmetric representation of large rank r , with dimension $d(r) = (r+1)(r+2)/2$. This contains components of charge $q = k$, where k is an integer ranging from 0 to r . The multiplicities of the charged states with $k \neq 0$ are found to be $N_k = 1 + [(r-k)/2]$, where $[\dots]$ denotes the integer part. This information allows us to compute the contribution of this field to the effective potential by summing up the contributions of all the charged components computed with eq. (24). We find that if the rank $r \gg 1$ and the parameter λ controlling the mass is not too large, we can have minima for $\alpha \sim 1/r$, and furthermore the second derivative V'' at the minimum grows very quickly with r , leading indeed to a substantial improvement of the situation. With $r = 6$, $\lambda \sim 2.2$ and $\delta = 1$, we obtain $\alpha \sim 0.13$, corresponding to $1/R \sim 0.6$ TeV and $m_H \sim 104$ GeV, while for $r = 8$, $\lambda \sim 3.5$ and $\delta = 0$, we get $\alpha \sim 0.096$, corresponding to $1/R \sim 0.8$ TeV and $m_H \sim 112$ GeV (see fig. 2).

It should however be noted that matter fields in large representations of the gauge group will induce electroweak quantum corrections that are enhanced by large group-theoretical factors $T(r)$. The scale at which the weak coupling becomes non-perturbative is therefore lowered: $\Lambda_w \rightarrow \Lambda_w/T(r)$. It is difficult to give a precise quantitative estimate of $T(r)$, because it is not universal. To get an order of magnitude, one can use the Dynkin index of the representation, which in our case is found to be $T(r) = r(r+1)(r+2)(r+3)/48$ [29]. This shows that Λ_w rapidly decreases as r increases too much. When Λ_w becomes comparable with Λ_c , the wave-function corrections to the physical masses get out of control, and only ratios of these masses can be predicted, as long as Λ_w does not get too close to $1/R$. We believe that values up to $r \sim 10$ could be reasonable.

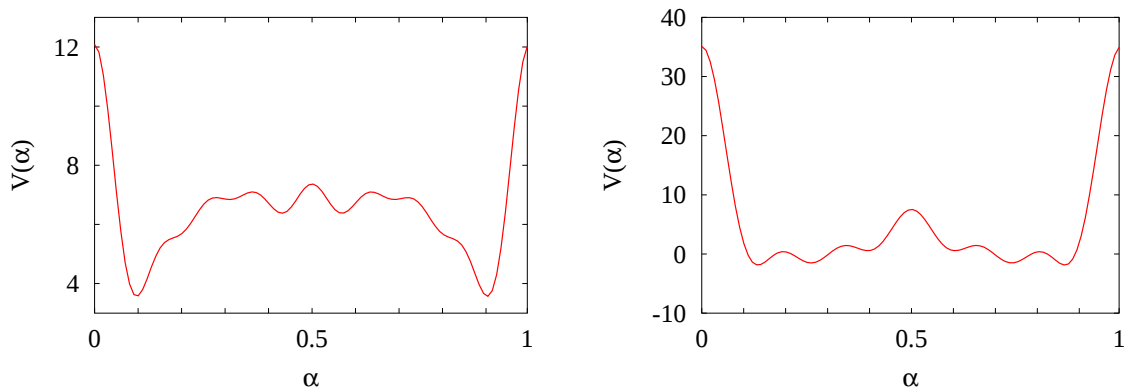


Figure 2: The full effective potential (in arbitrary units) in the presence of high-rank bulk fermions. Left: $r = 8$, $\lambda = 3.47$, $\epsilon_1 = \epsilon_2 = 9$ and $\delta = 0$, resulting in $m_H = 112$ GeV and $1/R = 830$ GeV. Right: $r = 6$, $\lambda = 2.23$, $\epsilon_1 = 7$, $\epsilon_2 = 1$ and $\delta = 1$, resulting in $m_H = 104$ GeV and $1/R = 600$ GeV.

6 Localized gauge kinetic terms

In the orbifold models we consider, gauge fields have certainly a bulk kinetic term, but no symmetry forbids the occurrence of additional localized kinetic terms. It is therefore interesting to consider the general case in which all of these are present with arbitrary coefficients, and study the consequences on model building. This kind of situation was first considered in [30], in the context of non-compact higher-dimensional theories, and more recently in [19] (see also [31]) for compact orbifolds⁵. We will consider for simplicity H -universal localized kinetic terms for 4D gauge fields only, described by two couplings $l_{1,2}$ with mass dimension -1 . Localized terms involving A_5 are allowed, but their presence considerably complicates the analysis, and we therefore discard them. Denoting 5D and 4D indices with M, N and μ, ν respectively, the Lagrangian for the $SU(3)_w$ gauge fields is then given by

$$\mathcal{L}_g = -\frac{1}{2} \text{Tr} F_{MN} F^{MN} - \left(l_1 \delta(y) + l_2 \delta(y - \pi R) \right) \text{Tr} F_{\mu\nu} F^{\mu\nu}. \quad (36)$$

As usual, it will be convenient to introduce dimensionless parameters relating the coefficients of the localized kinetic terms to the length of the orbifold: $c_i = (\pi R)^{-1} l_i$. For simplicity, we do not add localized terms associated with the $U(1)'$ gauge field. It actually turns out that their main effect would be a simple rescaling of the coupling constant g' and so we do not lose in generality by discarding them.

⁵Localized gauge kinetic terms do also naturally occur at tree level in certain string theory models; see e.g. [32].

6.1 Spectrum

The wave functions and KK spectrum for the 4D gauge fields $A_\mu^{(i)}$ are distorted by the localized couplings appearing in (36). The effect of the non-vanishing VEV α is most conveniently taken into account by adopting the point of view of twisted boundary conditions, in which each component with definite charge q (in a diagonal basis) satisfies $f_n(y + 2\pi R, q\alpha) = e^{2i\pi q\alpha} f_n(y, q\alpha)$. The differential equation defining the wave functions $f_n(y, q\alpha)$ of the mode with mass $m_n(q\alpha)$ is found by proceeding as in [19]. It reads:

$$\left\{ \partial_y^2 + m_n^2(q\alpha) \left[1 + 2\pi R c_1 \delta(y) + 2\pi R c_2 \delta(y - \pi R) \right] \right\} f_n(y; q\alpha) = 0. \quad (37)$$

The general solution of this equation in the interval $[-\pi R, \pi R]$ has the form

$$f_n(y; q\alpha) = \mathcal{N}_n(q\alpha) \begin{cases} \cos(m_n y) + \beta_n^- \sin(m_n y), & y \in [-\pi R, 0] \\ \cos(m_n y) - \beta_n^+ \sin(m_n y), & y \in [0, \pi R] \end{cases}. \quad (38)$$

The constant $\mathcal{N}_n$ is a normalization factor defined in such a way that $\int_0^{2\pi R} |f_n|^2 dy = 1$. The parameters $\beta_n^\pm$ are fixed by the twisted boundary conditions and the discontinuity at $y = 0$, and read:

$$\beta_n^\pm = e^{\pm i\pi q\alpha} \sec(\pi q\alpha) (\pi R m_n) c_1 \mp i \tan(\pi q\alpha) \cot(\pi R m_n). \quad (39)$$

Finally, the spectrum is determined by the discontinuity at $y = \pi R$, which enforces the following transcendental equation for the dimensionless eigenvalues $\xi_n = \pi R m_n$:

$$2(1 - c_1 c_2 \xi_n^2) \sin^2 \xi_n + (c_1 + c_2) \xi_n \sin 2\xi_n - 2 \sin^2(\pi q\alpha) = 0. \quad (40)$$

For $\alpha = 0$, eqs. (38)–(40) reduce to the equations derived in [19]. For $c_i = 0$, they also correctly reduce to the case of twisted gauge bosons; in particular, the solution of eq. (40) is then $\xi_n = \pi(n + q\alpha)$.

The deformation induced on the mass spectrum is one of the most important effects of localized kinetic terms. The mass of the lightest modes can be determined by solving the transcendental equation (40) in the limit $\xi_0 \ll 1$. One finds in this way $\xi_0 \sim \sin(\pi q\alpha)/\sqrt{1 + c_1 + c_2}$, and self-consistency of the assumption $\xi_0 \ll 1$ requires that $\alpha \ll 1$ and/or $c_i \gg 1$. This expression must be compared with the value $\xi_0 = \pi q\alpha$ of the standard case $c_i = 0$, and shows that the relation between masses of light gauge boson modes with different charges q is distorted if $c_i \neq 0$. The masses of heavy KK modes ($n \geq 1$) are deformed as well, and tend to become lighter; one finds that $\xi_n \rightarrow (n - 1)$ if $c_1 \sim c_2$ whereas $\xi_n \rightarrow (n - 1/2)$ if $c_1 \gg c_2$ or vice versa.

The deformation induced on the wave functions of the KK modes is also particularly relevant, because it affects the concept of 4D effective gauge coupling constant, which is no longer universal. More precisely, one can define an effective

gauge coupling $g_{4,q,n}(y)$ between matter fields localized at $y = 0, \pi R$ and the n -th KK mode of the gauge bosons with charge q ; defining the quantity $Z_n(q\alpha) = 1 + 2\pi R c_1 |f_n(0; q\alpha)|^2 + 2\pi R c_2 |f_n(\pi R; q\alpha)|^2$, this is found to be

$$g_{4,q,n}(y) = g_5 \frac{|f_n(y; q\alpha)|}{\sqrt{Z_n(q\alpha)}}. \quad (41)$$

Similarly, one can define a coupling g_{4,q_i,n_i} between three gauge bosons with KK modes n_i and charges q_i , which is given by⁶

$$g_{4,q_i,n_i} = g_5 \int_0^{2\pi R} dy \left[1 + 2\pi R c_1 \delta(y) + 2\pi R c_2 \delta(y - \pi R) \right] \prod_{i=1}^3 \frac{|f_{n_i}(y; q_i\alpha)|}{\sqrt{Z_{n_i}(q_i\alpha)}}. \quad (42)$$

The above equations describe an important distortion of the gauge coupling. In particular, the strength of the coupling depends on the type of modes and their location. Contrary to what happens for $\alpha = 0$ [19], eqs. (41) and (42) represent a distortion also for the zero-mode couplings, because the wave function f_0 becomes non-constant and q -dependent for $\alpha \neq 0$.

6.2 Contribution to the effective potential

The contributions of gauge fields to the Higgs effective potential are modified by the localized kinetic terms as well, and must be recomputed. It is convenient to use the background gauge-fixing condition $\bar{D}_M A^M = \partial_M A^M + i g_5 [\bar{A}_M, A^M] = 0$, where $\bar{A}_M = \delta_{M5}(\alpha/g_5 R)\lambda^7$ is the background field. This gauge-fixing is not affected by the localized boundary terms and the ghost fields η therefore have only a bulk kinetic term. After gauge-fixing, the bulk and ghost kinetic operators are diagonal and proportional to $\bar{D}^P \bar{D}_P$, but the boundary kinetic operator involves the transverse projector $\bar{D}^\rho \bar{D}_\rho \eta^{\mu\nu} - \bar{D}^\mu \bar{D}^\nu$. As usual, a change of basis is required to diagonalize the couplings to A_5 , and there are two modes with charge $q = 1$ and one mode with charge $q = 2$, both for ghost and gauge fields. After KK decomposition, the kinetic operator of the ghosts and the internal component of the gauge fields have a simple diagonal form, whereas the one of the four-dimensional components of the gauge fields is deformed in a non-trivial way. They are given by

$$K_{mn}^{A_5, \text{gh}} = \delta_{m,n} (p^2 + m_n^2), \quad (43)$$

$$K_{mn, \mu\nu}^{A_\mu} = \delta_{m,n} \eta_{\mu\nu} (p^2 + m_n^2) + (c_1 + (-1)^{m+n} c_2) (\eta_{\mu\nu} p^2 - p_\mu p_\nu). \quad (44)$$

The contribution to the effective action of each type of mode with fixed charge q is given by $\Gamma(q\alpha) = 1/2 \ln \det[K^{A_\mu} K^{A_5} (K^{\text{gh}})^{-2}(q\alpha)]$. The determinant over the KK

⁶In (42), the integral over y should be defined in the interval $[-\epsilon, 2\pi R - \epsilon]$ to correctly normalize the boundary contributions.

and vector indices can be explicitly performed. For K^{A_5} and K^{gh} this is trivial; for K^{A_μ} , the KK part can be done by considering a finite-dimensional truncation and recursively increasing the dimensionality, and the vector part just produces a factor of 3 in the exponent coming from the trace of the transverse projector. The result is finally

$$\text{Det} \left[\frac{K^{A_M}}{(K^{\text{gh}})^2} \right] = \prod_n (p^2 + m_n^2)^3 \left[\prod_{i=1}^2 \left(1 + c_i \sum_n \frac{p^2}{p^2 + m_n^2} \right) - \prod_{i=1}^2 \left(c_i \sum_n \frac{p^2 (-1)^n}{p^2 + m_n^2} \right) \right]^3. \quad (45)$$

As for the fermions, the contribution of each mode to the potential naturally splits into the standard bulk part and a boundary part encoding the effects of the localized interactions. The bulk part V_g^A is given by eq. (28). The boundary corrections can instead be written as

$$V_g^{c_i}(q\alpha) = \frac{3}{16\pi^6 R^4} \int_0^\infty dx x^3 \ln \left[\prod_{i=1}^2 \text{Re} [1 + c_i x f_0(x, q\alpha)] - \prod_{i=1}^2 \text{Re} [c_i x f_1(x, q\alpha)] \right]. \quad (46)$$

The total contribution of gauge fields to the one-loop effective potential is finally obtained by summing up the contributions of the three charged modes; it is given by

$$V_g(\alpha) = 2V_g^A(\alpha) + V_g^A(2\alpha) + 2V_g^{c_i}(\alpha) + V_g^{c_i}(2\alpha). \quad (47)$$

Again, the result is finite, thanks to the exponentially soft UV behaviour of the functions f_δ . Notice, moreover, that for $c_i \gg 1$, the α -dependence in the boundary contribution tends to exactly cancel the α -dependence in the bulk contribution (see fig. 3). This is most easily seen by putting (28) and (46) together and simplifying the argument of the logarithm, which becomes $\prod_i (\cosh x + c_i x \sinh x) - \cos^2(\pi\alpha)$. Note that this expression is proportional to eq. (40) after the analytic continuation $x \rightarrow i\xi_n$, and has therefore the same zeros. This constitutes a non-trivial consistency check of our result (47). Indeed, the latter could have been computed as the effective action of the new eigenstates, which would have led to a logarithm with an argument proportional to $\prod_n (x^2 + \xi_n^2)$, with zeros at $x = i\xi_n$. In this approach, however, performing the product over the KK modes is non-trivial (see for instance [33]). It is also interesting to observe that, as in the case of the fermions, the boundary contribution to the effective potential can be rewritten in terms of the determinant of a two-by-two matrix encoding the fixed-point-to-fixed-point propagation. The localized gauge kinetic terms (36) do not break the shift symmetry $A_5^a \rightarrow A_5^a + \partial_5 \xi^a$, and thus no direct divergence is expected in the gauge contribution to the potential at any order in perturbation theory.

6.3 Effects on model building

The deformations of the mass spectrum, gauge coupling constants and induced effective potential that we have described in the last two subsections have important

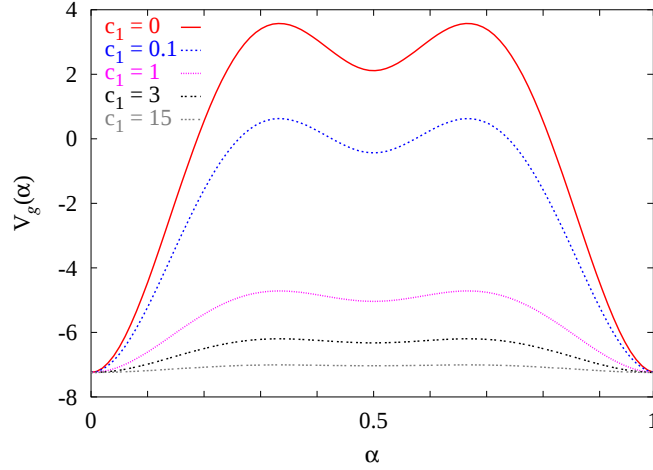


Figure 3: Gauge contribution to the effective potential (in arbitrary units) in the presence of localized gauge kinetic terms, with $c_2 = 0$ and increasing values of c_1 .

consequences on model building. The analysis for generic values of α and c_i is however quite involved, and we therefore consider only the special situation in which $c_1 \gg 1$ and $c_2 \ll 1$, for which a substantial simplification occurs. In this limit, the zero-mode wave-function f_0 reduces to a c -independent linear profile. Equations (41) and (42) then yield $g_{4,q,0}(0) \simeq g_4$, $g_{4,q,0}(\pi R) \simeq g_4 \cos(\pi q \alpha)$ and $g_{4,q,0} \simeq g_4$, with

$$g_4 = \frac{g_5}{\sqrt{c} \sqrt{2\pi R}}. \quad (48)$$

For non-zero modes, a stronger suppression factor is found for the couplings at $y = 0$, whereas those at $y = \pi R$ remain finite in the limit of large c . As in the case of [19], this phenomenon tends to suppress four-fermion operators induced from boundary fields at $y = 0$ by the exchange of heavy KK modes of the gauge bosons, in spite of the fact that these are now lighter. Correspondingly, the bounds on R become milder. The W and Z gauge bosons, that arise from states with charge 1 and 2, respectively, now have masses equal to⁷

$$m_W \simeq \frac{\sin(\pi\alpha)}{\pi\sqrt{c}R}, \quad m_Z \simeq \frac{\sin(2\pi\alpha)}{2\pi\sqrt{c}R} \sec \theta_W. \quad (49)$$

Finally, the expression for the Higgs mass is affected as well, because the relation between the 5D and 4D couplings is modified according to (48), and one finds:

$$m_H \simeq \frac{g_4 R}{2} \sqrt{c} \sqrt{V''(\alpha)}. \quad (50)$$

⁷Here we are neglecting the mixing between the Z and the anomalous A_X boson that results in a further deformation in m_Z .

We see that a sizable factor c results in further improvements. The W mass is lowered, so that the experimental bound on R can be satisfied with higher values of α and becomes therefore less restrictive. Taking this the other way around, large values of $1/R$ are easier to achieve. The problem of obtaining a reasonable value for m_{top} is basically solved for high enough values of c . As a rough estimate, one would need $c \sim 100$ if both the left- and right-handed components of the top field are at the same fixed-point, and $c \sim 10$ if they are at different ones. The Higgs mass is also enhanced, and gets higher at fixed VEV α . All the problems of our original 5D model, namely the too low values for $1/R$, m_H and m_{top} , can thus be solved by adding large localized kinetic terms. However, even in the limit of very large c , an unwanted distortion remains. As can be seen from (49), the ρ parameter does not depend on c , and it is given by

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} \simeq \sec^2(\pi\alpha) \neq 1. \quad (51)$$

Moreover, the effective gauge couplings of possible boundary fields located at πR are deformed by a factor $\cos(\pi\alpha)$ for charged interactions mediated by the W and $\cos(2\pi\alpha)$ for the neutral ones mediated by the Z . Once again, a phenomenologically acceptable situation therefore seems to require very low values of α , which in turn is dynamically determined by the radiatively generated Higgs effective potential.

As already mentioned, the contribution of the gauge fields to the effective potential is also strongly deformed by localized kinetic terms. For $c \gg 1$, its α -dependence gets suppressed and the total effective potential is dominated by the fermion contributions. In this case, the good features that we found at the end of section 4, namely the possibility of obtaining values for α lower than the usual ones, thanks to the boundary couplings, is ruined and one typically gets back values close to $\alpha \sim 0.3$. This clearly goes against what is needed to exploit the good features associated with localized gauge couplings. In particular, $\alpha \sim 0.3$ gives an unacceptable value for ρ . Owing to the enhancement of the bulk electroweak coupling, the scale where the latter becomes non-perturbative is lowered, $\Lambda_w \rightarrow \Lambda_w/c$, as for the case of high rank representations discussed in section 5.4. This represents another limitation to an increasing of c , but values up to $c \sim 15$ appear to be reasonable. Notice that for $c \sim 15$ and $\alpha \sim 0.3$, one would get $1/R \sim 1$ TeV, which turns out to be compatible with electroweak precision tests, thanks to the fact that the couplings between matter and KK modes of the gauge fields are suppressed.

Summarizing, we see that the presence of localized gauge kinetic terms can drastically improve the situation, but only if they are accompanied by low values of α , which allow some control on the unwanted deformations that these localized terms necessarily produce. Unfortunately, these low values for α do not appear to be generated in minimal situations. It would therefore be again of great help to have at our disposal some mechanism that provides an additional contribution to the potential

that could lower the VEV α , without distorting the electroweak symmetry breaking. As already mentioned in section 5.4, one possibility consists in introducing extra bulk fermions in large representations of $SU(3)_w$. Although rather unusual, such an additional large-rank heavy fermion would lead to an optimal situation when combined with localized gauge kinetic terms.

It would be interesting to study what happens for generic values of the c_i 's, because it is not excluded that all the deformations induced by these terms could conspire, in particular situations, to yield a phenomenologically viable model. On the other hand, it should be recalled that localized terms, even if not introduced at tree level, are radiatively generated in the theory and thus a proper study of their effects is necessary to draw definite conclusions on model building in this context.

Let us conclude this section by noting that the above considerations are valid as long as one introduces localized gauge kinetic terms for A_μ only. As already said, this is not a necessary restriction and a localized term involving the 5D field strength F_{MN} could be considered. The analysis of this case is complicated by the presence of derivatives along the internal directions and the computation of the effective potential seems much more involved. The KK spectrum and wave functions of the 5D gauge fields could be quite different, in particular for the zero-mode sector, and it is not excluded that this case could be phenomenologically interesting.

7 Outlook

We have studied in detail various aspects of orbifold models with unification of gauge and Higgs fields, ordinary matter localized at fixed-points, and additional heavy fermions in the bulk. We have also analysed the effect of having large localized gauge kinetic terms in these models. Electroweak symmetry breaking occurs at the quantum level through a rank-reducing Wilson-line symmetry breaking and is transmitted to matter at the boundaries by the massive bulk fermions. The main advantage of this mechanism is that the flavour structure of the SM can be achieved in an elegant way, without spoiling the stability of the Higgs potential.

We have presented a simple prototype example in 5D, based on the above structure and the gauge group $SU(3)_c \times SU(3)_w \times U(1)'$. For its minimal version, we find that $1/R$, m_H and m_{top} turn out to be too low, but acceptable values can be obtained, with a moderate tuning of the parameters, by adding extra heavy bulk fermions and/or localized gauge kinetic terms. By doing so, however, the predictive power of the model is lowered. Most importantly, we have seen that in the presence of localized gauge kinetic terms the electroweak sector of the theory is distorted in a non-universal and unwanted way.

At this stage, the prototype 5D models that we presented cannot be considered

neither as viable nor as ruled out. To this purpose, we think that a more careful phenomenological analysis is needed, which should take systematically into account the effect of localized kinetic terms for bulk fields and possible extra massive bulk fermions. On the other hand, the general structure that we have illustrated can be applied to similar constructions in more than five dimensions as well. The main new feature is the presence of a tree-level quartic potential for the Higgs fields arising from the decomposition of the higher-dimensional gauge kinetic term. The electroweak symmetry breaking still occurs radiatively, but the presence of the tree-level term can help in achieving a larger Higgs mass. In particular, 6D models represent the minimal version of this possibility, with two Higgs doublets [6, 7]. We plan to extend our analysis to this kind of models in a future work.

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A Mode decomposition

The mode decomposition of fields in various representations of the gauge group G in the presence of a projection P and twist T can be easily obtained as follows. If we denote by $\hat{\Psi}_{\mathcal{R}}(y)$ a field multiplet transforming in the representation $\mathcal{R}$ of G , we have

$$\hat{\Psi}_{\mathcal{R}}(-y) = \eta_{\Psi} \mathcal{R}(P) \hat{\Psi}_{\mathcal{R}}(y) , \quad \hat{\Psi}_{\mathcal{R}}(y + 2\pi R) = \mathcal{R}(T) \hat{\Psi}_{\mathcal{R}}(y) , \quad (52)$$

where $\mathcal{R}(P)$ and $\mathcal{R}(T)$ denote respectively the embedding of the projection and twist in the gauge group in the corresponding representation, and $\eta_{\Psi} = \pm 1$. In a basis in which P is diagonal, the first relation in (52) is easily solved in terms of single-valued fields $\Psi_{\mathcal{R}}$. One simply gets an expansion in cosines or sines for the various components, according to the eigenvalue of the projection matrix P . The second relation in (52) is then satisfied by taking

$$\hat{\Psi}_{\mathcal{R}}(y) = \mathcal{R}[\Omega(y)] \Psi_{\mathcal{R}}(y) . \quad (53)$$

In eq. (53), $\Omega(y) = \exp(i\alpha_a \tau^a y/R)$ when T is expressed as $T = \exp(2i\pi\alpha_a \tau^a)$, with τ^a the generators of the Lie algebra of the group G . The field $\hat{\Psi}_{\mathcal{R}}(y)$ automatically

solves also the first relation in (52) because $\mathcal{R}[\Omega(-y)] = \mathcal{R}[\Omega^{-1}(y)]$ and $TPT = P$ by consistency. Since (53) is simply a non-single valued gauge transformation, we can alternatively work with the untwisted fields $\Psi_{\mathcal{R}}$ only. In this gauge, the effect of the twist is encoded in the VEV for A_5 induced by the gauge transformation: $A_5 = (-i/g)\Omega^\dagger(y)\partial_5\Omega(y) = \alpha_a\tau^a/(gR)$. In this case, the problem of finding the mode decomposition of the field $\Psi_{\mathcal{R}}$ is reduced to the choice of a basis in which its coupling with $\langle A_5 \rangle$ is diagonal.

In the following, we will adopt this second point of view and consider the decomposition of the untwisted fields $\Psi_{\mathcal{R}}$, for the case in which $\mathcal{R}$ is the fundamental, symmetric and adjoint representation of $G = SU(3)$, with P and T taken as in (1) and (4). It will be convenient to introduce the factor η_n defined to be 1 for $n = 0$ and $1/\sqrt{2}$ for $n \neq 0$, as well as basic wave functions of even and odd modes:

$$f_n^+(y) = \frac{1}{\sqrt{2\pi R}} \cos \frac{ny}{R}, \quad f_n^-(y) = \frac{1}{\sqrt{2\pi R}} \sin \frac{ny}{R}. \quad (54)$$

We will mainly focus on matter fermions, but our results easily generalize to other fields. We will denote by $\Psi_{\pm}$ the left- and right-handed components, which satisfy the first equation in (52) with $\eta_{\Psi} = \pm$ respectively.

A.1 Fundamental

For the fundamental representation we have simply $\mathcal{R}(P) = P$ and $\mathcal{R}(T) = T$ in (52). It is convenient for later purposes to express $\Psi_{\pm}$ as a sum over all integer modes, both positive and negative; this is done by defining the negative modes of a given component as the reflection of the positive modes: $\Psi_{-n} = \pm\Psi_n$, depending on the parity of the component. In this way, the mode expansion for the untwisted fields $\Psi_{\pm}$ is given by

$$\Psi_{\pm}(y) = \sum_{n=-\infty}^{\infty} \eta_n \begin{pmatrix} f_n^{\mp}(y) \psi_n^{\pm u} \\ f_n^{\mp}(y) \psi_n^{\pm d} \\ \pm f_n^{\pm}(y) \chi_n^{\pm} \end{pmatrix}, \quad (55)$$

where we denoted by ψ^u, ψ^d the up and down components of the $SU(2) \times U(1)$ doublet, and by χ the $SU(2) \times U(1)$ singlet.

The basis in which the coupling of the three components of the triplet to the VEV of A_5 is diagonal is reached by defining the following new fields:

$$\Psi_n^{\pm(1)} = \psi_n^{\pm u}, \quad \Psi_n^{\pm(2)} = \eta_n (\psi_n^{\pm d} + \chi_n^{\pm}). \quad (56)$$

Notice that all the modes of $\Psi_n^{\pm(2)}$ are physical and correspond to orthogonal combinations of the physical modes $\psi_n^{\pm d}$ and $\chi_n^{\pm}$. In this new basis, the wave function is rewritten as

$$\Psi_{\pm}(y) = \sum_{n=-\infty}^{\infty} \begin{pmatrix} \eta_n f_n^{\mp}(y) \Psi_n^{\pm(1)} \\ f_n^{\mp}(y) \Psi_n^{\pm(2)} \\ \pm f_n^{\pm}(y) \Psi_n^{\pm(2)} \end{pmatrix}. \quad (57)$$

The action of the twist is now diagonal, and amounts to shifting $n \rightarrow n + \alpha$ in the coefficients of $\Psi_n^{\pm(2)}$, which describes both the down component of the doublet and the singlet. The 4D kinetic Lagrangian for the field $\Psi^{(i)} = \Psi_+^{(i)} + \Psi_-^{(i)}$, defined as $\mathcal{L}_{4D} = \int_0^{2\pi R} \mathcal{L}_{5D}$, is easily computed and reads

$$\mathcal{L}_{4D} = \sum_{n=0}^{\infty} \bar{\Psi}_n^{(1)} [i\partial_4 - m_n^{(1)}] \Psi_n^{(1)} + \sum_{n=-\infty}^{\infty} \bar{\Psi}_n^{(2)} [i\partial_4 - m_n^{(2)}] \Psi_n^{(2)}, \quad (58)$$

where

$$m_n^{(1)} = \frac{n}{R}, \quad m_n^{(2)} = \frac{n + \alpha}{R}. \quad (59)$$

A.2 Symmetric

For the symmetric representation, we have $\mathcal{R}(P) = P \otimes P^T$ and $\mathcal{R}(T) = T \otimes T^T$ in (52). Doubling again the modes for convenience, the untwisted wave functions read in this case:

$$\Psi_{\pm}(y) = \sum_{n=-\infty}^{\infty} \frac{\eta_n}{\sqrt{2}} \begin{pmatrix} \pm\sqrt{2}f_n^{\pm}(y)\phi_n^{\pm a} & \pm f_n^{\pm}(y)\phi_n^{\pm c} & f_n^{\mp}(y)\psi_n^{\pm u} \\ \pm f_n^{\pm}(y)\phi_n^{\pm c} & \pm\sqrt{2}f_n^{\pm}(y)\phi_n^{\pm b} & f_n^{\mp}(y)\psi_n^{\pm d} \\ f_n^{\mp}(y)\psi_n^{\pm u} & f_n^{\mp}(y)\psi_n^{\pm d} & \pm\sqrt{2}f_n^{\pm}(y)\chi_n^{\pm} \end{pmatrix}, \quad (60)$$

where, as before, we denoted the upper and lower components of the $SU(2) \times U(1)$ doublet by ψ^u and ψ^d , the singlet by χ and the three components of the triplet by ϕ^a , ϕ^b and ϕ^c . The diagonal basis is defined by the new fields

$$\begin{aligned} \Psi_n^{\pm(1)} &= \eta_n (\psi_n^{\pm u} - \phi_n^{\pm c}), \quad \Psi_n^{\pm(2)} = \eta_n \left(\psi_n^{\pm d} + \frac{\chi_n^{\pm} - \phi_n^{\pm b}}{\sqrt{2}} \right), \\ \Psi_n^{\pm(3)} &= \phi_n^{\pm a}, \quad \Psi_n^{\pm(4)} = \frac{\chi_n^{\pm} + \phi_n^{\pm b}}{\sqrt{2}}, \end{aligned} \quad (61)$$

where all the modes in $\Psi_n^{\pm(1)}$ and $\Psi_n^{\pm(2)}$ are now physical. In this way, calling for short $\Psi_n^{\pm(2\pm 4)} = \Psi_n^{\pm(2)} \pm \eta_n \Psi_n^{\pm(4)}$, one has

$$\Psi_{\pm}(y) = \frac{1}{\sqrt{2}} \sum_{n=-\infty}^{\infty} \begin{pmatrix} \pm\sqrt{2}\eta_n f_n^{\pm}(y)\Psi_n^{\pm(3)} & \mp f_n^{\pm}(y)\Psi_n^{\pm(1)} & f_n^{\mp}(y)\Psi_n^{\pm(1)} \\ \mp f_n^{\pm}(y)\Psi_n^{\pm(1)} & \mp f_n^{\pm}(y)\Psi_n^{\pm(2-4)} & f_n^{\mp}(y)\Psi_n^{\pm(2)} \\ f_n^{\mp}(y)\Psi_n^{\pm(1)} & f_n^{\mp}(y)\Psi_n^{\pm(2)} & \pm f_n^{\pm}(y)\Psi_n^{\pm(2+4)} \end{pmatrix}. \quad (62)$$

From the above expression, we see that the field $\Psi^{\pm(2)}$ again appears both in the down component of the doublet and the singlet. The action of the twist amounts to shifting $n \rightarrow n + \alpha$ in the coefficients of $\Psi_n^{\pm(1)}$ and $n \rightarrow n + 2\alpha$ in the coefficients of $\Psi_n^{\pm(2)}$. The 4D kinetic Lagrangian for the new fields is

$$\mathcal{L}_{4D} = \sum_{n=-\infty}^{\infty} \sum_{i=1,2} \bar{\Psi}_n^{(i)} [i\partial_4 - m_n^{(i)}] \Psi_n^{(i)} + \sum_{n=0}^{\infty} \sum_{i=3,4} \bar{\Psi}_n^{(i)} [i\partial_4 - m_n^{(i)}] \Psi_n^{(i)}, \quad (63)$$

where

$$m_n^{(1)} = \frac{n + \alpha}{R}, \quad m_n^{(2)} = \frac{n + 2\alpha}{R}, \quad m_n^{(3)} = \frac{n}{R}, \quad m_n^{(4)} = \frac{n}{R}. \quad (64)$$

A.3 Adjoint

For the adjoint representation, we have $\mathcal{R}(P) = P \otimes P^\dagger$ and $\mathcal{R}(T) = T \otimes T^\dagger$ in (52). The decomposition of untwisted fields reads

$$\Psi_\pm(y) = \sum_{n=-\infty}^{\infty} \frac{\eta_n}{\sqrt{2}} \begin{pmatrix} \pm f_n^\pm(y) (Z_n^\pm + \frac{1}{\sqrt{3}} \chi_n^\pm) & \pm f_n^\pm(y) Y_n^\pm & f_n^\mp(y) \psi_n^{\pm u} \\ \pm f_n^\pm(y) (Y_n^\pm)^\dagger & \mp f_n^\pm(y) (Z_n^\pm - \frac{1}{\sqrt{3}} \chi_n^\pm) & f_n^\mp(y) \psi_n^{\pm d} \\ f_n^\mp(y) (\psi_n^{\pm u})^\dagger & f_n^\mp(y) (\psi_n^{\pm d})^\dagger & \pm \frac{2}{\sqrt{3}} f_n^\pm(y) \chi_n^\pm \end{pmatrix}, \quad (65)$$

where, as before, we denoted the upper and lower complex components of the $SU(2) \times U(1)$ doublet by ψ^u and ψ^d , the singlet by χ and the three components of the triplet by Z and $Y, Y^\dagger$. The diagonal basis is defined by

$$\begin{aligned} \Psi_n^{\pm(1)} &= \eta_n (Y_n^\pm - \psi_n^{\pm u}), \quad \Psi_n^{\pm(2)} = \eta_n \left(\text{Re } \psi_n^{\pm d} + \frac{Z_n^\pm - \sqrt{3} \chi_n^\pm}{2} \right), \\ \Psi_n^{\pm(3)} &= \frac{\sqrt{3} Z_n^\pm + \chi_n^\pm}{2}, \quad \Psi_n^{\pm(4)} = \text{Im } \psi_n^{\pm d}, \end{aligned} \quad (66)$$

where all modes of $\Psi_n^{\pm(1)}$ and $\Psi_n^{\pm(2)}$ are physical and the first is a complex field. Defining for short $\Psi_n^{\pm(2\pm3)} = \Psi_n^{\pm(2)} \pm \frac{\eta_n}{\sqrt{3}} \Psi_n^{\pm(3)}$ and $\Psi_n^{\pm(2\pm4)} = \Psi_n^{\pm(2)} \pm i \eta_n \Psi_n^{\pm(4)}$, we obtain, in the new basis:

$$\Psi_\pm(y) = \frac{1}{\sqrt{2}} \sum_{n=-\infty}^{\infty} \begin{pmatrix} \pm \frac{2}{\sqrt{3}} \eta_n f_n^\pm(y) \Psi_n^{\pm(3)} & \pm f_n^\pm(y) \Psi_n^{\pm(1)} & f_n^\mp(y) \Psi_n^{\pm(1)} \\ \pm f_n^\pm(y) (\Psi_n^{\pm(1)})^\dagger & \mp f_n^\pm(y) \Psi_n^{\pm(2+3)} & f_n^\mp(y) \Psi_n^{\pm(2+4)} \\ f_n^\mp(y) (\Psi_n^{\pm(1)})^\dagger & f_n^\mp(y) (\Psi_n^{\pm(2+4)})^\dagger & \mp f_n^\pm(y) \Psi_n^{\pm(2-3)} \end{pmatrix}. \quad (67)$$

The action of the twist amounts to shifting $n \rightarrow n + \alpha$ in the coefficients of $\Psi_n^{\pm(1)}$ and $n \rightarrow n + 2\alpha$ in the coefficients of $\Psi_n^{\pm(2)}$. The 4D Lagrangian for the new fields is

$$\begin{aligned} \mathcal{L}_{4D} &= \sum_{n=-\infty}^{\infty} (\bar{\Psi}_n^{(1)})^\dagger [i \not{\partial}_4 - m_n^{(1)}] \Psi_n^{(1)} + \sum_{n=-\infty}^{\infty} \bar{\Psi}_n^{(2)} [i \not{\partial}_4 - m_n^{(2)}] \Psi_n^{(2)} \\ &\quad + \sum_{n=0}^{\infty} \sum_{i=3,4} \bar{\Psi}_n^{(i)} [i \not{\partial}_4 - m_n^{(i)}] \Psi_n^{(i)} \end{aligned} \quad (68)$$

where

$$m_n^{(1)} = \frac{n + \alpha}{R}, \quad m_n^{(2)} = \frac{n + 2\alpha}{R}, \quad m_n^{(3)} = \frac{n}{R}, \quad m_n^{(4)} = \frac{n}{R}. \quad (69)$$

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