

Scale independent spin effects in D-brane dynamics

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Abstract

We study spin interactions between two moving D-branes using the Green-Schwarz formalism of boundary states. We focus our attention on the leading terms for small velocities v , of the form v^{4-n}/r^{7-p+n} (v^{2-n}/r^{3-p+n}) for p-p (p-p+4) systems, with 16 (8) supercharges. In analogy with standard G-S computations of massless four-point one-loop amplitudes in Type I theory, the above terms are governed purely by zero modes, massive states contributions cancelling as expected by the residual supersymmetry. This implies the scale invariance of these leading spin-effects, supporting the relevant matrix model descriptions of supergravity interactions; in this context, we also discuss similar results for more general brane configurations. We then give a field theory interpretation of our results, that allows in particular to deduce the gyromagnetic ratio $g = 1$ and the presence of a quadrupole moment for D0-branes.

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I. INTRODUCTION

It is undoubt that D-branes [1] play at present a key role in the description of all non-perturbative phenomena in string theory. This has motivated the study of their dynamics [2–6] in different configurations and space-time dimensions; in particular, the so-called boundary state formalism [7,8] has proven to be a powerful tool for studying these problems, and has been succesfully applied to a number of problems, both in the covariant [9–16] and light-cone [17–20] formulations. However, since D-branes are usually analyzed in a semi-classical context, being heavy massive solitons at weak string coupling constant, most of these works considered their interactions in the approximation in which D-branes do not carry any spin.

The conjecture of [21] that the dynamics of M-theory in the infinite momentum frame is governed by the degrees of freedom of a large number of D0-branes further motivated the study of D0-brane dynamics. In particular, their spin-dependent long-range interactions have been computed through duality arguments, mapping the scattering of D0-branes to that of fundametal KK states [22]. In ref. [23], spin effects for generic p-branes have been analyzed in the Green-Schwarz formalism, by inserting broken supercharges in the partition function of two branes moving past each other. The structure of these interactions was then considered in a long range regime through the technique of boundary states and at short distances through a one-loop annulus analysis. This last open string point of view showed, in particular, how for a D p-brane all the terms of the form v^{4-n}/r^{7-p+n} with $n = 0, \dots, 4$, i.e. all the spin-effects associated to the universal v^4/r^{7-p} interaction, are determined purely by the *open* massless string states, meaning then that a one-loop M(atrrix) theory computation should be able to reproduce long range spin-dependent supergravity interactions. This has been indeed explicitly shown in [24] for the spin-orbit v^3/r^8 coupling.

We want then to study interactions between two branes moving with small relative velocity \vec{v} and at an impact parameter \vec{b} . We will work in the Green-Schwarz formalism of the boundary state. The choice of this formalism is justified by the huge simplification that takes over and that allows to compute spin effects basically by a simple analysis of zero modes, as we will see. We consider in the following supersymmetric configurations associated to the case of parallel p-p and p-p+4 systems of D-branes. Being interested only in leading effects for small velocity, we will not compute the full partition function associated to the configuration of two moving branes, but simply correlation functions of vertex operators, corresponding to the velocity, on the boundary of the world-sheet, together with insertions of broken supercharges encoding spin dependences. The approach we follow to compute n -point functions in the boundary state formalism is that derived by [7,8]. It is crucial to point out, however, that the results we obtain are *exact* for the particular

terms considered, being given by a full string computation, in analogy with one-loop 4-point functions between massless states of Type I theory in the G-S formalism [25]. In this context it is trivial to see some important issues. A p-p (p-p+4) system leaves unbroken sixteen (eight) supercharges meaning that our action will admit eight (four) fermionic zero modes S_0^a , in the notation of [25]. Since, as we will see, each insertion of a vertex operator associated to the velocity provides at most two of them, it follows that the potential between branes starts respectively with v^4 (v^2). On the other hand, the insertion of broken supercharges can allow non-vanishing results for terms with less powers of v , providing the lacking fermionic zero modes. Moreover, as has been shown in [19,23] and will be seen again in some detail, the insertion of supercharges induces polarization-dependent non-minimal couplings between D-branes, i.e. spin-effects. Alternatively, being all these terms related by supersymmetry [22], it is natural that they are produced by insertion of supercharges.

The main point we want to stress here is that for the amplitudes we consider, associated to the v^{4-n}/r^{7-p+n} terms discussed before *, our results will be governed by zero modes only, since massive non-BPS bosonic and fermionic string states contribution will always precisely cancel. This implies that the present results, that for large brane separations have a clear interpretation as spin-dependent interactions, due to supersymmetry, are valid at *all scales* and can be extrapolated to the substringy regime where, according to [6], the dominant degrees of freedom are given by the massless open string states living on the branes. This assertion is valid for a large class of interesting systems discussed in the final section, besides the more detailed studied p-p and p-p+4 system, and shows that several tests (if not all) performed until now to the proposal of [21], involving an agreement between one-loop Super Yang-Mills computation and classical supergravity, are basically determined by supersymmetry.

We then present a systematic way to construct the generic asymptotic form of spinning p-brane supergravity solutions, working out in detail the case of the D0-brane, where we perform a multi-pole expansion up to the quadrupole interaction. This analysis allows easily to show that the electric and gravitational dipole moments for D0-branes vanish, whereas there is a gravitational quadrupole term different from zero. The supersymmetric cancellation of some terms of brane-interactions, as discussed before, allows finally to fix uniquely the value of D0-brane's gyromagnetic ratio g to one and similarly the relative strength of the electric and gravitational quadrupole moments. This is consistent with their conjectured [26] 11-dimensional KK origin and interpretation as solitonic solution of 10-dimensional IIA supergravity, as recently discussed in ref. [30]. As a further check of this

*For the p-p+4 system the corresponding terms are of the form v^{2-n}/r^{3-p+n} , with $n = 0, 1, 2$.

correspondence, it would be interesting to work out, along the lines of [30], the quadrupole moments of 0-brane solitons in supergravity and compare them to the results found here for D0 branes. We then briefly mention how the knowledge of the asymptotic fields of the 0-brane solution allows to derive the 0-brane world-line effective action in an arbitrary Type IIA background, valid in the weak-field approximation.

The structure of this paper is as follows. In section 2 and 3 we review the main properties of G-S boundary states [18], introduce the set-up of the computation, and report one-point functions of higher spin boundary states with massless closed string states. In section 4 and 5, we compute spin-dependent interactions associated to the p-p and p-p+4 parallel brane configurations, and in section 6 the field theory interpretation of our results is given. In last section we discuss possible extensions of our results to more general brane configurations, relevant for one-loop tests of the matrix model conjecture, and give some conclusions. We include in an appendix the light-cone computations of the standard phase-shift for two moving D-branes.

II. BOUNDARY STATE FORMALISM

In this section we shall briefly review the boundary state description of spinning D-branes [23] in the G-S formalism [18]. Consider type II theory in the light-cone gauge $X^+ = x^+ + p^+ \tau$. X^- is completely determined in terms of the transverse X^i 's, in $\mathbf{8}_v$, and the left and right spinors S^a and \tilde{S}^a , in the $\mathbf{8}_s$ of the $SO(8)$ transverse rotation group [†]. In this gauge, the supercharges are

$$Q^a = \sqrt{2p^+} \oint d\sigma S^a, \quad Q^{\dot{a}} = \frac{1}{\sqrt{p^+}} \gamma_{\dot{a}a}^i \oint d\sigma \partial X^i S^a$$

with similar expressions for the right moving ones.

Dp-branes as defects can be described by suitable boundary states, implementing the usual Neumann-Dirichlet boundary conditions. In this frame, the two light-cone directions $\pm = 0 \pm 9$ satisfy automatically Dirichlet boundary conditions while the transverse directions $i = 1, \dots, 8$ can have either Neumann or Dirichlet boundary conditions. Since the time satisfies Dirichlet boundary conditions, we are actually dealing with Euclidean branes; however following [23], we can identify the “time” with one of the transverse directions, say X^1 . The

[†]In the following we shall concentrate on Type IIB theory, for which the notation is somewhat friendlier; the Type IIA case can be easily obtained by switching dotted and undotted indices in the right-moving fermions.

usual metric is then recovered through a double analytic continuation $0 \rightarrow i1$, $1 \rightarrow i0$ in the final result. We can therefore include in our analysis only branes with $p = -1, \dots, 7$. The BPS boundary state is defined to preserve a combination of left and right supersymmetries

$$\begin{aligned} Q_+^a |B\rangle = 0, \quad Q_+^{\dot{a}} |B\rangle = 0 &\Rightarrow Q_+^a, Q_+^{\dot{a}} \text{ unbroken} \\ Q_-^a |B\rangle \neq 0, \quad Q_-^{\dot{a}} |B\rangle \neq 0 &\Rightarrow Q_-^a, Q_-^{\dot{a}} \text{ broken} \end{aligned}$$

where

$$Q_{\pm}^a = \frac{1}{\sqrt{2}} (Q^a \pm i M_{ab} \tilde{Q}^b), \quad Q_{\pm}^{\dot{a}} = \frac{1}{\sqrt{2}} (Q^{\dot{a}} \pm i M_{\dot{a}\dot{b}} \tilde{Q}^{\dot{b}})$$

The boundary conditions are [18]

$$(\alpha_n^i + M_{ij} \tilde{\alpha}_{-n}^j) |B\rangle = 0, \quad (S_n^a + i M_{ab} \tilde{S}_{-n}^b) |B\rangle = 0, \quad (S_n^{\dot{a}} + i M_{\dot{a}\dot{b}} \tilde{S}_{-n}^{\dot{b}}) |B\rangle = 0 \quad (2.1)$$

with

$$M_{ij} = \begin{pmatrix} -I_{p+1} & 0 \\ 0 & I_{7-p} \end{pmatrix} \quad (2.2)$$

Consistency with the BPS condition implies the M 's to be orthogonal matrices related by triality

$$\begin{aligned} (MM^T)_{ab} &= \delta_{ab} \\ (M\gamma^i M^T)_{a\dot{a}} &= M_{ij} \gamma_{a\dot{a}}^j \end{aligned}$$

which yields

$$M_{ab} = (\gamma^1 \gamma^2 \dots \gamma^{p+1})_{ab}, \quad M_{\dot{a}\dot{b}} = (\gamma^1 \gamma^2 \dots \gamma^{p+1})_{\dot{a}\dot{b}} \quad (2.3)$$

The solution for the boundary state can then be found in a standard way as the eigenstate of the boundary conditions eqs.(2.1), and is written as

$$|B\rangle = \exp \sum_{n>0} \left(\frac{1}{n} M_{ij} \alpha_{-n}^i \tilde{\alpha}_{-n}^j - i M_{ab} S_{-n}^a \tilde{S}_{-n}^b \right) |B_0\rangle \quad (2.4)$$

the zero mode part being

$$|B_0\rangle = M_{ij} |i\rangle |\tilde{j}\rangle - i M_{\dot{a}\dot{b}} |\dot{a}\rangle |\tilde{\dot{b}}\rangle \quad (2.5)$$

The complete configuration space boundary state is

$$|B, \vec{x}\rangle = (2\pi\sqrt{\alpha'})^{4-p} \int \frac{d^{9-p}q}{(2\pi)^{9-p}} e^{i\vec{q}\cdot\vec{x}} |B\rangle \otimes |\vec{q}\rangle \quad (2.6)$$

where

$$\langle q|q'\rangle = V_{p+1} (2\pi)^{9-p} \delta^{(9-p)}(q - q')$$

and V_{p+1} is the space-time volume spanned by the p-brane. With these normalizations, the static force between two parallel branes is given by

$$\mathcal{A} = \frac{1}{16} \int_0^\infty dt \langle B, \vec{x} | e^{-2\pi t \alpha' p^+ (P^- - i\partial/\partial x^+)} | B, \vec{y} \rangle \quad (2.7)$$

where

$$P^- = \frac{1}{2p^+} \left[(p^i)^2 + \frac{1}{\alpha'} \sum_{n=1}^\infty (\alpha_{-n}^i \alpha_n^i + \tilde{\alpha}_{-n}^i \tilde{\alpha}_n^i + n S_{-n}^a S_n^a + n \tilde{S}_{-n}^a \tilde{S}_n^a) \right]$$

is the Hamiltonian in the light-cone gauge. The term $i\partial_+$ represents the subtraction of p^- (remember that in this gauge the effective Hamiltonian is $H - p^-$) and when applied to the boundary state, it reproduces simply the covariant p^2 . The factor $1/16$ is needed to normalize correctly the D-brane charge; indeed, from eq.(2.7) we obtain

$$\mathcal{A} = \frac{1}{16} V_{p+1} (4\pi^2 \alpha')^{4-p} \int_0^\infty dt \int \frac{d^{9-p} q}{(2\pi)^{9-p}} e^{i\vec{q} \cdot (\vec{x} - \vec{y})} e^{-\pi t \alpha' q^2} (8 - 8) \prod_{n=1}^\infty \frac{(1 - e^{-2\pi t n})^8}{(1 - e^{-2\pi t n})^8} \quad (2.8)$$

where the factor $(8 - 8)$ is due to the trace performed on the zero mode part of the boundary state, eq.(2.5). Performing the momenta and modulus integrations, one finds

$$\mathcal{A} = 2 V_{p+1} T_p^2 G_{9-p}(\vec{x} - \vec{y}) (1 - 1) \quad (2.9)$$

where $T_p = \sqrt{\pi} (4\pi^2 \alpha')^{(3-p)/2}$ is the tension of a p-brane in units of the ten-dimensional Planck constant k^2 of Type II supergravity [1] and $G_d(\vec{x})$ is the massless propagator of a scalar particle in d -dimensions

$$G_d(\vec{x}) = \frac{1}{4\pi^{d/2}} \frac{\Gamma(\frac{d-2}{2})}{|x|^{d-2}}$$

The generalization of the cylinder amplitude to the case of finite constant relative velocity v between two branes is reported for completeness in the appendix.

III. SUPERSYMMETRY AND HIGHER SPIN BPS STATES

As we have seen, Dp-branes correspond to solitonic BPS saturated solutions of Type IIA(B) supergravity, which preserve one half of the supersymmetries. The remaining half is realized on a short-multiplet containing 256 p-brane configurations lying in the **44+84+128** representations of the little group $SO(9)$ for massive states. The various components of the short-multiplet are related by supersymmetry transformations generated by the 16 broken supercharges.

In the formalism of previous section, the boundary state represents the semiclassical source formed by the “in” and “out” branes; its overlap $\langle B|\Psi\rangle$ with a string state $|\Psi\rangle$ represents semiclassical 3-point functions as shown in figure 1.

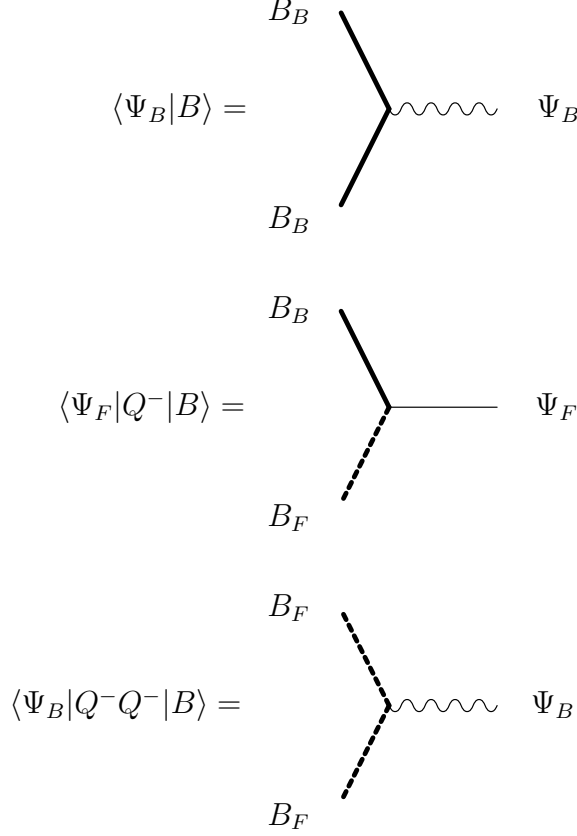


Fig. 1

Different components of the supermultiplet spanned by these sources, are obtained by applying supersymmetry transformations to the scalar boundary state $|B\rangle$ through the operator

$$U = e^{\epsilon Q^-} = \sum_{m=0}^{16} \frac{1}{m!} (\epsilon Q^-)^m \quad (3.1)$$

We have used the $SO(9)$ notation $\epsilon = (\eta^a, \tilde{\eta}^{\dot{a}})$ and $Q^- = (Q_-^a, Q_-^{\dot{a}})$. Different components of the supermultiplet, corresponding to the possible independent ϵ 's, can be thought as the semiclassical multipole expansion of the source. In particular, terms in $U|B\rangle$ with an even (odd) number of Q^- describe couplings to bosonic (fermionic) closed string states Ψ_B (Ψ_F). We shall restrict ourselves to terms with an even number of supercharges, the relevant for the study of semiclassical D-brane dynamics in the eikonal approximation. For instance, the usual boundary state corresponds to the universal part of the source, whereas applying two charges one obtains the part of the source due to angular momentum, and so on. As we

are going to see in the following, the field theory counterpart of this source expansion is a power series in the transferred momentum, each momentum corresponding to the insertion of a pair of supercharges.

Among the different terms in expansion (3.1) we will always work out the ones with an equal number of dotted-undotted $SO(8)$ components $(\eta_a Q^- \tilde{\eta}_{\dot{a}} Q^{-\dot{a}})^n$. All the other terms simply combine to reconstruct the covariant answer. We consider then boundary states of the form:

$$|B\rangle_{(n)} \equiv V_\eta^n |B\rangle \quad (3.2)$$

with

$$V_\eta = \eta_a Q^{-a} \tilde{\eta}_{\dot{a}} Q^{-\dot{a}} \quad (3.3)$$

The first interesting information we can extract from these higher spin boundary states is about their couplings to the massless bulk fields. This analysis for the D-instanton case was performed in the covariant NS-R formalism in ref. [19]. The formulae displayed in this section are “T-dual” versions of the ones reported in that reference.

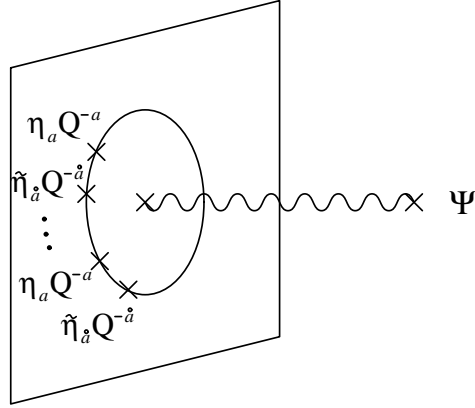


Fig. 2

In the following, we consider only terms with up to eight supercharges, $n = 0, \dots, 4$, in eq.(3.2). This covers all the physical information relevant to our considerations. The one-point functions of the massless bosonic states of NSNS and RR sectors (in R-NS terminology) are obtained simply by computing the boundary state overlap

$$\Psi_{(n)} \equiv \langle \Psi | B_0 \rangle_{(n)} \quad (3.4)$$

with the massless NSNS and RR closed string states

$$\begin{aligned} |\Psi_{NSNS}\rangle &= \xi_{mn} |m\rangle |\tilde{n}\rangle \Rightarrow \xi_{mn} \sim \phi \delta_{mn} + g_{mn} + b_{mn} \\ |\Psi_{RR}\rangle &= C_{\dot{a}\dot{b}} |\dot{a}\rangle |\tilde{\dot{b}}\rangle \Rightarrow C_{\dot{a}\dot{b}} \sim \sum_{k \text{ even}} \frac{1}{k!} C_{(k)}^{i_1 \dots i_k} \gamma_{\dot{a}\dot{b}}^{i_1 \dots i_k} \end{aligned}$$

$|B_0\rangle_{(n)}$ indicates the massless content of (3.2)

$$|B_0\rangle_{(n)} \equiv V_\eta^n |B_0\rangle = q_{i_1} \dots q_{i_n} \left[\eta_{[a_1} (\tilde{\eta} \gamma^{i_1})_{a_2} \dots \eta_{a_{2n-1}} (\tilde{\eta} \gamma^{i_n})_{a_{2n}} \right] S_0^{-a_1} \dots S_0^{-a_{2n}} |B_0\rangle \quad (3.5)$$

Using the boundary conditions eqs.(2.1), we can express everything in terms of the left-moving modes only. After applying the Fiertz identity

$$S_0^a S_0^b = \frac{1}{2} \delta^{ab} + \frac{1}{4} \gamma_{ab}^{ij} R_0^{ij}$$

we can further rewrite eq.(3.5) in terms of the $SO(8)$ generators $R_0^{ij} = 1/4 S_0^a \gamma_{ab}^{ij} S_0^b$. We are then left with the effective operator

$$V_\eta^n = q_{i_1} \dots q_{i_n} \omega_{j_1 \dots j_{2n}}^{i_1 \dots i_n}(\eta) R_0^{j_1 j_2} \dots R_0^{j_{2n-1} j_{2n}}, \quad (3.6)$$

where

$$\omega_{j_1 \dots j_{2n}}^{i_1 \dots i_n}(\eta) = \frac{1}{2^n} \left[\eta_{[a_1} (\tilde{\eta} \gamma^{i_1})_{a_2} \dots \eta_{a_{2n-1}} (\tilde{\eta} \gamma^{i_n})_{a_{2n}} \right] \gamma_{a_1 a_2}^{j_1 j_2} \dots \gamma_{a_{2n-1} a_{2n}}^{j_{2n-1} j_{2n}} \quad (3.7)$$

encodes the spin dependence. In this way, we can use standard results for Type I theory. The R_0^{ij} generators are represented in the $\mathbf{8}_v$ and $\mathbf{8}_c$ representations by

$$\begin{aligned} R_0^{mn} |i\rangle &= (\delta^{ni} \delta^{mj} - \delta^{mi} \delta^{nj}) |j\rangle \\ R_0^{mn} |\dot{a}\rangle &= \frac{1}{2} \gamma_{\dot{c}\dot{a}}^{mn} |\dot{c}\rangle \end{aligned} \quad (3.8)$$

and some simple algebra leads to

$$|B_0\rangle_{(n)} \equiv M_{(n)}^{ij} |i\rangle |\tilde{j}\rangle - i M_{\dot{a}\dot{b}}^{(n)} |\dot{a}\rangle |\tilde{\dot{b}}\rangle$$

with

$$M_{ij}^{(n)} = 2^n q_{i_1} \dots q_{i_n} \omega_{i k_1 k_1 \dots k_{n-1} k_{n-1} k_n}^{i_1 \dots i_n}(\eta) M_{k_n j} \quad (3.9)$$

$$M_{\dot{a}\dot{b}}^{(n)} = \frac{1}{2^n} q_{i_1} \dots q_{i_n} \omega_{j_1 \dots j_{2n}}^{i_1 \dots i_n}(\eta) (\gamma^{j_1 j_2} \dots \gamma^{j_{2n-1} j_{2n}} M)_{\dot{a}\dot{b}} \quad (3.10)$$

The 1-point functions can then be written as (up to numerical and α' factors)

$$\Psi_{(n)}^{NSNS} = q_{i_1} \dots q_{i_n} \xi^{ij} \omega_{i k_1 k_1 \dots k_{n-1} k_{n-1} k_n}^{i_1 \dots i_n}(\eta) M_{k_n j} \quad (3.11)$$

$$\Psi_{(n)}^{RR} = q_{i_1} \dots q_{i_n} \sum_k \frac{1}{k!} C_{(k)}^{m_1 \dots m_k} \omega_{j_1 \dots j_{2n}}^{i_1 \dots i_n}(\eta) \text{Tr}_S[\gamma^{m_1 \dots m_k} \gamma^{j_1 j_2} \dots \gamma^{j_{2n-1} j_{2n}} M] \quad (3.12)$$

Eqs.(3.11) and (3.12) contain all the non-minimal couplings of D-branes with the massless bosonic states of the corresponding supergravity theory. In particular, for even n the

boundary state couples potentially to the NSNS components $\phi, g_{\mu\nu}, g_{IJ}$ and $b_{\mu I}$ (μ, ν and I, J denoting Neumann and Dirichlet directions respectively), and to the remaining NSNS fields for odd n , as can be seen using the symmetry properties of $\omega^{i_1 \dots i_{2n}}$. As a source of RR fields, we can see that non-minimal couplings arise from the non-vanishing gamma-traces in eq.(3.12), corresponding to forms with $k = p+1-2n, \dots, p+1+2n$. The specific form of these couplings depends on the polarization details and will be explicated for the first terms in the following.

The first universal NSNS coupling is just

$$\Psi_{(0)}^{NSNS} = \xi_{ij} M_{ji} \quad (3.13)$$

We see that any p-brane couples to a specific linear combination of the dilaton ϕ and the diagonal graviton polarizations $g_{11} \dots g_{p+1, p+1}$, as it must be for an object with definite mass density [‡] (remember that $g_{11} \rightarrow -g_{00}$ after analytic continuation). The RR coupling is

$$\Psi_{(0)}^{RR} = \sum_{k \text{ even}} \frac{1}{k!} C_{(k)}^{i_1 \dots i_k} \text{Tr}_S[\gamma^{i_1 \dots i_k} M] \quad (3.14)$$

The gamma-trace vanishes unless $k = p+1$, giving the usual Dp-brane charge. The covariant expressions of eqs.(3.13) and (3.14) are

$$\Psi_{(0)}^{NSNS} = \xi_{\mu\nu} M^{\nu\mu}, \quad \Psi_{(0)}^{RR} = \sum_{k \text{ even}} \frac{1}{k!} C_{\mu_1 \dots \mu_k}^{(k)} \text{Tr}_S[\Gamma^{\mu_1 \dots \mu_k} M] \quad (3.15)$$

where Γ are $SO(1,9)$ gamma-matrices, $M^{\nu\mu}$ is the covariant extension of eq.(2.2), with diagonal entries only, -1 and +1 in Neumann and Dirichlet directions respectively, and $\mathcal{M} = \Gamma^0 \dots \Gamma^p$.

The first non-minimal NSNS coupling is given by

$$\Psi_{(1)}^{NSNS} = \xi_{ij} M_{jk} q_l \omega_{ki}^l = \xi_{ij} M_{jk} q_l \eta \gamma^{kil} \tilde{\eta} \quad (3.16)$$

where we have used the fact that $q_j \xi_{ij} = q_i \xi_{ij} = 0$ and $q_k M_{kj} = q_j$ (there is a non-vanishing momentum transfer only along the Dirichlet directions). As anticipated, eq.(3.16) represents a non-minimal coupling of the brane with the antisymmetric tensor and graviton polarizations $b_{\mu\nu}$, b_{IJ} and $g_{\mu I}$. The covariant expression of eq.(3.16) is simply

$$\Psi_{(1)}^{NSNS} = \xi_{\mu\sigma} M_\nu^\sigma q_\rho \bar{\psi} \Gamma^{\mu\nu\rho} \psi \quad (3.17)$$

[‡]The only exception is the D-instanton that has at this order only the coupling to the dilaton ($M_{ij} = \delta_{ij}$).

where ψ is the Majorana-Weyl fermionic parameter associated to the broken supersymmetry. In a chiral representation, it is simply $\psi = (\epsilon_0)$, where $\epsilon = (\eta^a, \tilde{\eta}^{\dot{a}})$. The corresponding RR coupling is

$$\Psi_{(1)}^{RR} = \sum_{k \text{ even}} \frac{1}{k!} C_{(k)}^{i_1 \dots i_k} \text{Tr}_S(\gamma^{i_1 \dots i_k} \gamma^{ij} M) q_l \eta \gamma^{ijl} \tilde{\eta} \quad (3.18)$$

where still the completely antisymmetric part in the fermion bilinear is the only non-vanishing contribution, since $q^{i_1} C_{(k)}^{i_1 \dots i_k} = 0$. The covariant form of eq.(3.18) is

$$\Psi_{(1)}^{RR} = \sum_{k \text{ even}} \frac{1}{k!} C_{\mu_1 \dots \mu_k}^{(k)} \text{Tr}_S(\Gamma^{\mu_1 \dots \mu_k} \Gamma_{\nu \rho} \mathcal{M}) q_\sigma \bar{\psi} \Gamma^{\nu \rho \sigma} \psi \quad (3.19)$$

The next coupling is $\Psi_{(2)}^{NSNS}$, that is

$$\Psi_{(2)}^{NSNS} = \xi_{ij} q_{j_1} q_{j_2} \omega_{i i_1 i_1 i_2}^{j_1 j_2} M^{i_2 j} \quad (3.20)$$

After some algebra eq.(3.20) can be rewritten, neglecting q^2 contact terms which are irrelevant for our semiclassical analysis, as

$$\Psi_{(2)}^{NSNS} = \tilde{\xi}_{ij} q_m q_n (\eta \gamma^{ink} \tilde{\eta} \eta \gamma^{jmk} \tilde{\eta} - \eta \gamma^{im} \eta \tilde{\eta} \gamma^{jn} \tilde{\eta}) \quad (3.21)$$

where $\tilde{\xi}_{ij} \equiv \xi_{ik} M_{kj} + \xi_{jk} M_{ki}$. Notice that the combination of spinors in eq.(3.21) is the right one reproducing the covariant expression [19]

$$\Psi_{(2)}^{NSNS} = \tilde{\xi}_{\mu\nu} q_\alpha q_\beta \bar{\psi} \Gamma^{\mu\alpha\rho} \psi \bar{\psi} \Gamma^{\nu\beta}_\rho \psi \quad (3.22)$$

The RR coupling is

$$\Psi_{(2)}^{RR} = \sum_{k \text{ even}} \frac{1}{k!} C_{(k)}^{i_1 \dots i_k} \text{Tr}_S(\gamma^{i_1 \dots i_k} \gamma^{j_1 j_2} \gamma^{j_3 j_4} M) q_{l_1} q_{l_2} \omega_{j_1 \dots j_4}^{l_1 l_2} \quad (3.23)$$

Using again the gauge condition $q^{i_1} C_{(k)}^{i_1 \dots i_k} = 0$ and after some manipulations, similar to those so far performed, it is not difficult to put eq.(3.23) into the covariant form

$$\Psi_{(2)}^{RR} = \sum_{k \text{ even}} \frac{1}{k!} C_{\mu_1 \dots \mu_k}^{(k)} \text{Tr}_S(\Gamma^{\mu_1 \dots \mu_k} \Gamma_{\nu_1 \nu_2} \Gamma_{\nu_3 \nu_4} \mathcal{M}) q_\alpha q_\beta (\bar{\psi} \Gamma^{\nu_1 \nu_2 \alpha} \psi \bar{\psi} \Gamma^{\nu_3 \nu_4 \beta} \psi) \quad (3.24)$$

Following the same procedure, it is possible to write down all the other terms.

IV. SPIN EFFECTS FOR THE P-P SYSTEM

Let us consider now spin interactions between two parallel slowly moving Dp-branes with impact parameter \vec{b} . Recall that we identify the “time” with X^1 ; accordingly, the boundary state of a brane moving with velocity v^i is obtained from the static one by applying the boost operator $e^{iv_i J^{1i}}$ [11], where

$$J^{1i} = \oint_{\tau=0} d\sigma \left(X^{[1} \partial_\sigma X^{i]} - \frac{i}{4} \bar{S} \rho^1 \gamma^{1i} S \right)$$

choosing to boost the boundary state defined in $\tau = 0$. Since, on the boundary, $iM^{(s)}\tilde{S} = S$, the vertex operator $V_B \equiv iv_i J^{1i}$ can be written as

$$V_B = v_i \oint_{\tau=0} d\sigma \left(X^{[1} \partial_\sigma X^{i]} + \frac{1}{2} S \gamma^{1i} S \right) \quad (4.1)$$

where now S is just the left-moving part of the world-sheet spinor. The same operator (4.1) could also have been derived from that of photons in Type I theory with a constant field strength background after a T-duality transformation [2].

As discussed in the introduction, leading orders in the expansion in powers of v can be read from correlations including the appropriate power of V_B insertions in the static brane-brane potential eq.(2.7). Before going on, it is important to point out that in computing leading orders of velocity-dependent potentials through correlation functions, we can actually directly extract potentials from the corresponding phase-shifts by simply dropping the overall time factor; this can be easily understood remembering that the bosonic coordinates along the velocity direction are not twisted and, according to eq.(2.6), the resulting zero mode integration turns then the phase shift into the potential, evaluated at the space-time $T = 0$, i.e. the time when the two D-branes are passing one each other at distance \vec{b} .

Given these preliminaries, we can evaluate correlation functions involving in general n V_B ’s and m V_η ’s. The corresponding amplitudes are given by

$$\mathcal{A}_{n,m} = \frac{1}{16n!(2m)!} \binom{2m}{m} \int_0^\infty dt \langle B_p, \vec{x} = 0 | e^{-2\pi t \alpha' p^+ (P^- - i\partial/\partial x^+)} (V_B)^n (V_\eta)^m | B_p, \vec{y} = \vec{b} \rangle \quad (4.2)$$

where the combinatorial factors come from the expansions of the boost and supersymmetry operator, eq.(3.1). There is an evident analogy between eq.(4.2) and four-point 1-loop amplitudes of massless states, in Type I string theory in the G-S formalism. In particular, the zero mode trace is vanishing unless all the eight zero modes S_0 are inserted [25], i.e.

$$\langle B_0 | R_0^N | B_0 \rangle = \text{Tr}_V[R_0^N] - \text{Tr}_S[R_0^N] = 0, \quad \text{for } N < 4$$

where the trace and matrix multiplication in both terms are over the vectorial and spinorial indices. Since V_B , as well as V_η , provides at most two of them, a total of $n + m \geq 4$ vertex insertions is needed in order to get a non-zero result. The first non-vanishing trace is

$$\begin{aligned}
t^{i_1 \dots i_8} &\equiv \text{Tr}_{S_0} R_0^{i_1 i_2} R_0^{i_3 i_4} R_0^{i_5 i_6} R_0^{i_7 i_8} \\
&= -\frac{1}{2} \epsilon^{i_1 \dots i_8} - \frac{1}{2} \left[\delta^{i_1 i_4} \delta^{i_2 i_3} \delta^{i_5 i_8} \delta^{i_6 i_7} + \text{perm.} \right] \\
&\quad + \frac{1}{2} \left[\delta^{i_2 i_3} \delta^{i_4 i_5} \delta^{i_6 i_7} \delta^{i_8 i_1} + \text{perm.} \right]
\end{aligned} \tag{4.3}$$

where “perm.” means permutations of the pairs $(i_{2n-1} i_{2n})$ plus antisymmetrization within all the pairs.

We will consider in the following the special case $n + m = 4$. This corresponds, for a fixed m , to the leading order in the velocity of the associated spin potential. The interest of this case lies in the fact that, being determined only by the fermionic zero mode part of both vertex insertions V_B and V_η , massive string contributions precisely cancel, exactly as in eq.(2.8). These amplitudes are therefore *scale invariant*, in the sense that their dependence on the distance \vec{b} is exact, keeping the same functional form at any finite distance. In the following, expressions similar to eq.(4.2) will be denoted simply by $\mathcal{A}_n \equiv \langle V_B^n V_\eta^{4-n} \rangle$, in order to lighten the notation. We wrote in eq.(4.2) all the supercharges applied to the same boundary state; being fixed simply by a zero modes analysis, the computation will not depend on the choice of the boundary, whereas the physical interpretation as polarization effects will be different. The polarizations of the two D-branes are indeed given by the supersymmetric parameters $\eta_1^a, \eta_1^{\dot{a}}$ and $\eta_2^a, \eta_2^{\dot{a}}$ associated to the two boundaries, as shown in figure 3.

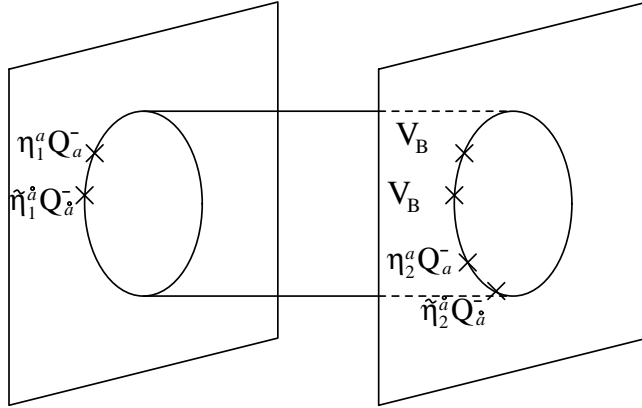


Fig. 3: example of a spin-dependent term at v^2 order, $m = n = 2$.

Let us start by inserting only the bosonic vertex operators V_B , that means to consider the universal v^4 potential $\mathcal{A}_4 = \langle V_B^4 \rangle$. From the above analysis, it follows that a non-vanishing

result is obtained only when we pick up the fermionic part of the operator (4.1) and in particular the zero modes for each operator S . The computation is straightforward and the result is

$$\mathcal{A}_4 = \frac{V_{p+1}}{64} T_p^2 |v|^4 G_{9-p}(\vec{b}) \quad (4.4)$$

As well known, possible contributions to a static force or to v^2 -potentials are absent due to a compensation between the gravitational and dilatonic fields (attractive) and the RR A_{p+1} field, that for two Dp-branes is repulsive, of course. In this formalism, it is immediately clear that supersymmetry implies a contribution starting only like v^4 .

The first spin effect is given by $\mathcal{A}_3 = \langle V_B^3 V_\eta \rangle$; going through the same steps and after some simple algebra, one obtains

$$\begin{aligned} \mathcal{A}_3 &= \frac{V_{p+1}}{8} |v|^2 (4\pi^2 \alpha')^{4-p} \int_0^\infty dt \int \frac{d^{9-p} q}{(2\pi)^{9-p}} e^{i\vec{q} \cdot (\vec{x} - \vec{y}) - 2\pi t \alpha' q^2} q_j \omega_{1i}^j(\eta) v^i \\ &= -i \frac{V_{p+1}}{16} T_p^2 |v|^2 v^i \tilde{\eta} \gamma^{1ij} \eta \partial_j G_{9-p}(\vec{b}) \end{aligned} \quad (4.5)$$

that represents a spin-orbit like coupling between branes. From eqs.(3.11) and (3.12) we can derive the NSNS and RR polarizations of the exchanged states, responsible of these non-minimal couplings. In order to perform the analytic continuation of eq.(4.5) to Minkowskian coordinates, it is convenient to write covariantly the term $\tilde{\eta} \gamma^{1ij} \eta$, whose $SO(1, 9)$ expression is $\bar{\psi} \Gamma^{1\mu\nu} \psi$. Performing the rotation we obtain $i \bar{\psi} \Gamma^{0\mu\nu} \psi$, and sending $v^i \rightarrow i v^i$ leads finally to

$$\mathcal{A}_3^{Mink.} = -\frac{V_{p+1}}{32} T_p^2 |v|^2 v_i \partial_j G_{9-p}(\vec{b}) J^{0ij} \quad (4.6)$$

where $i, j = 1, \dots, 9$ and $J^{\mu\nu\rho} \equiv i \bar{\psi} \Gamma^{\mu\nu\rho} \psi$. The next spin effect is $\mathcal{A}_2 = \langle V_B^2 V_\eta^2 \rangle$; in this case we have to distinguish two possible configurations, depending to which boundary state we apply the supercharges:

$$\mathcal{A}_2^{(1)} = \langle V_B^2 V_\eta^2 \rangle; \quad \mathcal{A}_2^{(2)} = \langle V_{\eta_1} V_B^2 V_{\eta_2} \rangle$$

These two contributions can be written as

$$\mathcal{A}_2^{(1)} = \frac{V_{p+1}}{32} T_p^2 \omega_{i_1 \dots i_4}^{ij}(\eta) t^{i_1 \dots i_4 1 k 1 l} v_k v_l \partial_i \partial_j G_{9-p}(\vec{b}) \quad (4.7)$$

$$\mathcal{A}_2^{(2)} = \frac{V_{p+1}}{32} T_p^2 \omega_{[i_1 i_2]}^i(\eta_1) \omega_{i_3 i_4]}^j(\eta_2) t^{i_1 \dots i_4 1 k 1 l} v_k v_l \partial_i \partial_j G_{9-p}(\vec{b}) \quad (4.8)$$

Noting that

$$t^{i_1 \dots i_4 1 k 1 l} v_k v_l = v^2 (4 \delta^{1i_2} \delta^{1i_4} \delta^{i_1 i_3} - \delta^{i_1 i_3} \delta^{i_2 i_4}) - 4 v^{i_1} v^{i_4} \delta^{i_2 i_3} \quad (4.9)$$

and working out the spinor algebra, it can be shown that eq.(4.7) reconstructs the covariant amplitude, that after analytic continuation, takes the following form:

$$\mathcal{A}_2^{(1)} = \frac{V_{p+1}}{768} T_p^2 v^2 (2J^{m0q} J_{0q}^n - J^{mpq} J_{pq}^n + 4J^{m\rho} J_{\rho j}^n \hat{v}^i \hat{v}^j) \partial_m \partial_n G_{9-p}(\vec{b}) \quad (4.10)$$

Latin letters i, j, k, \dots label $SO(9)$ indices running from 1 to 9, in contrast to $SO(1, 9)$ indices, denoted with Greek letters. In the same way, one can reconstruct the explicit covariant form of eq.(4.8) and all the remaining spin effects that will follow. We do not report the explicit relations for all the cases, being quite lengthy, as well as the analytic continuation. The remaining spin effects are

$$\mathcal{A}_1^{(1)} = \langle V_B V_\eta^3 \rangle = \frac{V_{p+1}}{144} T_p^2 \omega_{i_1 \dots i_6}^{ijk}(\eta) t^{i_1 \dots i_6 1l} v_l \partial_i \partial_j \partial_k G_{9-p}(\vec{b}) \quad (4.11)$$

$$\mathcal{A}_1^{(2)} = \langle V_{\eta_1} V_B V_{\eta_2}^2 \rangle = \frac{V_{p+1}}{144} T_p^2 \omega_{i_1 i_2}^i(\eta_1) \omega_{i_3 \dots i_6}^{jk}(\eta_2) t^{i_1 \dots i_6 1l} v_l \partial_i \partial_j \partial_k G_{9-p}(\vec{b})$$

and the static force

$$\mathcal{A}_0^{(1)} = \langle V_\eta^4 \rangle = \frac{V_{p+1}}{4(4!)^2} T_p^2 \omega_{i_1 \dots i_8}^{ijkl}(\eta) t^{i_1 \dots i_8} \partial_i \partial_j \partial_k \partial_l G_{9-p}(\vec{b})$$

$$\mathcal{A}_0^{(2)} = \langle V_{\eta_1} V_{\eta_2}^3 \rangle = \frac{V_{p+1}}{4(4!)^2} T_p^2 \omega_{i_1 i_2}^i(\eta_1) \omega_{i_3 \dots i_8}^{jkl}(\eta_2) t^{i_1 \dots i_8} \partial_i \partial_j \partial_k \partial_l G_{9-p}(\vec{b})$$

$$\mathcal{A}_0^{(3)} = \langle V_{\eta_1}^2 V_{\eta_2}^2 \rangle = \frac{V_{p+1}}{4(4!)^2} T_p^2 \omega_{i_1 \dots i_4}^{ij}(\eta_1) \omega_{i_5 \dots i_8}^{kl}(\eta_2) t^{i_1 \dots i_8} \partial_i \partial_j \partial_k \partial_l G_{9-p}(\vec{b})$$

In all these cases the one-point functions considered in last section allows to see which are, in each configuration, the massless string excitations responsible of all these polarization effects.

V. SPIN EFFECTS FOR THE P-P+4 SYSTEM

Let us now consider spin potentials for parallel p-p+4 brane configurations. Like more general p-q systems with mixed Neumann-Dirichlet boundary conditions in four directions, these BPS configurations preserves 1/4 of the initial supersymmetries, rather than the 1/2 of the p-p system. This residual supersymmetry protects as before the leading order term in the velocity from higher massive modes contributions; however, unlike the p-p system, the reduced amount of supersymmetry allows now a non-trivial metric in the D_p moduli space. In particular the D0-D4 system, studied in [6], was proposed in [27] as a matrix description for the scattering of an eleven dimensional supergraviton off the background of a longitudinal fivebrane. The aim of this section is to study the leading spin dependence

of the potential felt by slowly moving p-branes in the p+4 background. The relevant zero mode traces are in this case of the form

$$\langle B_p | \mathcal{O} | B_{p+4} \rangle = \text{Tr}_{S_0}[\mathcal{O}N] \equiv \text{Tr}_V[\mathcal{O}N] - \text{Tr}_S[\mathcal{O}N] \quad (5.1)$$

where \mathcal{O} is a product of R_0^{ij} arising from the zero mode part of the V_B and V_η vertex insertions and

$$\begin{aligned} N^{ij} &\equiv (M_p^T M_{p+4})^{ij} = \begin{pmatrix} I_{p+1} & 0 & 0 \\ 0 & -I_4 & 0 \\ 0 & 0 & I_{3-p} \end{pmatrix} \\ N_{\dot{a}\dot{b}} &\equiv (M_p^T M_{p+4})_{\dot{a}\dot{b}} = (\gamma^{p+2} \dots \gamma^{p+5})_{\dot{a}\dot{b}} \end{aligned} \quad (5.2)$$

By simple inspection of eq.(5.1), using the matrices (5.2) and the representation of the operators (3.8), we get vanishing traces for $\mathcal{O} = 1, R_0^{ij}$. The first non trivial result is

$$\begin{aligned} t^{i_1 \dots i_4} &\equiv \text{Tr}_{S_0} R_0^{i_1 i_2} R_0^{i_3 i_4} \\ &= 2 \epsilon^{i_1 \dots i_4 p+2 \dots p+5} \\ &\quad + 2 \left(\delta^{i_1 p+2} \delta^{i_2 p+3} \delta^{i_3 p+4} \delta^{i_4 p+5} + N^{i_2 i_4} \delta^{i_1 i_3} + \text{perm.} \right) \end{aligned} \quad (5.3)$$

where by “perm.” we mean as before an antisymmetrization within each pair (i_1, i_2) , (i_3, i_4) and symmetrization under the exchange of each of these pairs.

The relevant amplitudes describing leading spin-effects are then

$$\mathcal{A}_n = \frac{1}{16n!(4-2n)!} \binom{4-2n}{2-n} \int_0^\infty dt \langle B_p, \vec{x} = 0 | e^{-2\pi t \alpha' p^+ (P^- - i\partial/\partial x^+)} (V_B)^n (V_\eta)^{2-n} | B_{p+4}, \vec{y} = \vec{b} \rangle$$

where the total number of vertex insertions now is two providing the four zero modes required to get the first non-trivial result from eq.(5.3). The rest of the computation follow the lines of last section. We are left with the universal term

$$\mathcal{A}_2 = \frac{V_{p+1}}{4} T_p T_{p+4} |v|^2 G_{5-p}(\vec{b}) \quad (5.4)$$

and the leading spin potentials

$$\begin{aligned} \mathcal{A}_1 &= \frac{V_{p+1}}{4} T_p T_{p+4} \omega_{i_1 i_2}^i(\eta) t^{1j i_1 i_2} v_j \partial_i G_{5-p}(\vec{b}) \\ \mathcal{A}_0^{(1)} &= \frac{V_{p+1}}{16} T_p T_{p+4} \omega_{i_1 \dots i_4}^{ij}(\eta) t^{i_1 \dots i_4} \partial_i \partial_j G_{5-p}(\vec{b}) \\ \mathcal{A}_0^{(2)} &= \frac{V_{p+1}}{16} T_p T_{p+4} \omega_{i_1 i_2}^i(\eta_1) \omega_{i_3 i_4}^j(\eta_2) t^{i_1 \dots i_4} \partial_i \partial_j G_{5-p}(\vec{b}) \end{aligned} \quad (5.5)$$

The appearance of T_{p+4} and G_{5-p} instead of the T_p and G_{9-p} for the p-p system is due to the lack of four Dirichlet-Dirichlet transferred momentum integrations.

We recall that eqs.(5.4) and (5.5) are exact to any order in the brane separation \vec{b} . Of course this is a peculiar property only of these leading order terms and of the supersymmetric p-p, p-p+4 configurations. Higher order terms or more general brane configurations will involve contributions from the oscillator part of the vertices (4.1), (3.3) described by modular functions with non-trivial transformation properties which in general distinguish the large and short distance behaviors. We should say however that this property is shared by an amount of other interesting brane systems. In a final discussion we will go on through many of these examples showing how supersymmetry is enough to ensure the scale invariance of all their relevant leading interactions.

VI. FIELD THEORY INTERPRETATION AND D0-BRANE GYROMAGNETIC RATIO

In the present section we discuss the field theory interpretation of our results. We will show in particular that the knowledge of all the one-point functions of the massless fields of Type IIA/B supergravity allows to infer the complete and generic asymptotic form of the corresponding p-brane solution. Moreover, the spin-effects in scattering amplitudes that we have computed in section 4 and the supersymmetric cancellation of some of their leading orders proves to constitute an extremely efficient way to fix unambiguously the various coefficients entering the solution, and in particular the relative strength of the NSNS attraction and the RR repulsion (the fact that normalizations are better encoded in scattering amplitude than in one-point functions, especially through the vanishing of leading order, was already appreciated in Polchinski's computation of the Dp-brane charge [1]). As we will see, this approach yields a powerful technique to extract informations about a generic component of the p-brane multiplet. The analogous computation in supergravity would consist in performing supersymmetry transformations to the usual p-brane solution, to determine all the spinning superpartners; this requires looking up to eight variations, a program that, as can be appreciated from previous works [28–30], is out of reach within the component fields formalism.

Collecting the covariant one-point functions (3.15), (3.17), (3.19), (3.22) and (3.24), for up to four supercharge insertions, the NSNS and RR asymptotic fields for a generic component of the Dp-brane multiplet can be written as a multipole expansion in momentum space:

$$\xi^{\mu\nu} = \kappa^2 \left[A_0 M^{\mu\nu} + A_1 J^{\mu\sigma\alpha} M_\sigma^\nu q_\alpha + A_2 J^{\mu\alpha\rho} J_\rho^{\sigma\beta} M_\sigma^\nu q_\alpha q_\beta + \dots \right] \quad (6.1)$$

$$C_{(k)}^{\mu_1 \dots \mu_k} = \frac{\kappa^2}{k!} [B_0 \text{Tr}_S[\Gamma^{\mu_1 \dots \mu_k} \mathcal{M}] + B_1 \text{Tr}_S[\Gamma^{\mu_1 \dots \mu_k} \Gamma_{\nu_1 \nu_2} \mathcal{M}] J^{\nu_1 \nu_2 \alpha} q_\alpha \\ + B_2 \text{Tr}_S[\Gamma^{\mu_1 \dots \mu_k} \Gamma_{\nu_1 \nu_2} \Gamma_{\nu_3 \nu_4} \mathcal{M}] J^{\nu_1 \nu_2 \alpha} J^{\nu_3 \nu_4 \beta} q_\alpha q_\beta + \dots] \quad (6.2)$$

We have restored the ten-dimensional Plank constant κ^2 for convenience. Dots stand for six and eight supercharge insertions, corresponding to three and four powers of momentum, that we shall not consider. The constants A_i, B_i could in principle be fixed by correctly normalizing the one-point functions reported in section 3; however, this is awkward, and since any final conclusion will eventually depend in a crucial way on these constants, we will take advantage of our results for the scattering amplitude to fix them unambiguously.

From now on we specialize to the D0-brane, for which $M_0^0 = -1$, $M_j^i = \delta_j^i$ and $\mathcal{M} = \Gamma^0$; the other cases can be treated in the same way. Recall that in the NSNS sector, a generic field $\xi_{\mu\nu}$ is decomposed into trace, symmetric and antisymmetric parts ϕ , $h_{\mu\nu}$ and $b_{\mu\nu}$ as

$$\epsilon_{\mu\nu}^{(\phi)} = \frac{1}{4}(\eta_{\mu\nu} - q_\mu l_\nu - q_\nu l_\mu), \quad \epsilon_{\mu\nu}^{(h)} = \xi_{(\mu\nu)}, \quad \epsilon_{\mu\nu}^{(b)} = \xi_{[\mu\nu]}$$

where l^μ is a vector satisfying $q \cdot l = 1$, $l^2 = 0$. The asymptotic fields in the NSNS sector are then found to be

$$\begin{aligned} \phi &= \frac{3}{2}\kappa^2 M G_9(r) + \frac{1}{4}\kappa^2 C J^{mpq} J_{pq}^n \partial_m \partial_n G_9(r) + \dots \\ \begin{cases} h_{00} &= \kappa^2 M G_9(r) + \kappa^2 C J^{m0q} J_{0q}^n \partial_m \partial_n G_9(r) + \dots \\ h_{ij} &= \delta_{ij} \kappa^2 M G_9(r) + \kappa^2 C J_i^m J_{j\rho}^n \partial_m \partial_n G_9(r) + \dots \\ h_{0i} &= 2\kappa^2 A J_{0i}^m \partial_m G_9(r) + \dots \end{cases} \\ \begin{cases} b_{ij} &= \kappa^2 A J_{ij}^m \partial_m G_9(r) + \dots \\ b_{0i} &= 2\kappa^2 C J_{0q}^m J_i^q \partial_m \partial_n G_9(r) + \dots \end{cases} \end{aligned} \quad (6.3)$$

whereas eq.(6.2) in the RR sector become

$$\begin{aligned} \begin{cases} C_0 &= 2\kappa^2 Q G_9(r) + \kappa^2 D J^{m\rho\tau} J_{\rho\tau}^n \partial_m \partial_n G_9(r) + \dots \\ C_i &= 2\kappa^2 B J_{0i}^m \partial_m G_9(r) + \dots \end{cases} \\ \begin{cases} C_{0ij} &= \kappa^2 B J_{ij}^m \partial_m G_9(r) + \dots \\ C_{ijk} &= 2\kappa^2 D J_{0[i}^m J_{jk]}^n \partial_m \partial_n G_9(r) + \dots \end{cases} \end{aligned} \quad (6.4)$$

The constants A_i, B_i have been redefined and called M, A, B, Q, C, D for later convenience, and again, dots stand for higher derivative terms associated to further supercharge insertions.

Comparing eqs.(6.3) and (6.4) with the usual 0-brane solution [31] and the general result valid in D dimensions derived in [32], we conclude that M is the mass and Q the electric charge, whereas $2AJ_{0ij} = J_{ij}$ is the angular momentum and $BJ_{0ij} = \mu_{ij}$ the magnetic moment, so that the gyromagnetic ratio, defined by the relation $\mu^{ij} = (gQ)/(2M)J^{ij}$, is given by $g = (MB)/(QA)$. Also, the electric and gravitational dipole moments vanish, since they would correspond to one-derivative terms in C_0 and h_{00}, h_{ij} respectively. On the contrary, the presence of non-vanishing two-derivative terms in eqs.(6.3) shows the presence of a gravitational quadrupole moment for D-particles. Similarly, there are two-derivative terms also in eqs.(6.4) corresponding to gauge quadrupole effects. Actually, the one in C_0 vanishes due to a Fierz identity, showing that D-particles have a vanishing electric quadrupole moment with respect to the RR one-form. However, analogously to the gyromagnetic ratio g , we can define the ratio of the gauge and quadrupole moments by $\tilde{g} = 4(MD)/(QC)$.

It is now straightforward to show how the semiclassical analysis of the phase-shift between two of these configurations can be used to determine in a simple way the value of the gyromagnetic ratio g and its quadrupole analogue \tilde{g} associated to D0-branes. According to [33,30], massive Kaluza-Klein states present a common value $g = 1$, contrarily to the usual and “natural” [34–36] value $g = 2$ shared by all the known elementary particles (neglecting radiative corrections, of course). This particular signature of Kaluza-Klein states can be useful to establish the 11-dimensional nature of D0-branes, implying $g = 1$. This consistency check has been recently performed in [30] considering D0-branes as extended extremal 0-brane solution of IIA supergravity. We present an alternative and independent argument that relies on the “stringy” nature of D0-branes as points on which open strings can end; in particular, we show that $g = 1$ is the only possible value compatible with the cancellation of the linear term in velocity in the first spin effect, eq.(4.6) [§]. Similarly, we will show that our stringy analysis predicts for the quadrupole analog the value $\tilde{g} = 1$ from the cancellation of the static contribution to the second spin effect, eq.(4.7).

Consider first the scattering of a scalar 0-brane, taken as a probe, off a spinning 0-brane, acting as source. The effective action for the probe is (in the string frame)

$$\mathcal{S} = -M \int d\tau e^{-\phi} \sqrt{-g_{\mu\nu} \dot{X}^\mu \dot{X}^\nu} - Q \int d\tau C_\mu \dot{X}^\mu \quad (6.5)$$

For a trajectory with constant velocity $v = \tanh \pi\epsilon$, we can choose $X^0(\tau) = \tau \cosh \pi\epsilon$, $X^i(\tau) = \tau \hat{v}^i \sinh \pi\epsilon$. Expanding for small velocities and weak fields ($\kappa \rightarrow 0$), one finds, dropping a constant term, $\mathcal{S} = \int d\tau \sum_{n \geq 0} v^n \mathcal{L}_n$ with

[§]Actually the fact that the value of g is related to the cancellation of the leading term linear in v was implicit in ref. [23,24], even if its significance was not completely recognized.

$$\begin{aligned}
\mathcal{L}_0 &= M\phi + \frac{1}{2}Mh_{00} - QC_0 \\
\mathcal{L}_1 &= Mh_{0i}\hat{v}^i - QC_i\hat{v}^i, \quad \mathcal{L}_2 = \frac{1}{2}M(h_{00} + h_{ij}\hat{v}^i\hat{v}^j) - \frac{1}{2}QC_0 \\
\mathcal{L}_3 &= Mh_{0i}\hat{v}^i - \frac{1}{2}QC_i\hat{v}^i, \quad \mathcal{L}_4 = \frac{1}{2}M(h_{00} + h_{ij}\hat{v}^i\hat{v}^j) - \frac{3}{8}QC_0
\end{aligned} \tag{6.6}$$

We know from the amplitudes computed in section 4 that the leading non-vanishing contributions to the scattering amplitude behave like v^n/r^{7+n} , all lower orders in velocity cancelling by supersymmetry. Substituting the relevant asymptotic fields of the spinning 0-brane from eqs.(6.3) and (6.4), one then finds the following conditions:

$$\begin{aligned}
\mathcal{L}_0|_G = 0 &\Rightarrow M = Q, \quad \mathcal{L}_0|_{\partial^2 G} = 0 \Rightarrow MC = 4QD \\
\mathcal{L}_1|_{\partial G} = 0 &\Rightarrow MA = QB \\
\mathcal{L}_2|_G = 0 &\Rightarrow M = Q
\end{aligned} \tag{6.7}$$

Altogether, this yields

$$Q = M, \quad g = 1, \quad \tilde{g} = 1 \tag{6.8}$$

One can now use these results to write down the structure of the leading non-vanishing contributions to the scattering amplitude. Actually, thanks to the results obtained in section four we can write down the amplitude with all the coefficients fixed. Up to an overall factor, we get

$$\begin{aligned}
\mathcal{A} &= \kappa^2 M^2 v^4 G_9(r) + 2\kappa^2 M v^3 J_{0i}{}^m \hat{v}^i \partial_m G_9(r) \\
&\quad + \frac{1}{12} \kappa^2 M v^2 (2J^{m0q} J_{0q}^n - J^{mpq} J_{pq}^n + 4J^{m\rho}{}_i J_{\rho j}^n \hat{v}^i \hat{v}^j) \partial_m \partial_n G_9(r) + \dots
\end{aligned} \tag{6.9}$$

The matching of the tensor structure of the v^3 and v^2 terms with expressions (4.6) and (especially) (4.10) is a non-trivial check of the consistency of the whole picture.

A comment is in order on how our boundary state formalism for describing higher spin Dp-branes is related to the supergravity description, where p-branes appear as solitonic solutions. As already said, the asymptotic fields found by applying the procedure of this section correspond to supergravity solutions obtained by taking supersymmetric variations of the usual scalar ones. This has been partially done in [30] for the D0-brane solution, where the second supersymmetry variation was used to compute the angular momentum dependence of $h_{\mu\nu}$ and C_μ . Using the same strategy, we have similarly checked that the angular momentum contributions to the higher forms $b_{\mu\nu}$ and $C_{\mu\nu\rho}$ (which have not been considered in [30]) correctly reproduce those in eqs.(6.3) and (6.4). We have also checked

that the fourth supersymmetry variation reproduces all the two-derivative terms we find, but it is unrealistic to compute and trust the coefficient because of the increasing complexity of the involved expressions.

Finally, another interesting outcome of the knowledge of the asymptotic fields (6.3), (6.4) is the possibility to derive the supersymmetric completion of the 0-brane world-line effective action (6.5) *in an arbitrary Type IIA background*, at least for weak fields. For example, it is not difficult to verify that, in much the same way as the part of the asymptotic fields going like $1/r^7$ can be derived from the linearization of the action (6.5), the part of the fields going like $1/r^8$ can be derived from the following non-minimal couplings

$$\mathcal{S}_{2Q} = \int d\tau \left[-\partial_i h_{0j} J^{0ij} + \frac{1}{4} \partial_i b_{jk} J^{ijk} + \partial_i C_j J^{0ij} - \frac{1}{4} \partial_i C_{0jk} J^{ijk} \right] \quad (6.10)$$

The coefficients have been further checked by computing the static force contribution of order $1/r^9$ between two spinning 0-branes, that vanishes as expected.

Notice that the covariant form of eq.(6.10) should be obtained by replacing each 0 index by a contraction with the momentum \dot{X}^μ ; in such a way, the fields generated by a moving 0-brane are given by the boost of those produced by a static one. One obtains

$$\mathcal{S}_{2Q} = \int d\tau \left[\Gamma^\rho_{\sigma\mu} \dot{X}^\mu \dot{X}^\nu J_\rho{}^\sigma{}_\nu + \frac{1}{24} H_{\mu\nu\rho} J^{\mu\nu\rho} + \frac{g}{2} F_{\mu\nu} \dot{X}^\rho J^{\mu\nu}{}_\rho + \frac{g}{72} F_{\mu\nu\rho\sigma} \dot{X}^\mu J^{\nu\rho\sigma} \right] \quad (6.11)$$

where $F_{\mu\nu}$, $F_{\mu\nu\rho\sigma}$ and $H_{\mu\nu\rho}$ are the field-strengths of C_μ , $C_{\mu\nu\rho}$ and $b_{\mu\nu}$. We recognize a two-fermion gravitational term showing up through a coupling to the Christoffel connection $\Gamma^\mu_{\alpha\beta}$, coming from the (linearized) spin-connection entering the spinor covariant derivative, and similar non-minimal couplings to the gauge field curvatures. At next order, a four-fermion term manifesting itself through a coupling to the Riemann tensor is expected, among further non-minimal couplings. The coupling to the higher RR forms and the NSNS antisymmetric tensor seems to occur through more complicated terms which correctly disappear in the absence of fermionic background. From the eleven-dimensional point of view, eq.(6.11) is the Kaluza-Klein reduction (keeping J to be ten-dimensional) of an action containing only the first and last terms for the eleven-dimensional metric and three form. In order to work, this requires $g = 1$, as it is.

VII. FINAL REMARKS AND CONCLUSIONS

We studied in the present paper interactions between brane configurations associated to two parallel p-p and p-p+4 branes, using the Green-Schwarz boundary state formalism. We found explicit expressions for spin-dependent interactions between moving branes. Our

general strategy can be summarized as follows: instead of considering the full configuration of moving branes, where supersymmetry is broken, we perturb through appropriate vertex operators the supersymmetric vacuum associated to the static p-p (p-p+4) system, allowing in this way to easily derive important results on the structure of the exact (in powers of α') leading spin interactions in a velocity expansion. The cylinder computation of these terms collapses to its zero mode contribution, supporting an equivalent description in terms of either the open (matrix model) or closed (supergravity) massless degrees of freedom. However the analysis performed on the relevant cylinder correlations is quite general and can be easily extended to several other previously studied brane configurations (see for example [5,37–41]) where a similar long-short distance matching of their leading interactions were observed. They fall in general into two main groups: supersymmetric brane configurations, which include besides the examples studied above, the p-p+8 systems, bound states between p-p+2, p-p+2-p+4, p-p+4, ... D-branes, and any S or T-dual combinations of these systems; and brane configurations which are supersymmetric only in a certain limit of their moduli space. Bound states can be considered in general as fluxes for the gauge field living on the boundary of the biggest D-brane, modifying therefore its boundary conditions. The corresponding light-cone boundary state for a generic condensate was constructed in [18]. The cylinder amplitude defined by two of these boundary states, in the case that some of the supersymmetries are preserved (take for example two indetical p-p+2 bound states or the S-dual analog of two D-strings with equal electric fluxes turned on [10]) , will lead to similar vanishing traces as in eq.(5.1), unless $N/4$ velocity insertions soak the $N/2$ left zero modes, N being the number of supercharges left unbroken by the system. Again the spin-dependent dynamics can be studied by inserting supercharges on the cylinder, and once more an equivalent matrix-supergravity description for the slowly moving regime is guaranteed.

The second interesting class of configurations (and less straight) for which an analysis along the lines of this paper can be followed, is inspired from the brane systems studied in [38], which although not supersymmetric, become so in a given limit of the moduli space. In the analysis of these systems one can follow a strategy parallel to the one previously applied to the case of moving branes. In that case supersymmetry is broken for finite velocity v , but the existence of a supersymmetric limit $v = 0$ allow us to study leading orders by a simple analysis of the zero mode structure of amplitudes involving the insertion of vertex operators corresponding to the deformation (in that case v) from the supersymmetric point. Similarly, now we look at the neighborhood of a specific choice of flux for which some supersymmetry is restored. The fermionic part of the operators corresponding to deformations about this supersymmetric point coincide with the vertex (4.1), once we substitute the plane $(1i)$ defining the boost operation with the condensate euclidean plane (mn) , and therefore the results can be read directly from the ones quoted above for the moving brane systems. We

can illustrate this with the simple example of a Dp-brane, wrapped around a T^2 with a magnetic flux $f_1 = \frac{N}{2\pi R_m R_n}$ turned on. The boundary state for this specific condensate can be read from the more general one found in [18] to be defined by eqs.(2.4) and (2.5) through the matrices

$$M_{ij} = \sqrt{(1+f^2)} \begin{pmatrix} -I_{p-1} & 0 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha & 0 \\ 0 & \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 0 & I_{7-p} \end{pmatrix}$$

$$M_{\dot{a}\dot{b}} = (\gamma^{12} + f)\gamma^1 \dots \gamma^{p+1} \quad (7.1)$$

where $\cos \alpha \equiv -\frac{1-f^2}{1+f^2}$. Notice that eqs.(7.1) reduce, in the large f limit, to the matrices (2.2),(2.3) defining the D(p-2) brane, up to an overall f factor and the missing of two momentum modes corresponding to the Neumann-Dirichlet directions. As we discussed before, we can study the leading interactions of this bound state with a D(p-2) ordinary brane by simply perturbing the system by a small $c \equiv 1/f$ quantity from the supersymmetric $c = 0$ point. The universal potential is then defined by correlators involving insertions of $R_0^{i1}v^i$ and $R_0^{mn}\pi c$ in the D(p-2)-D(p-2) cylinder, and as before we have vanishing traces unless at least four of these insertions soak the eight zero modes, leaving

$$\langle B_{p-2}|B_p, c, v\rangle = \frac{1}{\pi c} \frac{V_{p-1}}{32} T_p T_{p-2} (v^4 + 2v^2(\pi c)^2 + (\pi c)^4) G_{7-p}(r). \quad (7.2)$$

The $1/c$ factor can be interpreted as the number of D(p-2) branes arising from the Dp-brane in the $c \rightarrow 0$ limit, while the two missing powers in r represents the reduced transverse space to the system, and the relative coefficients are fixed by the kinematical tensor (4.3) **. Given, as before, by an exact string computation at the relevant order in the v, c expansion, this potential is valid at any transverse distance r and in particular admits equivalent Super Yang-Mills and supergravity descriptions. The p=2 case is the relevant one for the analysis performed in [38]. In that reference the authors study the graviton-membrane, static membrane-antimembrane and orthogonal moving membranes scattering. In each case the infinite boost ($N \rightarrow \infty$ or equivalently $c \rightarrow 0$) represents a point where the 16 supercharges are recovered (for $v = c = 0$). The leading orders in v, c are given by eq.(7.2), and the scale invariance of these terms is guaranteed by our previous analysis, and checked explicitly in that reference. The case of orthogonal membranes is particular in the sense that contains two line of deformations $c \equiv c_1 + c_2 = 0$ and $c' \equiv c_1 - c_2 = 0$ (this is the case studied in [39]), c_1, c_2 being defined by the fluxes in each membrane, along which half

**A flip in the sign of the v^2 comes from the analytic continuation to the Minkowski plane.

the supersymmetries of the D0-D0 system are preserved. Along these lines, the potential starts then with v^2 as for the previously discussed p-p+4 system. The leading scale invariant interactions are in general given by

$$\langle c_2, B_{p2} | B_{p1}, c_1, v \rangle = \frac{1}{\pi^2 c_1 c_2} \frac{1}{32} T_2 T_2 (v^2 + (\pi c)^2) (v^2 + (\pi c')^2) G_5(r). \quad (7.3)$$

The absence of the static c^4 and c'^4 terms reflects the surviving of half-supersymmetries along the aforementioned lines. In [39,40] an exhaustive list of brane configurations was shown to present again agreement between the one-loop SYM and semi-classical supergravity descriptions of their potentials. Once more, homogenous polynomials of order four in the fluxes and velocities as in (7.2),(7.3) were found; an iteration of the analysis for the above discussed example provides a unified understanding of those results. We believe that this example can give a flavour of the generality of the analysis performed here, which extends to any supersymmetric (at least in a point of the moduli space) brane configuration and covers all (to our knowlegde) one-loop scattering tests of a given matrix description. We should say that scale invariance is however stronger than what a correct matrix description of supergravity interactions really requires. In fact higher loop potentials will not display a simple decoupling of their massive modes as in the examples studied here and only a matching between the two open-closed massless truncated computations can be at most expected. Understanding from the string point of view the results quoted in [42,43], where the v^6 potential arising at two loops in the super Yang-Mills effective action was shown to agree with the corresponding long range correction, or performing other higher loop tests of this correspondence between SYM and supergravity descriptions of D-brane interactions, is an essential step in the completion of the picture. It should be pointed out, however, that the light-cone formalism we used becomes awkward at higher loops, being affected by several disadvantages.

Finally, we applied our results about the spin-dependent D-brane interactions to the interesting case of the D-particle. The potential defined by a cylinder ending on two spinning D-particles allowed us to derive supersymmetric analogs of the universal BPS condition $Q = M$ for the rest of the components of the D0 supermultiplet. In particular we computed the gyromagnetic ratio $g = 1$ and a ratio of quadrupole moments $\tilde{g} = 1$. The value $g = 1$ is the consistent value for D0 branes considered as Kaluza-Klein modes of eleven dimensional supergravity and was reproduced by a corresponding supergravity computation in [30]. The results presented here can be considered as a further check of the identification of the 0-brane solitonic supergravity solution with the D-particle string definition as a point-defect where open strings can end. Moreover, the string analysis provides a compact and efficient way to obtain generic supergravity asymptotic solutions for spinning p-branes.

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APPENDIX A: PHASE-SHIFT COMPUTATION IN THE G-S FORMALISM

In this Appendix, we briefly show how the full phase-shift computation, performed in the covariant formalism in ref. [2] as an open string vacuum amplitude and in ref. [11] as an overlap of boundary states, is reproduced in the Green-Schwarz light-cone formalism. As throughout all this article, we will use the double analytic continuation described in [23] in order to define a moving brane; we will therefore work in Euclidean space-time and only at the end of the calculation we continue analytically our result back to Minkowski space-time^{††}.

1. Closed string channel

The moving boundary state can be obtained from the static one, eqs. (2.4)-(2.6), by performing a Lorentz transformation with negative rapidity ϵ . As explained, this will be performed in Euclidean space-time by a transverse rotation generated by the operator $J^{ij} = J_B^{ij} + J_F^{ij}$, where

$$J_B^{ij} = x^i p^j - x^j p^i - i \sum_{n=1}^{\infty} \frac{1}{n} \left(\alpha_{-n}^i \alpha_n^j - \alpha_{-n}^j \alpha_n^i - \tilde{\alpha}_{-n}^i \tilde{\alpha}_n^j + \tilde{\alpha}_{-n}^j \tilde{\alpha}_n^i \right) \quad (\text{A.1})$$

$$J_F^{ij} = -\frac{i}{4} \sum_{n=-\infty}^{\infty} \left(S_{-n}^a \gamma_{ab}^{ij} S_n^b + \tilde{S}_{-n}^a \gamma_{ab}^{ij} \tilde{S}_n^b \right) \quad (\text{A.2})$$

Taking the velocity in the X^8 direction, we have to compute $|B_v\rangle = e^{ivJ^{18}}|B\rangle$. The zero mode part of J_B^{18} breaks translational invariance along the X^8 direction and turns eq.(2.6) into

^{††}Throughout this appendix we fix $\alpha' = 1/2$.

$$|B_v, \vec{x}\rangle = (\sqrt{2\pi})^{4-p} \int \frac{d^{9-p}q}{(2\pi)^{9-p}} e^{i\vec{q}\cdot\vec{x}} |B_v\rangle \otimes |q_B\rangle \quad (\text{A.3})$$

with $q_B^i = (q^8 \sin \pi\epsilon, \vec{0}, q^8 \cos \pi\epsilon)$. The boosted boundary state (in momentum space) $|B_v\rangle$ is obtained by applying J^{18} to eqs.(2.4)-(2.5). The result corresponds to the replacement [23]

$$M^{ij} \rightarrow M^{ij}(v) = (\Sigma(v) M \Sigma^T(v))^{ij} \quad (\text{A.4})$$

$$M_{ab} \rightarrow M_{ab}(v) = (\Sigma(v) M \Sigma^T(v))_{ab} \quad (\text{A.5})$$

where $\Sigma(v)$ is the appropriate representation of the $SO(8)$ rotation:

$$\Sigma_v(v) = \begin{pmatrix} \cos \pi\epsilon & 0 & -\sin \pi\epsilon \\ 0 & I_6 & 0 \\ \sin \pi\epsilon & 0 & \cos \pi\epsilon \end{pmatrix} \quad (\text{A.6})$$

$$\Sigma_s(v) = \cos\left(\frac{\pi\epsilon}{2}\right) \delta_{ab} - \sin\left(\frac{\pi\epsilon}{2}\right) \gamma_{ab}^{18} \quad (\text{A.7})$$

After diagonalizing γ^{18} with a suitable global $SO(8)$ rotation of S_n^a and \tilde{S}_n^a , the cylinder amplitude between two Dp-branes moving with relative velocity v is found to be

$$\begin{aligned} \mathcal{A} = \frac{1}{16} V_p (2\pi^2)^{4-p} \int_0^\infty dt \int \frac{d^{8-p}q}{(2\pi)^{8-p}} e^{i\vec{q}\cdot(\vec{x}-\vec{y})} e^{-\frac{\pi}{2}tq^2} Z_0^F \\ \times \frac{1}{\sin \pi\epsilon} \prod_{n=1}^\infty \frac{|1 - e^{i\pi\epsilon/2} e^{-2\pi tn}|^8}{|1 - e^{i\pi\epsilon} e^{-2\pi tn}|^2 (1 - e^{-2\pi tn})^6} \end{aligned} \quad (\text{A.8})$$

The zero mode part Z_0^F no longer vanishes

$$Z_0^F = \langle B_0, v | B_0 \rangle = \text{Tr}_V M^T(v) M(0) - \text{Tr}_S M^T(v) M(0) = 16 \sin^4 \frac{\pi\epsilon}{2}$$

and after the analytic continuation $\epsilon \rightarrow i\epsilon$, the final result is

$$\mathcal{A} = \frac{V_p}{16\pi i} (2\pi^2)^{4-p} \int_0^\infty \frac{dt}{(2\pi t)^{\frac{8-p}{2}}} e^{-\frac{b^2}{2\pi i}} \frac{\vartheta_1^4(i\frac{\epsilon}{2}|2it)}{\vartheta_1(i\epsilon|2it)} \frac{\vartheta_1'(0|2it)}{\eta^{12}(2it)} \quad (\text{A.9})$$

which coincides with the well known result of ref. [2] after using the Riemann identity and the modular transformation $t \rightarrow 1/t$.

2. Open string channel

We compute now the phase-shift from the standard [2] open channel point of view in the light-cone gauge. The X^+ and X^- coordinates are here Neumann, as usual (and not

Dirichlet as before), and we could in principle consider only p-branes with $p \geq 2$, although the final result is clearly extendable to all branes.

Similarly to ref. [2], the action for two moving branes in the frame where one of them is at rest, is given by:

$$S = -\frac{1}{2\pi} \int d^2\sigma \partial_\alpha X^i \partial^\alpha X^i + \frac{i}{\pi} \int d^2\sigma \bar{S} \rho^a \partial_a S + \frac{v}{\pi} \oint_{\sigma=\pi} d\tau (X^1 \partial_\sigma X^8 - \frac{i}{4} \bar{S} \rho^1 \gamma^{18} S)$$

Varying this action, we require the boundary term to vanish and solve for the constraint; for the bosonic coordinates the result is identical to that of ref. [2] (with $\epsilon \rightarrow i\epsilon$), while the fermionic boundary conditions are found imposing $\delta S^a = -iM_{ab}\delta\tilde{S}^b$. The result is:

$$\begin{aligned} S^a(\tau, \sigma) &= P_+^{ab} \sum_{n=-\infty}^{+\infty} S_{-n}^b e^{-i(n+\epsilon/2)(\tau+\sigma)} + P_-^{ab} \sum_{n=-\infty}^{+\infty} S_{-n}^b e^{-i(n-\epsilon/2)(\tau+\sigma)} \\ \tilde{S}^a(\tau, \sigma) &= -iM_{ab} \left(P_+^{bc} \sum_{n=-\infty}^{+\infty} S_{-n}^c e^{-i(n+\epsilon/2)(\tau-\sigma)} + P_-^{bc} \sum_{n=-\infty}^{+\infty} S_{-n}^c e^{-i(n-\epsilon/2)(\tau-\sigma)} \right) \end{aligned} \quad (\text{A.10})$$

where $P_\pm = 1/2(1 \pm i\gamma^{18})$ and $\tan \pi\epsilon = v$.

The fermionic part of the normal ordered light-cone Hamiltonian P^- is then

$$\begin{aligned} P_F^- &= \frac{1}{2} \sum_{n=-\infty}^{+\infty} \left(S_{-n}^a (1 + i\gamma^{18})_{ab} S_n^b (n + \frac{\epsilon}{2}) + S_{-n}^a (1 - i\gamma^{18})_{ab} S_n^b (n - \frac{\epsilon}{2}) \right) \\ &= \sum_{n=1}^{\infty} (2n S_{-n}^a S_n^a + \epsilon S_{-n}^a i\gamma_{ab}^{18} S_n^b) + \frac{\epsilon}{2} S_0^a i\gamma_{ab}^{18} S_0^b \end{aligned} \quad (\text{A.11})$$

Note that since the S_n modes are space-time fermions, they are twisted by $\epsilon/2$, whereas the bosonic coordinate present a twist of ϵ . This implies that the eight $\epsilon/2$ -twisted fermions and the two ϵ -twisted bosons give a total contribution to the Hamiltonian equal to

$$8 \cdot \frac{1}{4} \frac{\epsilon}{2} (1 - \frac{\epsilon}{2}) - 2 \cdot \frac{1}{4} \epsilon (1 - \epsilon) = \frac{\epsilon}{2} \quad (\text{A.12})$$

As before, we can perform an $SO(8)$ rotation to the S_n oscillator modes to put the $S_{-n} i\gamma^{18} S_n$ term in a diagonal form; the total Hamiltonian P^- takes then the following form:

$$\begin{aligned} P^- &= \frac{1}{2p^+} \left[(p^i)^2 + \frac{b^2}{\pi^2} + 2 \sum_{a=1}^4 (n + \frac{\epsilon}{2}) S_{-n}^a S_n^a + 2 \sum_{a=5}^8 (n - \frac{\epsilon}{2}) S_{-n}^a S_n^a \right. \\ &\quad \left. + \frac{\epsilon}{2} S_0^a i\gamma_{ab}^{18} S_0^b + \epsilon + (\text{bos. osc.}) \right] \end{aligned} \quad (\text{A.13})$$

We can now compute the partition function

$$\begin{aligned}
\mathcal{A} &= \int_0^\infty \frac{dt}{t} \text{Tr} e^{-\pi t p^+ (P^- - i\partial_+)} \\
&= V_p \int_0^\infty \frac{dt}{t} \int \frac{d^p p}{(2\pi)^p} e^{-\frac{\pi t}{2}(p^2 + b^2/\pi^2)} \frac{e^{-\pi t \epsilon/2}}{1 - e^{-\pi t \epsilon}} \text{Tr}_{S_0} e^{-i\pi t \epsilon R_0^{18}} \times \\
&\quad \times \prod_{n=1}^\infty \frac{(1 - e^{-\pi t(n+\epsilon/2)})^4 (1 - e^{-\pi t(n-\epsilon/2)})^4}{(1 - e^{-\pi t n})^6 (1 - e^{-\pi(n+\epsilon)t}) (1 - e^{-\pi(n-\epsilon)t})}
\end{aligned} \tag{A.14}$$

The trace over the zero modes S_0 yields

$$\text{Tr}_{S_0} e^{-i\pi t \epsilon R_0^{18}} = 16 \sinh^4 \frac{\pi t \epsilon}{4} \tag{A.15}$$

Performing finally the integral over the momentum and the analytic continuation $\epsilon \rightarrow i\epsilon$, the result can be written in terms of ϑ -functions as:

$$\mathcal{A} = \frac{V_p}{2\pi i} \int_0^\infty \frac{dt}{t} (2\pi^2 t)^{-\frac{p}{2}} e^{-\frac{b^2 t}{2\pi}} \frac{\vartheta_1^4(\frac{\epsilon t}{4} | \frac{it}{2})}{\vartheta_1(\frac{\epsilon t}{2} | \frac{it}{2})} \frac{\vartheta_1'(0 | \frac{it}{2})}{\eta^{12}(\frac{it}{2})} \tag{A.16}$$

Again, eq.(A.16) reproduces the usual result after having performed the spin-structure sum.

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