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Closed string radiation from moving D-branes

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Abstract

We compute the amplitude for the radiation of massless NS-NS closed string states from the interaction of two moving D-branes. We consider particle-like D-branes with reference to 4-dimensional spacetime, in toroidal and orbifold compactifications, and we work out the relevant world sheet propagators within the moving boundary state formalism. We find no on-shell axion emission. For large inter-brane separation, we compute the space-time graviton emission amplitude and estimate the average energy radiated, whereas the spacetime dilaton amplitude is found to vanish in this limit. The possibility of emission of other massless states depends on the nature of the branes and of the compactification scheme.

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I. INTRODUCTION AND SUMMARY

The non-relativistic dynamics of Dirichlet branes [1], plays an essential role in the understanding of string theory at scales shorter than the Planck length [2–4]. Thus it is important to investigate the physical properties of the interactions between branes and/or branes and strings. These physical properties can be generically inferred from scattering amplitudes. For instance one can look at the interaction of strings with one brane, by considering a disk-like world sheet with appropriate boundary conditions and inserting Neveu-Schwarz-Neveu-Schwarz (NS-NS) or Ramond-Ramond (RR) string vertex operators [5].

Interactions between two branes can be studied by considering an annulus-like world sheet with one boundary on each brane. As is well known one finds in particular that there is no interaction between two identical branes at rest [6]. However, non zero amplitudes can occur when one includes in the system additional string states, by inserting appropriate vertex operators in the annulus, and/or when considering branes in relative motion. In the case of moving branes, the potential falls off like V^4/r^{D-3} for small relative velocity V in the maximally supersymmetric case, whereas it could generically vanish like V^2/r^{D-3} for compactifications breaking some supersymmetry. In addition to the universal force, there are also additional spin effects proportional to V^{4-n}/r^{D-3+n} (in the maximally supersymmetric case) which distinguish between the various components of the 256-dimensional brane BPS multiplet [7,8].

In previous works [9,10] we have used the boundary state formalism [11] to study, in particular, the case of two branes for a Type II superstring theory compactified on orbifolds, looking at the dependence on the relative velocity of the branes' scattering amplitude [10], which gives important information on the coupling of the massless fields to the branes. Here we study the interesting possibility of particle emission from interacting moving branes. Particle production is quantum mechanically allowed even within the eikonal approximation where recoil is neglected and the branes are assumed to move along straight trajectories. This is in particular more and more precise when dealing with emission of massless states in the limit of small momentum and large impact parameter. Here we present a systematic study of this case, namely by computing the amplitudes for the emission of a single NS-NS massless closed string state (graviton, dilaton or axion) from a system of two moving D0-branes, in both toroidal and orbifold compactifications [12] down to four dimensions. Actually we have considered also an interesting case of some Neumann boundary conditions in the compactified directions, technically describing a D3-brane, but which is still a particle-like D0-brane with reference to the uncompactified spacetime.

We find that the amplitude for axion emission, which gets contributions only from the odd spin structure in the twisted orbifold sector, vanishes exactly. On the contrary only the even spin structures contribute to the emission amplitudes for the dilaton and graviton. We study these amplitudes in the field theory limit, that is for large branes' separation.

Such a limiting process picks out the massless closed string states being exchanged between the branes. In this limit the "spacetime" dilaton emission amplitude also vanishes (by "spacetime" dilaton we refer to the particle described by the trace of the space-time part of the vertex operator polarisation tensor). However the graviton emission amplitude even in this limit is generically different from zero and we present its computation. Also the amplitude for emission of other massless particles, related to the compact directions of the polarisation tensor, can be different from zero, depending on the nature of the branes in the compactified directions.

We then compare the graviton emission amplitude in the field theory limit to Feynman diagrams when the two branes exchange a massless particle, which can either be a scalar, a vector or a graviton, and the outgoing graviton is emitted by it. We indeed find that the RR part of the amplitude, in the limit of large distance from both branes, corresponds to the coupling of the graviton to the RR vector being exchanged between them. Similarly we find that in the same limit the NS-NS part of the amplitude corresponds to the coupling of the emitted graviton to the graviton and the scalar being exchanged between the branes. It is interesting to note that our result for the graviton emission amplitude corresponds to the sum of various Feynman diagram contributions, as is usual in string theory, including both bremsstrahlung-like processes where the graviton is directly emitted by the branes, and processes where it is emitted far from them. We also evaluate the average energy $< p >$ radiated by the two branes when they pass each other at impact parameter b and relative velocity V, finding $< p > \sim g_s^2 l_s^2 \frac{V^{1+2n}}{b^3}$, where g_s, l_s are the string coupling and length, and the integer $n = 2, 4$ depends on the brane nature and compactification scheme, that is essentially on the amount of supersymmetry. If we extrapolate down to the eleven-dimensional Planck length $b \sim g_s^{1/3} l_s$ we would find a maximal radiated energy $\sim g_s \frac{V^{1+2n}}{l_s}$, which one can compare with the estimate of ref. [4]. Actually, this extrapolation would be valid for small velocity $V < g_s^{2/3}$, see Sec. IVB

Further, let us note that the graviton amplitude, when the graviton coming out of the interaction of two branes is off-shell, could be also regarded as a first perturbative correction $\mathcal{O}(1/r^2)$ in the evaluation of the gravitational field at large distance r from a system of branes. In fact, the perturbation expansion of the classical solution in terms of tree diagrams, where the branes are sources, would give at the second order the graphs of Figs. 1,2,3. A related issue is the result for the scalar emission amplitude. In fact the pattern of scalar couplings is of crucial importance for the question of whether a non-zero horizon is produced. From our result, this seems possible for a system of just one species of D3-branes on an orbifold, since we find these D3-branes to be uncoupled to any scalars.

The propagators for the world-sheet bosons and fermions in this system are an important ingredient of the calculation. Since they are not available in the literature, we outline the computation of these technical tools and present the resulting expressions which are interesting in their own right and constitute an important output of our work.

The paper is organised as follows. To make sure that the arguments of this paper can be followed without being distracted by too many computations we have kept in the main text only what is strictly necessary for its understanding and have relegated technical tools to appendices. In Sec. II we construct the general amplitude for particle emission, carefully separating out the zero mode contributions and setting up the kinematics. In Sec. III we discuss the axion emission amplitude and show that it is zero. Sec. IV is devoted to the construction of the dilaton and graviton emission amplitudes. In Sec. IVA we show that the dilaton amplitude vanishes in the field theory limit (by dilaton here we mean the massless scalar corresponding to the trace part of the polarisation tensor, whose traceless part describes the four dimensional spacetime graviton). In Sec. IVB we present the computation and the results for the graviton emission amplitude and the estimate of the energy radiated in this process. In Sec. V we consider the field theory interpretation of the graviton emission amplitude. In Sec. VI we discuss the case of the emission of other types of massless particles, corresponding to other components of the ten dimensional polarisation tensor. Appendix A outlines the construction of the spacetime part of the boundary state for a moving D-brane. In Appendix B we use the boosted boundary states to compute the uncompactified part of the partition functions. This appendix also contains the calculation of the propagators for bosons and fermions on the cylindrical world sheet representing the exchange of closed string states between two relatively moving D-branes.

II. GENERAL AMPLITUDE

Consider the interaction of two zero branes, moving with velocities V_1 and V_2 respectively, say along the 1 direction only. We will call Transverse (T) the other two uncompactified space directions 2,3. In the closed string picture the interaction between two branes is viewed as the exchange of a closed string between two boundary states, geometrically describing a cylinder. In the present work we use τ for the coordinate along the length of the cylinder, $0 \leq \tau \leq l$, and σ as the periodic coordinate running from 0 to 1. We will always consider particle like D-branes, that is the time coordinate satisfies Neumann boundary conditions, whereas the three uncompactified space coordinates satisfy Dirichlet boundary conditions. The emission of a closed string state from these interacting branes is described by the matrix element of the appropriate vertex operator sandwiched between the boundary states describing the branes:

$$\mathcal{A} = \int_0^\infty dl \int_0^l d\tau \sum_s (\pm) < B, V_1, Y_1 | e^{-lH} V(\tau,\sigma) | B, V_2, Y_2 >^s , \tag{1}$$

where the $\sum_s(\pm)$ represents the sum over the spin structures with the appropriate signs (GSO projections).

The vertex operator for a massless NS-NS state (this state represents a massless particle propagating in 4-dimensional uncompactified spacetime, with momentum p^μ, $\mu = 0, 1, 2, 3$ and $p^\mu p_\mu = 0$) is given by

$$V(z, \bar{z}) = e_{ij}(\partial X^i - \frac{1}{2}p \cdot \psi\psi^i)(\bar{\partial}X^j + \frac{1}{2}p \cdot \bar{\psi}\bar{\psi}^j)e^{ip\cdot X} \ , \tag{2}$$

with $z = \sigma + i\tau$ and $\partial = \partial_z$. For most of this work we consider a polarisation tensor $e_{\mu\nu}$ with components in the uncompactified directions only. In this case in the vertex operator, only bosonic and fermionic noncompact coordinates appear. For relative normalisation of X and ψ see Appendix A. We can take $e_{\mu\nu}$ to have only space components, which we denote as i, j, as this is allowed by the gauge invariance of the vertex operator, and moreover $p^i e_{ij} = 0$. The various cases we will consider are:

1) the axion, described by $e_{ij} = b_{ij}$ with $b_{ij} = -b_{ji}$,

2) the dilaton, described by $e_{ij} = \delta_{ij} - \frac{p_i p_j}{\vec{p}^2}$,

3) the graviton, described by $e_{ij} = h_{ij}$ with $h_{ij} = h_{ji}$ and $\delta^{ij}h_{ij} = 0$. In Sect. VI we discuss more general polarisation tensors, having also components in the compactified directions.

As is well known, the D-brane is described by an appropriate boundary state [11]. We write the boundary state for the moving D-brane as [13]

$$|B, V_2, Y_2 >_s = \int \frac{d^3q}{(2\pi)^3} e^{-i\vec{Y}_2 . \vec{q}} |q_B^\mu > \otimes |B, sm >^s \ , \tag{3}$$

where $|B, sm >^s$ is the boundary state constructed from the Fock space of the bosonic and fermionic string modes (sm) (see Appendix A). Here q_B^μ is the boost of the momentum $(0, q^1, q^2, q^3)$:

$$q_B^\mu = (\sinh v_2 q^1, \cosh v_2 q^1, \vec{q}_T) = (\gamma_2 V_2 q^1, \gamma_2 q^1, \vec{q}_T) \ , \tag{4}$$

v_2 being the rapidity of the brane 2. Similarly k_B^μ is defined as the boost of the vector $(0, k^1, k^2, k^3)$ with rapidity v^1. In our notation the integration measure is always defined as $d^3q = dq^1 d^2q_\perp$ and similarly in the following $d^3k = dk^1 d^2k_\perp$. In eq. (3) we take different from zero only the spacetime part of the momentum emitted by the brane (we will be mainly concerned with the case of large distances, where we can neglect the configurations having momentum or winding in the compactified directions different from zero). Separating the zero modes for $\mu = 0, 1, 2, 3$

$$X^\mu(\sigma, \tau) = X^\mu_{0m}(\tau) + X^\mu_{osc}(\sigma, \tau) \ , \tag{5}$$

where $X^\mu_{0m}(\tau) = X^\mu_0 - iQ^\mu\tau$, we can write:

$$\begin{aligned} e^{ipX} &= e^{ipX_{0m}} \circ e^{ipX_{osc}} \ , \\ \partial X e^{ipX} &= \partial X_{0m} e^{ipX_{0m}} \circ e^{ipX_{osc}} + e^{ipX_{0m}} \circ \partial X_{osc} e^{ipX_{osc}} \ , \end{aligned} \tag{6}$$

etc. Since in general we have terms of the kind:

$$F(X_{0m})e^{ipX_{0m}} \circ G(sm)e^{ipX_{osc}} \ , \tag{7}$$

where F is an expression ($F(X) = 1$ or ∂X^i or $\bar{\partial} X^j$ or $\partial X^i \bar{\partial} X^j$) containing X_{0m} only and $G(sm)$ contains everything else, that is generically all the remaining string mode, both bosonic and fermionic, we can split the computation:

$$\begin{aligned} &< B_1, V_1, Y_1 | e^{-lH} F(X_{0m}) e^{ipX_{0m}} \circ G(sm) e^{ipX_{osc}} | B_2, V_2, Y_2 > \\ &\quad =< F(X_{0m)} e^{ipX_{0m}} >< B_1, V_1, sm | e^{-lH(sm)} G(sm) e^{ipX_{osc}} | B_2, V_2, sm >^s \ . \end{aligned} \tag{8}$$

We have defined:

$$< F(X_{0m})e^{ipX_{0m}} >\equiv \int \frac{d^3\vec{k}}{(2\pi)^3}\frac{d^3\vec{q}}{(2\pi)^3} e^{i\vec{Y}_1\vec{k} - i\vec{Y}_2\vec{q}} < k^\mu_B | e^{-lH(X_{0m})} F(X_{0m}) e^{ipX_{0m}} | q^\mu_B > \ . \tag{9}$$

To avoid ambiguity let us stress here that $\vec{q}$ ($\vec{k}$) refers to the space components of the vector $q^\mu (k^\mu)$ defined in eq. (4). As mentioned above, we have four possibilities for $F(X_{0m})$.

Consider first $F = 1$, giving

$$< e^{ip\cdot X_{0m}} >_o= \int \frac{d^3\vec{k}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{Y}_1} \int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{Y}_2} e^{-\frac{l}{2}k_B^2 + \tau p\cdot q_B} < k^\mu_B | (p + q_B)^\mu > \ . \tag{10}$$

Notice now that

$$\begin{aligned} &< k^\mu_B | (p + q_B)^\mu >= (2\pi)^4 \delta^{(4)}(p^\mu - k^\mu_B + q^\mu_B)) \\ &\quad = \frac{(2\pi)^4}{\sinh|v_1 - v_2|} \delta\left(k^1 - \frac{p^{(2)}}{\sinh(v_1 - v_2)}\right) \delta\left(q^1 - \frac{p^{(1)}}{\sinh(v_1 - v_2)}\right) \delta^{(2)}(\vec{p}_T - \vec{k}_T + \vec{q}_T) \ . \end{aligned} \tag{11}$$

We have used $(V_1 - V_2)\gamma_1\gamma_2 = \sinh(v_1 - v_2)$ and defined the boosted energies to be

$$p^{(1,2)} = \gamma_{1,2}(1 - V_{1,2}\cos\theta)p \ , \tag{12}$$

where $p = p^0$ and $\cos\theta = \frac{p^1}{p}$. From now we drop the subscript "B" on the momenta and use the notation $q^\mu = (q^0, q^1, \vec{q}_T)$, $k^\mu = (k^0, k^1, \vec{k}_T)$ where:

$$\begin{aligned} &k^0 = V_1 k^1 \ , \quad q^0 = V_2 q^1 \ , \\ &k^1 = \frac{p}{V_1 - V_2}(1 - V_2\cos\theta) \ , \quad q^1 = \frac{p}{V_1 - V_2}(1 - V_1\cos\theta) \ , \\ &\vec{k}_T - \vec{q}_T = \vec{p}_T \ . \end{aligned} \tag{13}$$

Introducing the impact parameter $\vec{b} = \vec{Y}_1 - \vec{Y}_2$ and defining $l' = l - \tau$, finally we get

$$< e^{ip\cdot X_{0m}} >= \frac{1}{\sinh|v_1 - v_2|} \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} e^{-\frac{q^2}{2}\tau} e^{-\frac{k^2}{2}l'} \ . \tag{14}$$

The other three possible matrix elements $F(X_{0m})$ are easily evaluated since they correspond to further insertions of momentum operators Q^i, and involve additional k^i factors in the integral. They are:

$$
\begin{aligned}
< \partial X^i_{0m} e^{ip\cdot X_{0m}} >&= \frac{1}{\sinh|v_1-v_2|}\int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} e^{-\frac{q^2}{2}\tau} e^{-\frac{k^2}{2}l'} (-\frac{1}{2}k^i) \;, \\
< \bar{\partial} X^j_{0m} e^{ip\cdot X_{0m}} >&= \frac{1}{\sinh|v_1-v_2|}\int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} e^{-\frac{q^2}{2}\tau} e^{-\frac{k^2}{2}l'} (\frac{1}{2}k^j) \;, \\
< \partial X^i_{0m} \bar{\partial} X^j_{0m} e^{ip\cdot X_{0m}} >&= \frac{1}{\sinh|v_1-v_2|}\int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}\cdot\vec{b}} e^{-\frac{q^2}{2}\tau} e^{-\frac{k^2}{2}l'} (-\frac{1}{4}k^i k^j) \;.
\end{aligned} \tag{15}
$$

It will prove very convenient to change the integration variables of the amplitude to τ and l' which will be interpreted as the proper times of the particles mediating the interaction from each of the two branes to the vertex:

$$
\int_0^\infty dl \int_0^l d\tau = \int_0^\infty d\tau \int_0^\infty dl' \;. \tag{16}
$$

Notice that $\tau = 0$ corresponds to the emission from the second brane $|B, V_2, Y_2>$, whereas $l' = 0$, corresponds to the emission from the first brane $< B, V_1, Y_1|$; conversely, $\tau, l' > 0$ corresponds instead to emission far from both branes.

As for the factor containing the string modes, we write

$$
< B_1, V_1, sm| e^{-lH(sm)} G(sm) e^{ipX_{osc}} |B_2, V_2, sm >^s \equiv < G(sm) e^{ipX_{osc}} >^s \cdot Z^b Z^{fs} \tag{17}
$$

where Z^b and Z^{fs} are the partition functions defined by

$$
Z^b \equiv < B_1, V_1| e^{-lH} |B_2, V_2 >_{bosonic\ osc} \;, \quad Z^{fs} \equiv < B_1, V_1| e^{-lH} |B_2, V_2 >^s_{fermionic\ modes} \;. \tag{18}
$$

For the matrix element of an operator, as defined in eq. (17), we distinguish the case of even and odd spin structures. In the boundary state formalism, the various spin structures correspond to the GSO projections and are obtained by the possibility of inserting the operator $(-1)^F$ in the matrix element of the boundary states, in the Ramond-Ramond or Neveu-Schwarz-Neveu-Schwarz case. Since the boundary state is of the form $|B >= e^{i\phi}|0>$, where ϕ is an expression quadratic in the left and right moving fermionic modes and $|0>$ is a suitable Fock vacuum (see Appendix A), the insertion of $(-1)^F$ has the effect of changing the sign of ϕ in one of the boundary states. To take it into account, we define $|B, \eta >= e^{i\eta\phi}|0>$ where $\eta = \pm 1$ (actually, only the relative sign $\eta_1\eta_2$ is relevant, $\eta_{1,2}$ referring to the $B_{1,2}$ boundary state). The odd spin structure corresponds to $\eta_1\eta_2 = -1$ for the RR case. For the even case for any operator $\mathcal{O}$ we define

$$
< \mathcal{O}(\sigma,\tau) >^{even} \equiv \frac{< B_1, V_1, \eta_1| e^{-lH} \mathcal{O}(\sigma,\tau) |B_2, V_2, \eta_2 >}{< B_1, V_1, \eta_1| e^{-lH} |B_2, V_2, \eta_2 >} \;. \tag{19}
$$

For the odd case in general there are fermionic zero modes which make the result zero unless they are soaked up by an equal number of insertions. Since in our vertex only spacetime fermionic coordinates appear, the overall result will be zero in the odd case whenever there are zero modes for the compactified fermionic coordinates. Thus, we restrict the discussion of the odd spin structure to the case of the Z_3 orbifold when the branes are at the fixed point. In this case, for the twisted sector, there are no zero modes in the compactified directions [9,10]. In the boundary state formalism this is seen from Appendix A, because

$$< 0|e^{i\eta\tilde{b}^*b}e^{i\eta b^*\tilde{b}}|0>= 0 \tag{20}$$

where $b = (-i\gamma^A + \gamma^{A+1})/2$, with $A \geq 2$, is a fermionic zero mode in a transverse direction, whereas

$$< 0|e^{i\eta\tilde{b}^*b}bb^*e^{i\eta b^*\tilde{b}}|0>= 1 \tag{21}$$

and also different from zero on inserting $b\tilde{b}^*$, $\tilde{b}b^*$, and $\tilde{b}\tilde{b}^*$. Notice instead that in the longitudinal direction, $a = (\gamma^0 + \gamma^1)/2$,

$$\frac{e^{v_2-v_1}}{2} < 0|e^{-i\eta e^{2v_1}\tilde{a}a}e^{-i\eta e^{-2v_2}a^*\tilde{a}^*}|0>= \sinh(v_2 - v_1)\ . \tag{22}$$

Thus in order to have a nonvanishing result, the operator $\mathcal{O}$ must contain $\psi_0^2\psi_0^3$ or $\psi_0^2\tilde{\psi}_0^3$ or $\tilde{\psi}_0^2\psi_0^3$ or $\tilde{\psi}_0^2\tilde{\psi}_0^3$. In this case we define (for the RR case with $\eta_1\eta_2 = -1$):

$$\ll \mathcal{O}(\sigma,\tau) \gg^{odd} \equiv \frac{< B_1, V_1, \eta_1|e^{-lH}\mathcal{O}(\sigma,\tau)|B_2, V_2, \eta_2 >}{< B_1, V_1, \eta_1|e^{-lH}\psi_0^2\bar{\psi}_0^3|B_2, V_2, \eta_2 >}\ . \tag{23}$$

The fermionic partition function for the odd spin structure is accordingly defined with the zero modes insertion, as in the denominator of (23). If $\mathcal{O}$ does not contain zero modes the result will be zero.

Since $|B>$ can be written as a direct product for pairs of directions and therefore also the expectation value can be accordingly factorized, we can evaluate $< \mathcal{O} >$ by using the Wick theorem for each factor. In the odd case the factor relative to the $2,3$ directions will be zero if $\mathcal{O}$ does not contain the zero modes. The relevant rules are given in Appendix B. In particular

$$< G_B(X_{osc})G_F(\psi)e^{ipX_{osc}} >=< G_B(X_{osc})e^{ipX_{osc}} >< G_F(\psi) > \tag{24}$$

and for the expressions containing X_{osc} we have

$$\begin{aligned} < \partial X^i_{osc}e^{ipX_{osc}} > &= i < \partial X^i p\cdot X >_{osc}< e^{ipX_{osc}} >\ , \\ < \bar{\partial} X^i_{osc}e^{ipX_{osc}} > &= i < \bar{\partial} X^i p\cdot X >_{osc}< e^{ipX_{osc}} >\ . \end{aligned} \tag{25}$$

We thus get, finally, that the general amplitude can be written as :

$$\mathcal{A}=\frac{1}{\sinh|v_1-v_2|}\int_0^\infty d\tau\int_0^\infty dl'\int\frac{d^2\vec{k}_T}{(2\pi)^2}e^{i\vec{k}\cdot\vec{b}}e^{-\frac{q^2}{2}\tau}e^{-\frac{k^2}{2}l'}<e^{ip\cdot X}>_{osc}\mathcal{N} \tag{26}$$

where

$$\mathcal{N}=Z^b\sum_s(\pm)Z^{fs}\mathcal{M}^s \tag{27}$$

and

$$\begin{aligned}\mathcal{M}^s=e_{ij}\Big\{&<\partial X^i\bar\partial X^j>_{osc}-<\partial X^i p\cdot X>_{osc}<\bar\partial X^j p\cdot X>_{osc}\\&+\frac{1}{4}\left(<p\cdot\psi p\cdot\bar\psi>^s<\psi^i\bar\psi^j>^s-<p\cdot\psi\psi^i>^s<p\cdot\bar\psi\bar\psi^j>^s\right.\\&\qquad\left.+<p\cdot\bar\psi\psi^i>^s<p\cdot\psi\bar\psi^j>^s\right)\\&+\frac{i}{2}\left(<\partial X^i p\cdot X>_{osc}<p\cdot\bar\psi\bar\psi^j>^s-<\bar\partial X^j p\cdot X>_{osc}<p\cdot\psi\psi^i>^s\right)\\&-\frac{1}{2}k^i\left(i<\bar\partial X^j p\cdot X>_{osc}+\frac{1}{2}<p\cdot\bar\psi\bar\psi^j>^s\right)\\&+\frac{1}{2}k^j\left(i<\partial X^i p\cdot X>_{osc}-\frac{1}{2}<p\cdot\psi\psi^i>^s\right)-\frac{1}{4}k^ik^j\Big\}\ .\end{aligned} \tag{28}$$

In the case of the odd spin structure, terms not containing $<\psi^2\psi^3>$ (or $\bar\psi^{2,3}$) at least once are zero, see Appendix B eqs. (B14) and (B15).

Notice that the unphysical longitudinal part b^1 of the impact parameter appears in the amplitude only in the irrelevant constant overall phase $e^{ik^1b^1}$; one can put $b^1=0$ without loss of generality. In order to get some preliminary physical information from the amplitude, we can explicitly carry out the kinematical integration, obtaining

$$\int\frac{d^2\vec{k}_T}{(2\pi)^2}e^{i\vec{k}_T\cdot\vec{b}_T}e^{-\frac{q^2}{2}\tau}e^{-\frac{k^2}{2}l'}=\frac{1}{2\pi l}e^{\frac{1}{2l}(\vec{p}_T\tau-i\vec{b}_T)^2}e^{-\frac{p^{(1)2}\tau+p^{(2)2}l'}{2\sinh^2(v_1-v_2)}}\ . \tag{29}$$

Because of the term $e^{-\frac{\vec{b}_T^2}{2l}}$, at fixed transverse distance $\vec{b}_T$, world sheets with $l<\vec{b}_T^2$ are exponentially suppressed; in particular, the large distance limit $|\vec{b}_T|\to+\infty$ implies $l\to+\infty$, and selects the part of the amplitude where the fields are massless.

In order to complete the computation one has to choose the compactification scheme. We will consider either toroidal compactification and D0-branes (that is TypeIIA theory) or Z_3 orbifold and either D0-branes (TypeIIA), or D3-branes (TypeIIB), see our earlier paper [10].

As we will see in the next section the contributions of the even spin structures to axion emission are zero. The amplitude for the emission of an axion receives contributions only from the odd spin structure ($RR-$) sector where one has to insert two transverse zero modes. Of course, in the case of toroidal compactification from 10 dimensions to 4 dimensions or, in the case of orbifold compactification, when the D-branes are on a generic point of the orbifold, the axion production amplitude is trivially zero, due to the lack of zero mode insertions in the compactified fermionic coordinates integration. But when the D-branes are on the fixed point of a Z_3 orbifold there is the twisted sector contribution where there are no zero modes in the compactified directions. In this case, the compactified part of the amplitude turns out to be "1", since the bosonic part exactly cancels the oscillators of the $RR-$ part and we are left with the zero modes' contribution $Z^B Z^{R^-} = 4\sinh(v_1 - v_2)$.

In the case of the dilaton and the graviton, the situation is different since the three even spin structures contribute. Now we consider the D0-brane or also the D3-brane case. Let us first take the D0-brane on a generic point of the Z_3 orbifold or on T_6. We will be interested in the $l \to \infty$ limit, in which case (see [10]):

$$
\begin{aligned}
Z_{\pm}^{NS} &\to e^{2\pi l} \pm 2[\cosh 2(v_1 - v_2) + \sum_a \cos 2\pi z_a] \ , \\
Z_{+}^{R} &\to 16\cosh(v_1 - v_2)\prod_a \cos\pi z_a \ .
\end{aligned} \tag{30}
$$

The case of the D0-brane corresponds to taking all $z_a = 0$, whereas for the D3-branes, with mixed Dirichlet/Neumann boundary conditions in each of the compactified pairs of coordinates, we take either $z_a = 0$ or $g(z_a = 1/3, 1/3, -2/3)$ and one has to sum over all possibilities $1 + g + g^2$. In this case since $\mathcal{M}_{\pm}^{NS} = S \pm e^{-2\pi l}T$, where S and T are in general functions of τ and l', we finally have

$$
\begin{aligned}
\mathcal{N} &= Z_{+}^{R}\mathcal{M}^{R+} - Z_{+}^{NS}\mathcal{M}^{NS+} + Z_{-}^{NS}\mathcal{M}^{NS-} \\
&\to 16\cosh(v_1 - v_2)\prod_a \cos\pi z_a \mathcal{M}^{R+} - 4S[\cosh 2(v_1 - v_2) + \sum_a \cos 2\pi z_a] - 2T \ .
\end{aligned} \tag{31}
$$

In the case of a D0-brane on a fixed point of the orbifold there are both the untwisted sector, where the result is the same as eq. (31), and the twisted sector where one has to make the combination

$$
\mathcal{N} = Z_{+}^{R}\mathcal{M}^{R+} - Z_{+}^{NS}\mathcal{M}^{NS+} - Z_{-}^{NS}\mathcal{M}^{NS-} \ . \tag{32}
$$

Since in this case, for $l \to \infty$, $Z_{+}^{R} \to 2\cosh(v_1 - v_2)$, $Z_{\pm}^{NS} \to 1$, the sum over the even spin structures gives

$$
\mathcal{N} \to 2\cosh(v_1 - v_2)\mathcal{M}^{R+} - 2S \ . \tag{33}
$$

III. AXION AMPLITUDE

The axion is described by an antisymmetric and transverse polarisation tensor satisfying $b^{ij} = -b^{ji}$ and $p_i b^{ij} = p_j b^{ij} = 0$. Thus up to a constant $b^{ij} = \frac{1}{2}\epsilon^{ijk}\frac{p_k}{p}$.

Using the general properties and the definitions given in Appendix B (remembering that $\epsilon^{ij} \equiv \epsilon^{1ij}$) we have

$$
\begin{aligned}
&< \partial X^i p \cdot X >_{osc} = - < \bar{\partial} X^i p \cdot X >_{osc} , \\
&< \psi^i \bar{\psi}^j >^{even} - < \psi^j \bar{\psi}^i >^{even} = 0 , \quad < p \cdot \psi \psi^j >^{even} - < p \cdot \bar{\psi} \bar{\psi}^j >^{even} = 0 , \\
&< \psi^i \bar{\psi}^j >^{odd} - < \psi^j \bar{\psi}^i >^{odd} = \epsilon^{ij} , \quad < p \cdot \psi \psi^j >^{odd} - < p \cdot \bar{\psi} \bar{\psi}^j >^{odd} = i p_k \epsilon^{kj} .
\end{aligned} \tag{34}
$$

Also from the rules of eq. (B16) it is easy to see that

$$
b_{ij} < p \cdot \bar{\psi} \psi^i >^{even} < p \cdot \psi \bar{\psi}^j >^{even} = 0 . \tag{35}
$$

Thus one can see that the whole amplitude (28), with $e_{ij} = b_{ij} = -b_{ji}$, is zero for even spin structures. Further noticing that

$$
b_{ij} < p \cdot \bar{\psi} \psi^i >^{odd} < p \cdot \psi \bar{\psi}^j >^{odd} = b_{ij} p_k \epsilon^{kj} < p \cdot \bar{\psi} \psi^i >^{odd} , \tag{36}
$$

the amplitude is found to reduce to

$$
\begin{aligned}
\mathcal{M}^{odd} = \frac{1}{8} b_{ij} \Big\{ & \epsilon^{ij} < p \cdot \psi p \cdot \bar{\psi} >^{odd} \\
& + 4 p_k \epsilon^{kj} \left(< \partial X^i p \cdot X >_{osc} - \frac{1}{2} < \psi^i p \cdot \bar{\psi} >^{odd} - \frac{i}{2} < \psi^i p \cdot \psi >^{odd} + \frac{i}{2} k^i \right) \Big\} .
\end{aligned} \tag{37}
$$

By explicit calculation it is seen that the oscillator parts of the last set of four terms add up to zero, as expected from world-sheet supersymmetry, and we are left with

$$
\mathcal{M}^{odd} = \frac{1}{8} b_{ij} \left\{ \epsilon^{ij} < p \cdot \psi p \cdot \bar{\psi} >^{odd} + 2 i p_k \epsilon^{kj} \left(k^i + i < \psi^i p \cdot \bar{\psi} >_o^{odd} - < \psi^i p \cdot \psi >_o^{odd} \right) \right\} , \tag{38}
$$

where the subscript o on the fermionic propagator indicates the zero mode contribution.

Of course, in the case of toroidal compactification from 10 dimensions to 4 dimensions, this axion production amplitude is trivially zero, due to the lack of zero mode insertions in the compactified fermionic coordinates integration. But in the case of a Z_3 orbifold compactification, when the 0-branes are on the fixed points of the orbifold (see Appendix B of [9]), there are no zero modes in the compactified directions. We will thus consider this case. Using the explicit form of the polarisation tensor, and evaluating the zero modes, the amplitude in the $RR-$ sector reduces to

$$
\begin{aligned}
\mathcal{M}^{R-} = & \frac{1}{8} \cos\theta < p \cdot \psi p \cdot \bar{\psi} >_{osc}^{R-} + \frac{i}{8} \left[\cos\theta \vec{p}_T \cdot \vec{k}_T - \sin^2\theta p k^1 \right] \\
& + \frac{i}{8} p^2 \left[2 \cos\theta F_v^{oR-} - (1 + \cos^2\theta) G_v^{oR-} + \sin^2\theta U_v^{oR-} \right] .
\end{aligned} \tag{39}
$$

Finally, using the results of Appendix B and remembering the kinematics, the last expression can be shown to simplify to

$$\mathcal{M}^{R-}=\frac{1}{8}\cos\theta\left[<p\cdot\psi p\cdot\bar{\psi}>^{R-}_{osc}+\frac{i}{2}(k^2-q^2)\right] \tag{40}$$

and using eq. (B21) one ends up with

$$\mathcal{M}^{R-}=\frac{i}{8}\cos\theta\left[-\partial_\tau<p\cdot X(z)p\cdot\bar{X}(\bar{z})>_{osc}+\frac{1}{2}(k^2-q^2)\right]\ . \tag{41}$$

From eqs. (B3) and (B13) of Appendix B we see that for the uncompactified parts of the partition functions we have

$$Z^bZ^{R-}=4\sinh(v_1-v_2)$$

and observing that $\partial_\tau|_l=\partial_\tau|_{l'}-\partial_{l'}|_\tau$ the final integrated amplitude, eq.(26), for axion emission is seen to be a total derivative

$$\begin{aligned}\mathcal{A}^{ax}&=\frac{i}{2}\cos\theta\int_0^\infty d\tau\int_0^\infty dl'\int\frac{d^2\vec{k}_T}{(2\pi)^2}e^{i\vec{k}\cdot\vec{b}}(\partial_\tau-\partial_{l'})\left\{e^{-\frac{q^2}{2}\tau}e^{-\frac{k^2}{2}l'}<e^{ip\cdot X}>_{osc}\right\}\\&=0\ .\end{aligned} \tag{42}$$

Here, as in the following, possible surface terms at $\tau,l'=0$ have been dropped by making an analytic continuation from $p^2<0$ of formula (B8) for $<e^{ip\cdot X}>_{osc}$.

Thus, finally, we find that there is no on-shell axion emission during the interaction of two moving branes, even in the case of the Z_3 orbifold compactification. This result is not in contradiction with our previous work [9]. There we computed the amplitude for axion production due to the interaction of an incoming graviton with two parallel branes at rest. Indeed we found no pole in the axion-graviton momentum transfer squared and thus there is no on-shell axion coming out of the two brane system.

IV. DILATON AND GRAVITON AMPLITUDES

The graviton is described by a symmetric, transverse and traceless polarisation tensor, satisfying $h^{ij}=h^{ji}$, $p_ih^{ij}=0$ and $h^i_i=0$. Consequently, there are two physical transverse polarisations. The dilaton, instead, can be thought of as the trace part of the graviton and is described by a symmetric and transverse polarisation tensor, satisfying $h^{ij}=h^{ji}$ and $p_ih^{ij}=0$, which can be taken to be $h^{ij}=\delta^{ij}-\frac{p^ip^j}{p^2}$.

In these cases one can verify that, due to the symmetry of the polarisation tensor, the amplitudes are non-vanishing in the even spin structure sectors only.

It will prove of great help in this case to integrate by parts the two-derivative bosonic term; by using $\bar{\partial}=\frac{i}{2}\partial_\tau|_l=\frac{i}{2}(\partial_\tau|_{l'}-\partial_{l'}|_\tau)$, since $\bar{\partial}$ acts on a function of $z-\bar{z}=2i\tau$, and observing that the partition function behaves like a constant with respect to the latter derivative since it depends only on $l=\tau+l'$, one gets

$$
\begin{aligned}
&\int_0^\infty d\tau \int_0^\infty dl' e^{-\frac{q^2}{2}\tau} e^{-\frac{k^2}{2}l'} < e^{ip\cdot X} >_{osc} h_{ij} < \partial X^i(z) \bar\partial \bar X^j(\bar z) >_{osc} \\
&\quad = -\frac{i}{2}\int_0^\infty d\tau \int_0^\infty dl' h_{ij} < \partial X^i(z) \bar X^j(\bar z) >_{osc} (\partial_\tau - \partial_{l'}) \left\{ e^{-\frac{q^2}{2}\tau} e^{-\frac{k^2}{2}l'} < e^{ip\cdot X} >_{osc} \right\} \\
&\quad = -\int_0^\infty d\tau \int_0^\infty dl' e^{-\frac{q^2}{2}\tau} e^{-\frac{k^2}{2}l'} < e^{ip\cdot X} >_{osc} h_{ij} < \partial X^i(z) \bar X^j(\bar z) >_{osc} \\
&\qquad\qquad \times \left\{ < p\cdot \partial X(z) p\cdot \bar X(\bar z) >_{osc} + \frac{i}{4}(k^2 - q^2) \right\} \ . \qquad (43)
\end{aligned}
$$

Taking into account also the symmetry of h^{ij} for both the graviton and the dilaton and the property (see appendix B)

$$
< \partial X^i p\cdot X >_{osc} = - < \bar\partial X^i p\cdot X >_{osc} \ , \qquad (44)
$$

the amplitude $\mathcal{M}^s$ in (28) can be taken to be (writing $X_{osc}(z,\bar z) = X_{osc}(z) + \bar X_{osc}(\bar z)$)

$$
\begin{aligned}
\mathcal{M}^s = h_{ij}\Big\{ &- < \partial X^i \bar X^j >_{osc} < p\cdot \partial X p\cdot \bar X >_{osc} \\
&+ < \partial X^i p\cdot (X + \bar X) >_{osc} < \partial X^j p\cdot (X+\bar X) >_{osc} \\
&+ \frac{1}{4}\Big(< p\cdot\psi p\cdot\bar\psi >^s < \psi^i \bar\psi^j >^s - < p\cdot\psi\psi^i >^s < p\cdot\bar\psi\bar\psi^j >^s \\
&\qquad + < p\cdot\bar\psi\psi^i >^s < p\cdot\psi\bar\psi^j >^s \Big) \\
&+ \frac{1}{2}\left(i < \partial X^i p\cdot (X+\bar X) >_{osc} + \frac{1}{2}k^i \right) \left(< p\cdot\psi\psi^j >^s + < p\cdot\bar\psi\bar\psi^j >^s \right) \\
&+ i k^i < \partial X^j p\cdot (X+\bar X) >_{osc} - \frac{i}{4}(k^2 - q^2) < \partial X^i \bar X^j >_{osc} - \frac{1}{4}k^i k^j \Big\} \ . \qquad (45)
\end{aligned}
$$

We will focus on the large distance limit $l \to +\infty$, in which only the massless modes will contribute and we expect the low energy effective field theory to reproduce all the results. Since $l = \tau + l'$, in this limit at least one among τ and l' is large and thus a massless state is propagating between the two branes, which are far away from each other. If $l' \to \infty$ and τ is finite, the particle is emitted near the second brane; if $\tau \to \infty$ and l' is finite, it is emitted near the first brane. If both $\tau, l' \to \infty$, the particle is emitted far from both branes.

In the large distance limit $l \to \infty$ the bosonic exponential reduces to (see eqs. (B7) and (B8))

$$
< e^{ip\cdot X} >_{osc} = \left[1 - e^{-4\pi\tau}\right]^{-\frac{p^{(2)2}}{2\pi}} \left[1 - e^{-4\pi l'}\right]^{-\frac{p^{(1)2}}{2\pi}} \ . \qquad (46)
$$

After having evaluated the limiting forms of $\mathcal{N}$, one has to integrate in eq. (26) over the proper times τ and l',

$$
\int_0^\infty d\tau \int_0^\infty dl' e^{-\frac{q^2}{2}\tau} e^{-\frac{k^2}{2}l'} < e^{ip\cdot X} >_{osc} \mathcal{N} \ . \qquad (47)
$$

These last integrations will eventually produce factors like $1/q^2$ or $1/k^2$ or both, corresponding to the denominator of the propagators of the massless particles emitted by the branes.

A. Dilaton

Using the explicit form for the polarisation tensor and recalling the notation defined in the Appendix B, the amplitude is found to be

$$
\begin{aligned}
\mathcal{M}^s = \frac{p^2}{4} \Big\{ & \sin^2\theta \left[(K_v - K)^2 - (F_v^s - F^s)^2 - L_v^2 + G_v^{s2} - (U_v^s - W_v)^2 \right. \\
& \left. -2(U_v^s - W_v)[L_v - \cos\theta(K_v - K)] \right] \\
& +4(KK_v - F^sF_v^s) - 4\cos\theta(KL_v - F^sG_v^s) \Big\} \\
& + \frac{1}{8}(k^2 - q^2)[\sin^2\theta K_v + (1+\cos^2\theta)K] \\
& + \frac{p}{2} h_{i1}k^i [L_v - \cos\theta(K_v - K) + (U_v^s - W_v)] - \frac{1}{4} h_{ij}k^ik^j \ . \qquad (48)
\end{aligned}
$$

For the three even spin structures, this expression can be further simplified using the results of Appendix B in the $l \to +\infty$ limit. The non exponential terms $-\frac{(v_1 - v_2)}{2\pi l}$, present in both U_v^s and W_v, cancel in all the three even spin structures.

By using the kinematics and the results of Appendix B, the $\mathcal{M}^{R+}$ amplitude for $l \to \infty$ reduces to

$$
\begin{aligned}
\mathcal{M}^{R+} = & -\frac{1}{4} h_{ij}k^ik^j \\
& - \left[p^{(2)2} + V_2\gamma_2 p^{(2)} h_{i1}k^i + \frac{1}{4}(k^2 - q^2)(1 + V_2^2\gamma_2^2 \sin^2\theta) \right] f(\tau) \\
& - \left[p^{(1)2} - V_1\gamma_1 p^{(1)} h_{i1}k^i - \frac{1}{4}(k^2 - q^2)(1 + V_1^2\gamma_1^2 \sin^2\theta) \right] f(l') \\
& + \frac{1}{2}\tanh(v_1 - v_2)\cos\theta \left\{ -\frac{1}{4}(k^2 - q^2) + p^{(2)2} f(\tau) - p^{(1)2} f(l') \right\} \ . \qquad (49)
\end{aligned}
$$

We define here and in the following

$$
f(\tau) = \frac{e^{-4\pi\tau}}{1 - e^{-4\pi\tau}} \ , \quad f(l') = \frac{e^{-4\pi l'}}{1 - e^{-4\pi l'}} \ . \qquad (50)
$$

The last term in $\mathcal{M}^{R+}$ is easily seen to be a total derivative; in fact by inserting into (47) both eq. (46) and these last terms in eq. (49) we get

$$
\begin{aligned}
& \int_0^\infty d\tau \int_0^\infty dl' e^{-\frac{q^2}{2}\tau} e^{-\frac{k^2}{2}l'} \left[1 - e^{-4\pi\tau} \right]^{-\frac{p^{(2)2}}{2\pi}} \left[1 - e^{-4\pi l'} \right]^{-\frac{p^{(1)2}}{2\pi}} \\
& \qquad \times \left\{ -\frac{1}{4}(k^2 - q^2) + p^{(2)2}\frac{e^{-4\pi\tau}}{1 - e^{-4\pi\tau}} - p^{(1)2}\frac{e^{-4\pi l'}}{1 - e^{-4\pi l'}} \right\} \\
& = -\frac{1}{2}\int_0^\infty d\tau \int_0^\infty dl' \, (\partial_\tau - \partial_{l'}) \left\{ e^{-\frac{q^2}{2}\tau} e^{-\frac{k^2}{2}l'} \left[1 - e^{-4\pi\tau} \right]^{-\frac{p^{(2)2}}{2\pi}} \left[1 - e^{-4\pi l'} \right]^{-\frac{p^{(1)2}}{2\pi}} \right\} \\
& = 0 \ . \qquad (51)
\end{aligned}
$$

For later use, notice that this kind of integration by parts implies the following equivalence relations in the amplitude $\mathcal{M}^s$ (see the remark made after eq. (42))

$$\frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}} \sim -\frac{1}{4}\frac{q^2}{p^{(2)2}} \; , \quad \frac{e^{-4\pi l'}}{1-e^{-4\pi l'}} \sim -\frac{1}{4}\frac{k^2}{p^{(1)2}} \; . \tag{52}$$

From kinematics one finds the relations

$$\begin{aligned}
p^{(2)2} + V_2\gamma_2 p^{(2)2} h_{i1}k^i &= -\frac{1}{2}\cos\theta(k^2-q^2)V_2\gamma_2\frac{p^{(2)}}{p} + \frac{k^0}{p}p^{(2)2} \; , \\
p^{(1)2} - V_1\gamma_1 p^{(1)2} h_{i1}k^i &= \frac{1}{2}\cos\theta(k^2-q^2)V_1\gamma_1\frac{p^{(1)}}{p} - \frac{q^0}{p}p^{(1)2} \; , \\
h_{ij}k^ik^j &= -\frac{1}{4p^2}(k^2-q^2)^2 + \frac{k^0}{p}q^2 - \frac{q^0}{p}k^2 \; .
\end{aligned} \tag{53}$$

By using these relations and again the equivalence relations (52) one can see that also the remaining terms in (49) cancel, and thus there is no contribution from the RR sector in $l \to \infty$ limit.

In the NSNS$\pm$ spin structures the amplitude is found to be

$$\begin{aligned}
\mathcal{M}^{NS\pm} = &-\frac{1}{4}h_{ij}k^ik^j \\
&-\left[p^{(2)2} + V_2\gamma_2 p^{(2)}h_{i1}k^i + \frac{1}{4}(k^2-q^2)(1+V_2^2\gamma_2^2\sin^2\theta)\right] f(\tau) \\
&-\left[p^{(1)2} - V_1\gamma_1 p^{(1)}h_{i1}k^i - \frac{1}{4}(k^2-q^2)(1+V_1^2\gamma_1^2\sin^2\theta)\right] f(l') \\
&\mp e^{-2\pi l}\Big\{p^2\sin^2\theta\sinh^2(v_1-v_2) + p^{(1)2} + p^{(2)2} - p h_{i1}k^i \sinh 2(v_1-v_2) \\
&\qquad -2\cos\theta\sinh 2(v_1-v_2)p^{(2)2}f(\tau) + 2\cos\theta\sinh 2(v_1-v_2)p^{(1)2}f(l')\Big\} \; .
\end{aligned} \tag{54}$$

Here we have used the relation

$$\frac{1}{1-e^{-4\pi\tau}}\cdot\frac{1}{1-e^{-4\pi l'}} = 1 + f(\tau) + f(l') \; , \tag{55}$$

up to terms $\mathcal{O}(e^{-4\pi l})$ which we neglect in the large distance limit.

The first three rows in equation (54) are identical to the first three rows of eq. (49) and thus they also cancel in the integration of eq. (47). Moreover:

$$\begin{aligned}
&p^2\sin^2\theta\sinh^2(v_1-v_2) + p^{(1)2} + p^{(2)2} - p h_{i1}k^i\sinh 2(v_1-v_2) \\
&\quad = \frac{1}{2}\cos\theta\sinh 2(v_1-v_2)(k^2-q^2) \; .
\end{aligned} \tag{56}$$

Thus modulo terms which cancel in the integration of eq. (47), we are left with $\mathcal{M}^{NS\pm} = \pm\, e^{-2\pi l}T$ (following the notation of eq. (31)) where

$$T = -\cos\theta\sinh 2(v_1-v_2)\left[\frac{1}{2}(k^2-q^2) - 2p^{(2)2}f(\tau) + 2p^{(1)2}f(l')\right] \; , \tag{57}$$

which is also seen to be zero using the equivalence relations (52). In conclusion, there is no dilaton emission from interacting moving branes far from each other.

B. Graviton

Using the properties of the polarisation tensor for the graviton, the general amplitude is found to be

$$\begin{aligned}\mathcal{M}^s = \frac{p^2}{4}h_{11}&\left\{(K_v^2 - K^2 - L_v^2) - (F_v^{s2} - F^{s2} - G_v^{s2}) - (U_v^s - W_v)^2\right.\\&\left.-2(U_v^s - W_v)[L_v - \cos\theta(K_v - K)]\right\}\\&+\frac{1}{8}(k^2 - q^2)h_{11}[K_v - K]\\&+\frac{p}{2}h_{i1}k^i[L_v - \cos\theta(K_v - K) + (U_v^s - W_v)] - \frac{1}{4}h_{ij}k^ik^j\ .\end{aligned} \tag{58}$$

For the three even spin structures, this expression can be further simplified using the results of Appendix B in the $l \to +\infty$ limit. Just as for the dilaton the non exponential terms $-\frac{(v_1-v_2)}{2\pi l}$, present in both U_v^s and W_v, cancel in all the three even spin structures.

By using the kinematics, the amplitude for $l \to \infty$ reduces to

$$\begin{aligned}\mathcal{M}^{R+} &= -\frac{1}{4}h_{ij}k^ik^j - \left[V_2\gamma_2 p^{(2)}h_{i1}k^i + \frac{1}{4}(k^2 - q^2)V_2^2\gamma_2^2 h_{11}\right]f(\tau)\\&+\left[V_1\gamma_1 p^{(1)}h_{i1}k^i + \frac{1}{4}(k^2 - q^2)V_1^2\gamma_1^2 h_{11}\right]f(l')\\&+\frac{p}{2}\tanh(v_1 - v_2)\left\{\frac{1}{2}h_{i1}k^i + V_2\gamma_2 p^{(2)}h_{11}f(\tau) - V_1\gamma_1 p^{(1)}h_{11}f(l')\right\}\ .\end{aligned} \tag{59}$$

For the NS$\pm$ sectors, in the $l \to \infty$ limit, we get:

$$\begin{aligned}\mathcal{M}^{NS\pm} &= -\frac{1}{4}h_{ij}k^ik^j - \left[V_2\gamma_2 p^{(2)}h_{i1}k^i + \frac{1}{4}(k^2 - q^2)V_2^2\gamma_2^2 h_{11}\right]f(\tau)\\&+\left[V_1\gamma_1 p^{(1)}h_{i1}k^i + \frac{1}{4}(k^2 - q^2)V_1^2\gamma_1^2 h_{11}\right]f(l')\\&\mp e^{-2\pi l}\left\{p^2 h_{11}\sinh^2(v_1 - v_2) - p h_{i1}k^i\sinh 2(v_1 - v_2)\right.\\&\quad -2h_{11}V_2\gamma_2\sinh 2(v_1 - v_2)pp^{(2)}f(\tau)\\&\quad \left.+2h_{11}V_1\gamma_1\sinh 2(v_1 - v_2)pp^{(1)}f(l')\right\}\ .\end{aligned} \tag{60}$$

The graviton emission amplitude is generically different from zero. We can always use the relations (52) to reduce the final integration over the two proper times τ and l' to the expression (see eq. (46))

$$\int_0^\infty d\tau\int_0^\infty dl' e^{-\frac{q^2}{2}\tau}e^{-\frac{k^2}{2}l'} < e^{ip\cdot X} >_{osc} = I_1 I_2 \tag{61}$$

where

$$I_1 = -\frac{1}{4\pi}\frac{\Gamma[\frac{k^2}{8\pi}]\Gamma[-\frac{p^{(1)2}}{2\pi}+1]}{\Gamma[\frac{k^2}{8\pi}-\frac{p^{(1)2}}{2\pi}+1]} \to -\frac{2}{k^2}\ ,\quad I_2 = -\frac{1}{4\pi}\frac{\Gamma[\frac{q^2}{8\pi}]\Gamma[-\frac{p^{(2)2}}{2\pi}+1]}{\Gamma[\frac{q^2}{8\pi}-\frac{p^{(2)2}}{2\pi}+1]} \to -\frac{2}{q^2}\ . \tag{62}$$

We have indicated the limiting expressions of $I_{1,2}$ for the relevant case where the energy p of the emitted graviton is much smaller than the string scale.

Finally the amplitude at fixed impact parameter $\vec{b}_T$ can be written as

$$\mathcal{A} = \frac{4}{\sinh|v_1 - v_2|} \int \frac{d^2\vec{k}_T}{(2\pi)^2} e^{i\vec{k}_T\cdot\vec{b}_T} \left(\frac{B}{q^2k^2} + \frac{C}{q^2} + \frac{D}{k^2} \right) \tag{63}$$

remembering that

$$k^2 = \vec{k}_T^2 + \frac{p^{(2)2}}{\sinh^2(v_1 - v_2)} \ , \quad q^2 = (\vec{k}_T - \vec{p}_T)^2 + \frac{p^{(1)2}}{\sinh^2(v_1 - v_2)} \ . \tag{64}$$

For the untwisted sector one has:

$$\begin{aligned}
B = & -h_{ij}k^ik^j \left(4\cosh(v_1 - v_2) \prod_a \cos\pi z_a - [\cosh 2(v_1 - v_2) + \sum_a \cos 2\pi z_a] \right) \\
& + p h_{i1} k^i \left(4\sinh(v_1 - v_2) \prod_a \cos\pi z_a - 2\sinh 2(v_1 - v_2) \right) \\
& + 2p^2 h_{11} \sinh^2(v_1 - v_2) \ , \\
C = & -V_1\gamma_1 \frac{h_{i1}k^i}{p^{(1)}} \left(4\cosh(v_1 - v_2) \prod_a \cos\pi z_a - [\cosh 2(v_1 - v_2) + \sum_a \cos 2\pi z_a] \right) \\
& + V_1\gamma_1 \frac{p h_{11}}{p^{(1)}} \left(2\sinh(v_1 - v_2) \prod_a \cos\pi z_a - \sinh 2(v_1 - v_2) \right) \ , \\
D = & \ V_2\gamma_2 \frac{h_{i1}k^i}{p^{(2)}} \left(4\cosh(v_1 - v_2) \prod_a \cos\pi z_a - [\cosh 2(v_1 - v_2) + \sum_a \cos 2\pi z_a] \right) \\
& - V_2\gamma_2 \frac{p h_{11}}{p^{(2)}} \left(2\sinh(v_1 - v_2) \prod_a \cos\pi z_a - \sinh 2(v_1 - v_2) \right) \ .
\end{aligned} \tag{65}$$

In C and D we neglected terms of order k^2 and q^2.

For the twisted sector we have:

$$\begin{aligned}
B &= -\frac{1}{2} h_{ij}k^ik^j \left(\cosh(v_1 - v_2) - 1\right) + \frac{p}{2} h_{i1}k^i \sinh(v_1 - v_2) \ , \\
C &= -\frac{1}{2} V_1\gamma_1 \frac{h_{i1}k^i}{p^{(1)}} \left(\cosh(v_1 - v_2) - 1\right) + \frac{1}{4} V_1\gamma_1 \frac{p h_{11}}{p^{(1)}} \sinh(v_1 - v_2) \ , \\
D &= \frac{1}{2} V_2\gamma_2 \frac{h_{i1}k^i}{p^{(2)}} \left(\cosh(v_1 - v_2) - 1\right) - \frac{1}{4} V_2\gamma_2 \frac{p h_{11}}{p^{(2)}} \sinh(v_1 - v_2) \ .
\end{aligned} \tag{66}$$

This result, which exhibits the graviton emission amplitude from interacting moving branes at large distance, comes from a single string diagram. It receives contributions from three distinct physical processes, namely the bremsstrahlung-like case where the graviton is emitted directly from one of the branes (C and D terms with a single pole in either q^2 or k^2), and the case where it is emitted far from both branes as depicted in the figures of next section (B term with poles in both q^2 and k^2).

The result simplifies for $\theta = 0$, where

$$B = -h_{ij}k^i k^j \left(4\cosh(v_1 - v_2)\prod_a \cos\pi z_a - [\cosh 2(v_1 - v_2) + \sum_a \cos 2\pi z_a]\right) \tag{67}$$

for the untwisted sector,

$$B = -\frac{1}{2}h_{ij}k^i k^j \left(\cosh(v_1 - v_2) - 1\right) \tag{68}$$

for the twisted sector and

$$C = D = 0 \, . \tag{69}$$

In this case, we find that the bremsstrahlung terms cancel.

We discuss now the result for small relative velocity $V = \tanh(v_1 - v_2)$. In the case of the 0-brane, all the z_a's can be set to zero (untwisted sector) and one finds

$$\begin{aligned}
B &\simeq \frac{V^4}{2}h_{ij}k^i k^j - 2V^3 p h_{i1}k^i + 2V^2 p^2 h_{11} \, , \\
C &\simeq \frac{V_1\gamma_1}{p^{(1)}}\left(\frac{V^4}{2}h_{i1}k^i - V^3 p h_{11}\right) \, , \\
D &\simeq -\frac{V_2\gamma_2}{p^{(2)}}\left(\frac{V^4}{2}h_{i1}k^i - V^3 p h_{11}\right) \, .
\end{aligned} \tag{70}$$

Observe that for small velocities $p^{(1,2)} \to p$.

In the orbifold twisted sector we have

$$\begin{aligned}
B &\simeq -\frac{1}{4}V^2 h_{ij}k^i k^j + \frac{1}{2}V p h_{i1}k^i \, , \\
C &\simeq \frac{V_1\gamma_1}{p^{(1)}}\left(\frac{1}{4}V^2 h_{i1}k^i - \frac{1}{4}V p h_{11}\right) \, , \\
D &\simeq -\frac{V_2\gamma_2}{p^{(2)}}\left(\frac{1}{4}V^2 h_{i1}k^i - \frac{1}{4}V p h_{11}\right) \, .
\end{aligned} \tag{71}$$

In the case of the 3-brane of the Z_3 orbifold, averaging over the orbifold relative twists z_a [10], one gets $< \prod_a \cos\pi z_a >= \frac{1}{4}$, $< \sum_a \cos 2\pi z_a >= 0$. Thus for $V \to 0$ one finds

$$\begin{aligned}
B &\simeq \frac{3}{2}V^2 h_{ij}k^i k^j - 3V p h_{i1}k^i + 2V^2 p^2 h_{11} \, , \\
C &\simeq \frac{V_1\gamma_1}{p^{(1)}}\left(\frac{3}{2}V^2 h_{i1}k^i - \frac{3}{2}V p h_{11}\right) \, , \\
D &\simeq -\frac{V_2\gamma_2}{p^{(2)}}\left(\frac{3}{2}V^2 h_{i1}k^i - \frac{3}{2}V p h_{11}\right) \, .
\end{aligned} \tag{72}$$

Finally, by looking at the nearest singularity in k_T^2 in the integrand of eq. (63) one can estimate that the amplitude is maximal for $\theta = 0$ and that for small V

$$\mathcal{A} \lesssim V^{n-1} g_s l_s f(b_T \cdot p/V) e^{-b_T \cdot p/V} \ . \tag{73}$$

where f is some mildly varying function, $n = 2$ for the D3-brane on the orbifold while generically $n = 4$, and we have inserted explicitly the appropriate factors of the string coupling constant g_s and the string length l_s (assuming a compactification radius of the order of l_s). The cross-section for radiating a particle is

$$\sigma = \int d^2\vec{b}_T \int \frac{d^3p}{p} |\mathcal{A}|^2 \ . \tag{74}$$

The probability that two interacting branes at impact parameter $\vec{b}_T$ and with relative velocity V would radiate a particle is thus

$$dP(p, \vec{b}_T, V) = |\mathcal{A}|^2 \frac{d^3p}{p} \ . \tag{75}$$

Thus the amount of radiated energy is

$$< p > \sim g_s^2 l_s^2 \frac{V^{3+2(n-1)}}{b_T^3} \ . \tag{76}$$

If we extrapolate down to the eleven-dimensional Planck length $b_T \sim l_{11} = g_s^{1/3} l_s$ we get a maximal radiated energy

$$< p > \sim g_s \frac{V^{1+2n}}{l_s} \tag{77}$$

which could be compared with the estimate of ref. [4] (the power of V in our expression is due to the amount of supersymmetry cancellation). Actually, there are dynamical effects, that we have so far disregarded in our approximations, which could invalidate the above extrapolation; but for small velocity they are in fact negligible. First, the dynamics would change due to open string pair creation [1] (a process which is encoded in the poles of the partition function eq. (B3)) but this effect is suppressed even at $b_T \sim l_{11}$ for $V < g_s^{2/3}$ [4]. Further, the factor $(V/b_T)^3$ in (76) is due the "kinematical" cutoff on the maximal emitted energy $p_{max} \sim V/b_T$, encoded in the expression for $dP(p, \vec{b}_T, V)$ through the exponential factor of eq. (73); but there is also the string cutoff $1/l_s$ which does not appear here, due to our neglect of the exchange of massive string states. However the string cutoff is larger than the kinematical one, at $b_T \sim l_{11}$, if $V < g_s^{2/3} < g_s^{1/3}$. Note also that in this case p_{max} is much smaller than the brane momentum $M_{br} V = V/(g_s l_s)$ and thus the eikonal approximation holds.

V. FIELD THEORY INTERPRETATION

Let us consider the terms in the graviton emission amplitude which have simultaneously a pole in k^2 and q^2. They correspond to diagrams in which the two branes exchange a massless particle, which can either be a scalar, a vector or a graviton, and the outgoing graviton is emitted by it (see Figs. 1,2,3).

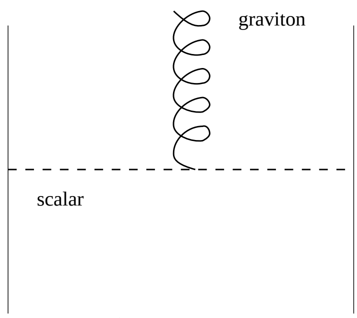

Figure 1

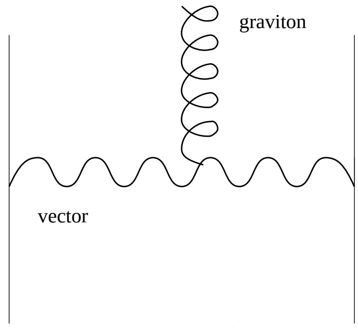

Figure 2

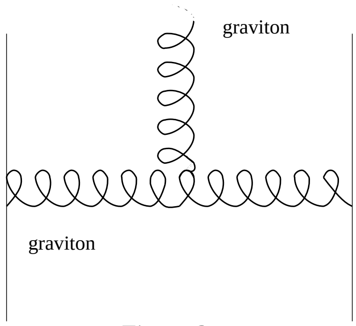

Figure 3

The kinematics for Feynman diagrams is the following. The incoming momenta of the two branes are

$$B_1^\mu = (\gamma_1, V_1\gamma_1, 0) \ , \ \ B_2^\mu = (\gamma_2, V_2\gamma_2, 0) \ . \tag{78}$$

Observe that

$$k \cdot B_1 = q \cdot B_2 = 0 \ . \tag{79}$$

The current, J^μ, and energy-momentum tensor, $T^{\mu\nu}$, of the branes are (neglecting corrections due to the small momentum transfer)

$$\begin{aligned} &J_1^\mu = B_1^\mu \ , \ \ J_2^\mu = B_2^\mu \ , \\ &T_1^{\mu\nu} = B_1^\mu B_1^\nu \ , \ \ T_2^{\mu\nu} = B_2^\mu B_2^\nu \ , \end{aligned} \tag{80}$$

and their conservation follows from eq. (79).

To select the double poles in k^2 and q^2 we have to discard, in eqs. (59) and (60), those terms which are proportional to k^2 or q^2 or $f(\tau)$ or $f(l')$. In the RR sector we find, after multiplying by the $l \to \infty$ limit of Z^{R+}, a result proportional to

$$\frac{1}{k^2 q^2} \left[h_{ij} k^i k^j - p \tanh(v_1 - v_2) h_{i1} k^i \right] \cosh(v_1 - v_2) \ . \tag{81}$$

This is in fact seen to correspond to the diagram, Fig. 2, where the on-shell outgoing graviton is coupled to the RR vector exchanged by the branes through the minimal coupling

$$\mathcal{L}_{Int} = h_{ij}T^{ij} \ . \tag{82}$$

T^{ij} is the symmetrized energy-momentum tensor of the two RR vectors. The latter is given by

$$T^{ij} = F_1^{i\alpha}F_{2\alpha}^{j} - \frac{1}{4}\eta^{ij}F_{1\alpha}^{\alpha}F_{2\beta}^{\beta} \ . \tag{83}$$

The second piece does not contribute upon contraction with the traceless polarisation tensor h^{ij} of the graviton. The first part can be computed using the fields

$$A_1^{\mu} = \frac{1}{k^2}J_1^{\mu} \ , \quad A_2^{\mu} = \frac{1}{q^2}J_2^{\mu} \ . \tag{84}$$

The Feynman diagram is then found to give

$$\begin{aligned}\mathcal{L}_{Int} = \frac{1}{k^2q^2}\big\{&h_{ij}k^ik^j\cosh(v_1-v_2) - ph_{i1}k^i\sinh(v_1-v_2)\\ &-k\cdot q h_{11}\sinh v_1 \sinh v_2\big\} \ .\end{aligned} \tag{85}$$

Since $k\cdot q = \frac{1}{2}(k^2+q^2)$, the last term does not contribute to the double pole and we find an expression proportional to eq. (81).

In the NS-NS sector, there are two contributions, one coming from dilaton exchange and one from graviton exchange. For the dilatons, one has a minimal coupling of the form (82). The symmetrized energy-momentum tensor of the two dilatons is given by

$$T^{ij} \sim \partial^i\phi_1\partial^j\phi_2 \ . \tag{86}$$

Using the fields

$$\phi_1 = \frac{1}{k^2} \ , \quad \phi_2 = \frac{1}{q^2} \ , \tag{87}$$

the interaction is found to be

$$\mathcal{L}_{Int} = \frac{1}{k^2q^2}h_{ij}k^ik^j \ . \tag{88}$$

For the gravitons, the interaction can be deduced from the three gravitons vertex in the harmonic gauge [15]

$$\begin{aligned}\mathcal{L}_{Int} = {}&\partial^{\mu}h_{\beta}^{\alpha}h_{\alpha}^{\nu}\partial_{\nu}h_{\mu}^{\beta} - \frac{1}{2}h_{\beta}^{\alpha}\partial^{\mu}h_{\nu}^{\beta}\partial_{\mu}h_{\alpha}^{\nu} + \frac{1}{2}h_{\beta}^{\alpha}\partial^{\nu}h_{\alpha}^{\mu}\partial_{\mu}h_{\nu}^{\beta} + \frac{1}{2}h_{\beta}^{\alpha}\partial_{\mu}h_{\alpha}^{\beta}\partial^{\mu}h\\ &+\frac{1}{4}\partial_{\mu}h_{\beta}^{\alpha}\partial^{\nu}h_{\alpha}^{\beta}h_{\nu}^{\mu} + \frac{1}{2}\partial_{\nu}\partial^{\mu}hh_{\mu}^{\beta}h_{\beta}^{\nu} - \frac{1}{4}h\partial_{\nu}h_{\beta}^{\alpha}\partial^{\beta}h_{\alpha}^{\nu} + \frac{1}{8}h\partial^{\beta}h_{\nu}^{\mu}\partial_{\beta}h_{\mu}^{\nu}\\ &-\frac{1}{8}h\partial^{\mu}h\partial_{\mu}h - \frac{1}{4}h\partial^{\nu}\partial_{\beta}hh_{\nu}^{\beta} + \frac{1}{2}h_{\nu}^{\mu}\partial^{\alpha}\partial_{\beta}h_{\mu}^{\nu}h_{\alpha}^{\beta} \ .\end{aligned} \tag{89}$$

One has to choose in all possible ways one of the gravitons to be on-shell, with polarisation tensor satisfying $\partial_i h^{ij} = 0$ and $h^i_i = 0$, and the other two to be the off-shell gravitons $h_1^{\mu\nu}$ and $h_2^{\mu\nu}$ coming from the two branes. Using the fields in the harmonic gauge

$$h_1^{\mu\nu} = \frac{1}{k^2}(T_1^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}T_1) \ , \ \ h_2^{\mu\nu} = \frac{1}{q^2}(T_2^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}T_2) \ , \tag{90}$$

and discarding terms containing q^2 or k^2, in the numerator, the interaction is seen to be

$$\mathcal{L}_{Int} = \frac{1}{k^2 q^2}\Big\{ -\frac{1}{4}h_{ij}k^i k^j \cosh 2(v_1 - v_2) + \frac{p}{2}\sinh 2(v_1 - v_2) h_{1i} k^i \\ -\frac{p^2}{2} h_{11}\sinh^2(v_1 - v_2)\Big\} \ . \tag{91}$$

Now let us consider $(\mathcal{M}_{grav}^{NS+}Z^{NS+} - \mathcal{M}_{grav}^{NS-}Z^{NS-})$ and look for the double pole in k^2 and q^2. In the $l \to \infty$ limit we find an expression proportional to

$$\frac{1}{k^2 q^2}\left\{ -\frac{1}{4}h_{ij}k^i k^j \cosh 2(v_1 - v_2) + \frac{p}{2}\sinh 2(v_1 - v_2) h_{1i}k^i - \frac{p^2}{2}h_{11}\sinh^2(v_1 - v_2)\right\} \\ + \frac{1}{k^2 q^2}\left\{ -\frac{1}{4}h_{ij}k^i k^j \sum_a \cos 2\pi z_a \right\} \ . \tag{92}$$

The first bracket matches the contribution from field theory where the emitted graviton couples to the graviton exchanged between the branes via the three graviton coupling. The second bracket matches the emission of the graviton from scalar exchange, the factor $\sum_a cos 2\pi z_a$ indicating that this possible scalar is related to the compactified coordinates. In particular, for the D3-brane case of ref. [10], the invariant projection over the orbifold group gives $< \sum_a \cos 2\pi z_a = 0 >$, and there is no scalar emission from the branes.

VI. AMPLITUDES FOR OTHER MASSLESS PARTICLES

Concerning other massless particles, corresponding to other components of the ten dimensional polarisation tensor h_{AB} $(A, B = 0, 1, \cdots, 9)$, we can restrict to the case where A, B are space i, j (transverse to $\vec{p}$) or internal indices a, b. (For the internal indices we use a complexified notation like $a = (4 + i5), (6 + i7), (8 + i9)$ meaning $X^4 + iX^5$, etc. and similarly for a^*. We do the same for the fermionic coordinates, see ref. [10].) The case where they are both space has been already discussed. In the case where $A = i$ and $B = b$ the matrix element of the vertex is zero for the branes we have considered and disregarding non zero compactified momenta for the large distance limit. In the case where $A = a$ and $B = a^*$ we can have a non zero result for D0-branes compactified either on T^6 or on an orbifold, with the same Neumann or Dirichlet boundary conditions for both members of pairs of compactified coordinates (described by the boundary state of eq. (10) of ref. [10]). This is consistent with our previous work [10] where we have seen that for these compactifications the branes are coupled to spacetime scalars, which can then be emitted. Technically, the non-zero result comes because there is no term $h_{ij}k^ik^j$ in the amplitude eq. (48), and thus the previously seen cancellation does not occur.

Finally in the case of orbifold compactification with mixed Neumann-Dirichlet boundary conditions for the pairs of compactified coordinates (corresponding to D3-branes described by the boundary state of eqs (12), (14) of ref. [10]), the non zero matrix element occurs for $A = a$ and $B = a$. But in this case the projection over the orbifold invariant states multiply the vertex by $1 + g_a^2 + g_a^4 = 0$ (with $g_a = exp(\pm i2\pi/3)$), thus there is no emission, consistently with the analysis of ref. [10].

Let us finally recall that we showed in Sec. IV that there is no emission of the spacetime dilaton in all cases. By spacetime dilaton we mean the massless scalar corresponding to the trace part of the four dimensional polarisation tensor. The spacetime dilaton looks to be uncoupled to the branes, consistently with an analysis appearing in ref. [14]. There can only occur, in some case, the emission of dilaton-like scalars corresponding to the trace of the compactified (internal) components of the polarisation tensor.

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APPENDIX A: SPACETIME BOUNDARY STATE

In this section we briefly recall the construction of the spacetime part of the boundary state for a D0-brane. Starting from the static case, in which the boundary conditions are Neumann for the time direction and Dirichlet for the space directions, the boost required to get the dynamical case is easily implemented as an imaginary rotation. We will use the complex variable $z = \sigma + i\tau$, with σ periodic and ranging from 0 to 1, and τ ranging from 0 to l.

Starting with the bosonic coordinates, recall the mode expansion

$$\begin{aligned} X^\mu(z) &= \frac{X_o^\mu}{2} - \frac{z}{2}Q^\mu + \frac{i}{\sqrt{4\pi}} \sum_{n>0} \frac{1}{\sqrt{n}} (a_n^\mu e^{2\pi niz} - a_n^{\dagger\mu} e^{-2\pi niz}) \ , \\ \bar{X}^\mu(\bar{z}) &= \frac{X_o^\mu}{2} + \frac{\bar{z}}{2}Q^\mu + \frac{i}{\sqrt{4\pi}} \sum_{n>0} \frac{1}{\sqrt{n}} (\tilde{a}_n^\mu e^{-2\pi ni\bar{z}} - \tilde{a}_n^{\dagger\mu} e^{2\pi ni\bar{z}}) \ , \end{aligned} \tag{A1}$$

with the standard commutation relations $[a_m^\mu, a_n^{\dagger\nu}] = [\tilde{a}_m^\mu, \tilde{a}_n^{\dagger\nu}] = \eta^{\mu\nu}\delta_{mn}$, and $[X_o^\mu, Q^\nu] = i\eta^{\mu\nu}$.

The static boundary conditions for the oscillators are

$$(a_n^0 + \tilde{a}_n^{\dagger 0})|B>_{osc}^B = 0 \ , \ \ (a_n^i - \tilde{a}_n^{\dagger i})|B>_{osc}^B = 0 \ . \tag{A2}$$

Pairing the X^0, X^1 coordinates in the light-cone combinations $X^\pm = X^0 \pm X^1$, whose oscillators ($\alpha_n = a_n^0 + a_n^1, \beta_n = a_n^0 - a_n^1$) have as the only non-vanishing commutation relations $[\alpha_m, \beta_n^\dagger] = [\beta_m, \alpha_n^\dagger] = -2\delta_{mn}$, the boundary conditions become

$$\begin{aligned}
&(\alpha_n + \tilde{\beta}_n^\dagger)|B>_{osc}^b = 0 \ , \ \ (\beta_n + \tilde{\alpha}_n^\dagger)|B>_{osc}^b = 0 \ , \\
&(a_n^{Ti} - \tilde{a}_n^{T\dagger i})|B>_{osc}^b = 0 \ ,
\end{aligned} \tag{A3}$$

and the oscillator part of the bosonic boundary state is written as

$$|B>_{osc}^b = \exp\sum_{n=1}^\infty \{\frac{1}{2}(\alpha_n^\dagger \tilde{\alpha}_n^\dagger + \beta_n^\dagger \tilde{\beta}_n^\dagger) + a_n^{T\dagger}\tilde{a}_n^{T\dagger}\}|0> \ . \tag{A4}$$

For the zero modes, the boundary conditions are

$$Q^0|B,Y>_o^b = 0 \ , \ \ (X_o^i - Y^i)|B,Y>_o^b = 0 \ , \tag{A5}$$

where Y^i is the transverse position of the brane. These are solved taking

$$|B,Y>_o^b = \delta^{(3)}(X_o^i - Y^i)|0> = \int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{Y}\cdot\vec{q}}|\vec{q}> \ . \tag{A6}$$

In the dynamical case, it is convenient to introduce the rapidity v related to the velocity by $V = \tanh v$. The boundary conditions then become in terms of the light-cone combinations

$$\begin{aligned}
&(e^{-v}\alpha_n + e^v \tilde{\beta}_n^\dagger)|B,V>_{osc}^b = 0 \ , \ \ (e^v\beta_n + e^{-v}\tilde{\alpha}_n^\dagger)|B,V>_{osc}^b = 0 \ , \\
&(a_n^{Ti} - \tilde{a}_n^{T\dagger i})|B,V>_{osc}^b = 0 \ .
\end{aligned} \tag{A7}$$

Thus, the oscillator part of the boosted bosonic boundary state is

$$|B,V>_{osc}^b = \exp\sum_{n=1}^\infty \{\frac{1}{2}(e^{-2v}\alpha_n^\dagger \tilde{\alpha}_n^\dagger + e^{2v}\beta_n^\dagger \tilde{\beta}_n^\dagger) + a_n^{T\dagger}\tilde{a}_n^{T\dagger}\}|0> \ . \tag{A8}$$

For the zero modes, the new boundary conditions are

$$\begin{aligned}
&(\cosh v Q^0 - \sinh v Q^1)|B,V,Y>_o^b = 0 \ , \\
&(\cosh v X_o^1 - \sinh v X_o^0 - Y^1)|B,V,Y>_o^b = 0 \ , \\
&(X_o^{Ti} - Y^{Ti})|B,V,Y>_o^b = 0 \ ,
\end{aligned} \tag{A9}$$

where Y^i is the transverse position of the brane. These are solved taking

$$
\begin{aligned}
|B,V,Y>_o^b &= \delta(\cosh vX_o^1 - \sinh vX_o^0 - Y^1)\delta^{(2)}(X_o^{Ti} - Y^{Ti})|0> \\
&= \int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{Y}\cdot\vec{q}}|q^\mu> ,
\end{aligned} \tag{A10}
$$

where $q^\mu = (\sinh vq^1, \cosh vq^1, \vec{q}_T) = (\gamma V q^1, \gamma q^1, \vec{q}_T)$

In a more formal way, the moving boundary state is obtained from the static one with a boost of opposite velocity [13]:

$$
|B,V,Y>= e^{-ivJ^{01}}|B,Y> , \tag{A11}
$$

where

$$
J^{\mu\nu} = X_o^\mu Q^\nu - X_o^\nu Q^\mu - i\sum_{n=1}^{\infty}(a_n^{\dagger\mu}a_n^\nu - a_n^{\dagger\nu}a_n^\mu + \tilde{a}_n^{\dagger\mu}\tilde{a}_n^\nu - \tilde{a}_n^{\dagger\nu}\tilde{a}_n^\mu) . \tag{A12}
$$

Now consider the fermionic part. The mode expansions are

$$
\begin{aligned}
\psi^\mu(z) &= \sum_{n>0}(\psi_n^\mu e^{2\pi niz} + \psi_n^{\dagger\mu}e^{-2\pi niz}) , \\
\bar{\psi}^\mu(\bar{z}) &= \sum_{n>0}(\tilde{\psi}_n^\mu e^{-2\pi ni\bar{z}} + \tilde{\psi}_n^{\dagger\mu}e^{2\pi ni\bar{z}}) ,
\end{aligned} \tag{A13}
$$

where the sums are over half integer or integer depending on whether we have NSNS or RR fermions. In the RR sector there are also zero modes ψ_o^μ and $\tilde{\psi}_o^\mu$. The anticommutation relation for the oscillators are the standard ones, $\{\psi_m^\mu, \psi_n^{\dagger\nu}\} = \{\bar{\psi}_m^\mu, \bar{\psi}_n^{\dagger\nu}\} = \eta^{\mu\nu}\delta_{mn}$, in both the NSNS and RR sectors, whereas the RR zero modes satisfy the Clifford algebra $\{\psi_o^\mu, \psi_o^\nu\} = \{\tilde{\psi}_o^\mu, \tilde{\psi}_o^\nu\} = \eta^{\mu\nu}$.

The static boundary conditions, consistent with the mode expansion (A13) are

$$
(\psi_n^0 + i\eta\tilde{\psi}_n^{\dagger 0})|B,\eta>_{osc}^f = 0 , \quad (\psi_n^i - i\eta\tilde{\psi}_n^{\dagger i})|B,\eta>_{osc}^f = 0 , \tag{A14}
$$

where $\eta = \pm 1$ has been introduced to deal with the GSO projection.

Pairing the ψ^0, ψ^1 fields in the light-cone combinations $\psi^A = \psi^0 + \psi^1$, $\psi^B = \psi^0 - \psi^1$, whose oscillators ψ_n^A, ψ_n^B have as the only non-vanishing commutation relation $\{\psi_m^A, \psi_n^{B\dagger}\} = \{\psi_m^B, \psi_n^{A\dagger}\} = -2\delta_{mn}$, both in the NSNS and RR sectors, the boundary conditions become

$$
\begin{aligned}
&(\psi_n^A + i\eta\tilde{\psi}_n^{B\dagger})|B,\eta>_{osc}^f = 0 , \quad (\psi_n^B + i\eta\tilde{\psi}_n^{A\dagger})|B,\eta>_{osc}^f = 0 , \\
&(\psi_n^T - i\eta\tilde{\psi}_n^{T\dagger})|B,\eta>_{osc}^f = 0 ,
\end{aligned} \tag{A15}
$$

and the oscillator part of the fermionic boundary state is written as

$$
|B,\eta>_{osc}^f = \exp i\eta\sum_{n>0}^{\infty}\{\frac{1}{2}(\psi_n^{A\dagger}\tilde{\psi}_n^{A\dagger} + \psi_n^{B\dagger}\tilde{\psi}_n^{B\dagger}) - \psi_n^{T\dagger}\tilde{\psi}_n^{T\dagger}\}|0> , \tag{A16}
$$

with appropriate moding for each sector. For the RR zero modes, the boundary conditions are

$$(\psi_o^0 + i\eta\tilde{\psi}_o^{\dagger 0})|B,\eta>_o^R = 0\ , \quad (\psi_o^i - i\eta\tilde{\psi}_o^{\dagger i})|B,\eta>_o^R = 0\ . \tag{A17}$$

It is convenient to define $a = (\gamma^0 + \gamma^1)/2$, $a^* = (\gamma^0 - \gamma^1)/2$ and $b = (-i\gamma^2 + \gamma^3)/2$, $b^* = (-i\gamma^2 - \gamma^3)/2$ such that $\{a, a^*\} = \{b, b^*\} = 1$, and similarly for $\tilde{a}$,$\tilde{b}$, all other anticommutators being zero. The boundary conditions for the zero modes can then be written as

$$\begin{aligned}
&(a + i\eta\tilde{a}^*)|B,\eta>_o^R = 0\ , \quad (a^* + i\eta\tilde{a}^)|B,\eta>_o^R = 0\ , \\
&(b - i\eta\tilde{b})|B,\eta>_o^R = 0\ , \quad (b^* - i\eta\tilde{b}^*)|B,\eta>_o^R = 0\ .
\end{aligned} \tag{A18}$$

Defining a "vacuum" $|0> \otimes |\tilde{0}>$ by $a|0> = b|0> = \tilde{a}|\tilde{0}> = \tilde{b}^*|\tilde{0}> = 0$, we find the zero mode stationary boundary state

$$|B,\eta>_0^R = \frac{1}{\sqrt{2}} e^{-i\eta(a^*\tilde{a}^* - b^*\tilde{b})}|0> \otimes |\tilde{0}>\ \ . \tag{A19}$$

Notice that the boundary conditions imply that for $z = \bar{z}$, i.e. $\tau = 0$,

$$\psi^0(z) = -i\eta\bar{\psi}^0(\bar{z})\ , \quad \psi^i(z) = i\eta\bar{\psi}^i(\bar{z})\ . \tag{A20}$$

In the dynamical case, the boundary conditions become in terms of the light-cone combinations $\psi^{A,B}$,

$$\begin{aligned}
&(e^{-v}\psi_n^A + i\eta e^v\tilde{\psi}_n^{B\dagger})|B,\eta,V>_{osc}^f = 0\ , \quad (e^v\psi_n^B + i\eta e^{-v}\tilde{\psi}_n^{A\dagger})|B,\eta,V>_{osc}^f = 0\ , \\
&(\psi_n^T - i\eta\tilde{\psi}_n^{T\dagger})|B,\eta,V>_{osc}^f = 0\ ,
\end{aligned} \tag{A21}$$

so that the oscillator part of the boosted fermionic boundary state is

$$|B,\eta,V>_{osc}^f = \exp i\eta\sum_{n>0}^{\infty}\{\frac{1}{2}(e^{-2v}\psi_n^{A\dagger}\tilde{\psi}_n^{A\dagger} + e^{2v}\psi_n^{B\dagger}\tilde{\psi}_n^{B\dagger}) - \psi_n^{T\dagger}\tilde{\psi}_n^{T\dagger}\}|0>\ , \tag{A22}$$

with appropriate moding for each sector.
For the RR zero modes, the new boundary conditions are

$$\begin{aligned}
&(e^{-v}a + i\eta e^v\tilde{a}^*)|B,\eta>_o^R = 0\ , \quad (e^v a^* + i\eta e^{-v}\tilde{a}^)|B,\eta>_o^R = 0\ , \\
&(b - i\eta\tilde{b})|B,\eta>_o^R = 0\ , \quad (b^* - i\eta\tilde{b}^*)|B,\eta>_o^R = 0\ .
\end{aligned} \tag{A23}$$

The boosted zero mode boundary state then becomes

$$|B,\eta,V>_o^R = \frac{1}{\sqrt{2}} e^v e^{-i\eta(e^{-2v}a^*\tilde{a}^* - b^*\tilde{b})}|0> \otimes |\tilde{0}>\ \ . \tag{A24}$$

Notice furthermore that the new boundary conditions imply that for $z = \bar{z}$, i.e. $\tau = 0$,

$$\begin{aligned}
&\psi^0(z) = -i\eta(\cosh 2v\bar{\psi}^0(\bar{z}) - \sinh 2v\bar{\psi}^1(\bar{z}))\ , \\
&\psi^1(z) = i\eta((\cosh 2v\bar{\psi}^1(\bar{z}) - \sinh 2v\bar{\psi}^0(\bar{z})))\ , \\
&\psi^T(z) = i\eta\bar{\psi}^T(\bar{z})\ .
\end{aligned} \tag{A25}$$

APPENDIX B: PARTITION FUNCTIONS AND PROPAGATORS

In this section, we will use the boosted boundary states to compute the uncompactified part of the partition functions and the correlation functions on the cylinder. The contribution to the partition function of the $(2,3)$ bosonic and fermionic coordinates cancels with the ghost contributions (except that in the odd spin structure case the $\beta-\gamma$ ghosts always contain the zero mode insertion). The net effect of the velocity is a twist. We shall define the modular parameter $q=e^{-2\pi l}$.

For the bosonic field, we need only to consider the oscillator part, with the Hamiltonian

$$H_{osc}=2\pi\sum_{n=1}^{\infty}n\left\{-\frac{1}{2}(\alpha_n^{\dagger}\beta_n+\beta_n^{\dagger}\alpha_n+\tilde{\alpha}_n^{\dagger}\tilde{\beta}_n+\tilde{\beta}_n^{\dagger}\tilde{\alpha}_n)+a_n^{T\dagger}a_n^{T}+\tilde{a}_n^{\dagger T}\tilde{a}_n^{T})\right\}\ . \tag{B1}$$

Here a^T includes all the transverse directions, both uncompactified and compactified. The uncompactified part of the partition function

$$Z^b=<B,V_1|e^{-lH_{osc}}|B,V_2>^b_{osc} \tag{B2}$$

is then computed to be (taking into account the ghost contribution)

$$Z^b(unc)=\prod_{n=1}^{\infty}\frac{1}{\left(1-q^{2n}e^{-2(v_1-v_2)}\right)\left(1-q^{2n}e^{2(v_1-v_2)}\right)}\ . \tag{B3}$$

The complete partition function has been explicitly written in ref. [10], eq. (16) (for the toroidal compactification or in general for D0-branes putting $z_a=0$) and in eq. (19) (for D0-branes on a Z_3 orbifold fixed point, twisted sector).

The correlation functions, as defined in eq. (19) require a bit more work. Define

$$\begin{aligned}
A_v(\tau,l)&\equiv<X^0(z)\bar{X}^0(\bar{z})>_{osc}=<X^1(z)\bar{X}^1(\bar{z})>_{osc}\ ,\\
A(\tau,l)\delta^{ij}&\equiv<X^{Ti}(z)\bar{X}^{Tj}(\bar{z})>_{osc}\ ,\\
B_v(\tau,l)&\equiv<X^0(z)\bar{X}^1(\bar{z})>_{osc}=<X^1(z)\bar{X}^0(\bar{z})>_{osc}\ ,\\
C_v(l)&\equiv<X^0(z)X^0(z)>_{osc}=<\bar{X}^0(\bar{z})\bar{X}^0(\bar{z})>_{osc}\\
&=-<X^1(z)X^1(z)>_{osc}=-<\bar{X}^1(\bar{z})\bar{X}^1(\bar{z})>_{osc}\ ,\\
-C(l)\delta^{ij}&\equiv<X^{Ti}(z)X^{Tj}(z)>_{osc}=<\bar{X}^{Ti}(\bar{z})\bar{X}^{Tj}(\bar{z})>_{osc}\ ,
\end{aligned} \tag{B4}$$

with $A(\tau,l)=A_v(\tau,l)|_{v_1=v_2=0}$ and $C(\tau,l)=C_v(\tau,l)|_{v_1=v_2=0}$. Doing the oscillator algebra, and using the formulae

$$\sum_{k=0}^{\infty}x^k=\frac{1}{1-x}\ ,\quad\sum_{k=1}^{\infty}\frac{x^k}{k}=-\ln(1-x)\ , \tag{B5}$$

we write the results as

$$
\begin{aligned}
A_v &= \frac{1}{4\pi}\sum_{n=0}^{\infty}\Big\{\cosh 2[(v_1-v_2)n-v_2]\ln(1-q^{2n}e^{-4\pi\tau}) \\
&\qquad\qquad + \cosh 2[(v_2-v_1)n-v_1]\ln(1-q^{2n}e^{-4\pi l'})\Big\}\ , \\
B_v &= -\frac{1}{4\pi}\sum_{n=0}^{\infty}\Big\{\sinh 2[(v_1-v_2)n-v_2]\ln(1-q^{2n}e^{-4\pi\tau}) \\
&\qquad\qquad + \sinh 2[(v_2-v_1)n-v_1]\ln(1-q^{2n}e^{-4\pi l'})\Big\}\ , \\
C_v &= \frac{1}{2\pi}\sum_{n=1}^{\infty}\cosh 2[(v_1-v_2)n]\ln(1-q^{2n})\ .
\end{aligned}
\tag{B6}
$$

In the last expression, we have discarded a normal ordering constant that will never contribute in the amplitude because of $p^2=0$.

The bosonic exponential correlation is given by

$$
< e^{ip\cdot X} >_{osc}= e^{-\frac{1}{2}p_\mu p_\nu <(X+\bar{X})^\mu_{osc}(X+\bar{X})^\nu_{osc}>} = e^{-[(p_0^2+p_1^2)A_v+\vec{p}_T^{\,2}(A+C_v-C)+2p_0p_1B_v]} \tag{B7}
$$

and, using $p=p^0$ and $\cos\theta=\frac{p^1}{p}$, can be recast in the following form

$$
\begin{aligned}
< e^{ip\cdot X} >_{osc}= &\prod_{n=1}^{\infty}\Big[1-q^{2n}\Big]^{-\frac{p^2}{\pi}\sinh^2[(v_1-v_2)n]\sin^2\theta} \\
&\times\prod_{n=0}^{\infty}\Big[1-q^{2n}e^{-4\pi\tau}\Big]^{-\frac{p^2}{2\pi}\cosh^2[(v_1-v_2)n-v_2]\{1+\tanh[(v_1-v_2)n-v_2]\cos\theta\}^2} \\
&\times\prod_{n=0}^{\infty}\Big[1-q^{2n}e^{-4\pi l'}\Big]^{-\frac{p^2}{2\pi}\cosh^2[(v_2-v_1)n-v_1]\{1+\tanh[(v_2-v_1)n-v_1]\cos\theta\}^2}\ .
\end{aligned}
\tag{B8}
$$

Consider now correlations involving only one derivative, and define

$$
\begin{aligned}
\frac{i}{2}K_v(\tau,l) &\equiv < \partial X^0(z)\bar{X}^0(\bar{z}) >_{osc}=< \partial X^1(z)\bar{X}^1(\bar{z}) >_{osc} \\
&= -< \bar{\partial}\bar{X}^0(\bar{z})X^0(z) >_{osc}= -< \bar{\partial}\bar{X}^1(\bar{z})X^1(z) >_{osc}\ , \\
\frac{i}{2}\delta^{ij}K(\tau,l) &\equiv < \partial X^{Ti}(z)\bar{X}^{Tj}(\bar{z}) >_{osc}= -< \bar{\partial}\bar{X}^{Tj}(\bar{z})X^{Ti}(z) >_{osc}\ , \\
\frac{i}{2}L_v(\tau,l) &\equiv < \partial X^0(z)\bar{X}^1(\bar{z}) >_{osc}=< \partial X^1(z)\bar{X}^0(\bar{z}) >_{osc} \\
&= -< \bar{\partial}\bar{X}^1(\bar{z})X^0(z) >_{osc}= -< \bar{\partial}\bar{X}^0(\bar{z})X^1(z) >_{osc}\ , \\
\frac{i}{2}W_v(l) &\equiv < \partial X^0(z)X^1(z) >_{osc}= -< \bar{\partial}\bar{X}^0(\bar{z})\bar{X}^1(\bar{z}) >_{osc}\ ,
\end{aligned}
\tag{B9}
$$

with $K(\tau,l)\equiv K_v(\tau,l)|_{v_1=v_2=0}$. One obtains

$$
\begin{aligned}
K_v = -\sum_{n=0}^{\infty}\Bigg\{&\cosh 2[(v_1-v_2)n-v_2]\frac{q^{2n}e^{-4\pi\tau}}{1-q^{2n}e^{-4\pi\tau}} \\
&-\cosh 2[(v_2-v_1)n-v_1]\frac{q^{2n}e^{-4\pi l'}}{1-q^{2n}e^{-4\pi l'}}\Bigg\}\ ,
\end{aligned}
$$

$$
\begin{aligned}
L_v = \sum_{n=0}^{\infty} \Bigg\{ &\sinh 2[(v_1 - v_2)n - v_2] \frac{q^{2n} e^{-4\pi\tau}}{1 - q^{2n} e^{-4\pi\tau}} \\
&- \sinh 2[(v_2 - v_1)n - v_1] \frac{q^{2n} e^{-4\pi l'}}{1 - q^{2n} e^{-4\pi l'}} \Bigg\} , \\
W_v = -\frac{v_1 - v_2}{2\pi l} &- 2 \sum_{n=1}^{\infty} \sinh 2[(v_1 - v_2)n] \frac{q^{2n}}{1 - q^{2n}} .
\end{aligned} \tag{B10}
$$

Turn now to the fermions, whose Hamiltonian is

$$
H = 2\pi \sum_{n>0}^{\infty} n \left\{ -\frac{1}{2} (\psi_n^{A\dagger} \psi_n^B + \psi_n^{B\dagger} \psi_n^A + \tilde{\psi}_n^{A\dagger} \tilde{\psi}_n^B + \tilde{\psi}_n^{B\dagger} \tilde{\psi}_n^A) + \psi_n^{T\dagger} \psi_n^T + \tilde{\psi}_n^{\dagger T} \tilde{\psi}_n^T) \right\} \tag{B11}
$$

with appropriate moding in each sector. The uncompactified part of the partition functions

$$
\begin{aligned}
Z_{\pm}^R &= < B, \pm, V_1 | e^{-lH} | B, +, V_2 >^R , \\
Z_{\pm}^{NS} &= < B, \pm, V_1 | e^{-lH} | B, +, V_2 >^{NS} ,
\end{aligned} \tag{B12}
$$

are found to be (taking into account the ghost contribution)

$$
\begin{aligned}
Z_+^R(unc) &= 4 \cosh(v_1 - v_2) \prod_{n=1}^{\infty} \left(1 + q^{2n} e^{-2(v_1 - v_2)} \right) \left(1 + q^{2n} e^{2(v_1 - v_2)} \right) , \\
Z_-^R(unc) &= 4 \sinh(v_1 - v_2) \prod_{n=1}^{\infty} \left(1 - q^{2n} e^{-2(v_1 - v_2)} \right) \left(1 - q^{2n} e^{2(v_1 - v_2)} \right) , \\
Z_{\pm}^{NS}(unc) &= \prod_{n=1}^{\infty} \left(1 \pm q^{2n-1} e^{-2(v_1 - v_2)} \right) \left(1 \pm q^{2n-1} e^{2(v_1 - v_2)} \right) .
\end{aligned} \tag{B13}
$$

The complete partition functions for the even spin structures have been written in ref. [10] eqs. (28) (toroidal or generic orbifold case) and (32) (D0-branes on a Z_3 orbifold fixed point, twisted sector). The odd spin structure case has already been separately discussed in Sec. I.

Let us discuss the fermionic correlation functions. In order to treat the odd spin structure case, we have to make some preliminary observation. Referring to our definition eq. (23), we have to consider two cases, when $\mathcal{O}$ is quadratic in the fermion operators and when it is quartic. In the quadratic case we define $< \mathcal{O} >^{odd} \equiv \ll \mathcal{O} \gg^{odd}$, and thus the only non vanishing correlators are

$$
\frac{1}{2} \epsilon^{ij} = < \psi^{Ti}(z) \bar{\psi}^{Tj}(\bar{z}) >^{odd} = -i < \psi^{Ti}(z) \psi^{Tj}(z) >^{odd} = i < \bar{\psi}^{Ti}(\bar{z}) \bar{\psi}^{Tj}(\bar{z}) >^{odd} , \tag{B14}
$$

with $\epsilon^{ij} = \epsilon^{1ij}$. For the quartic case, the only non vanishing result is when $\mathcal{O}$ contains the zero modes in the two transverse directions and each only once. Thus, $\mathcal{O}$ can be written as a product of a quadratic term, say $\psi\psi$ times the 2-3 zero modes (which can be either both left or right or mixed): $\mathcal{O} = \psi\psi \cdot (0 - modes)^{2,3}$. We then define the odd propagator $< \psi\psi >^{odd}$ by

$$< \psi\psi >^{odd} < (0-modes)^{2,3} > \equiv \ll \psi\psi \circ (0-modes)^{2,3} \gg^{odd} , \tag{B15}$$

where $< (0-modes)^{2,3} >$ is given by eq. (B14) and $\ll \cdots \gg$ by eq. (23).

We thus have (here we always put $\eta_1 = +1$), in all cases for $s = even$ and in the quartic case for $s = odd$,

$$\begin{aligned}
iF_v^s(\tau, l) &\equiv < \psi^0(z)\bar{\psi}^0(\bar{z}) >^s = < \psi^1(z)\bar{\psi}^1(\bar{z}) >^s , \\
i\delta^{ij} F^{s(even)}(\tau, l) &\equiv < \psi^{Ti}(z)\bar{\psi}^{Tj}(\bar{z}) >^{s\ even} , \\
i\delta^{ij} F^{odd}(\tau, l) &= < \psi^{Ti}(z)\bar{\psi}^{Tj}(\bar{z}) >^{odd} , \\
iG_v^s(\tau, l) &\equiv < \psi^0(z)\bar{\psi}^1(\bar{z}) >^s = < \psi^1(z)\bar{\psi}^0(\bar{z}) >^s , \\
U_v^s(l) &\equiv < \psi^0(z)\psi^1(z) >^s = < \bar{\psi}^0(\bar{z})\bar{\psi}^1(\bar{z}) >^s , \\
0 &= < \psi^{Ti}(z)\psi^{Tj}(z) >^{even} = - < \bar{\psi}^{Ti}(\bar{z})\bar{\psi}^{Tj}(\bar{z}) >^{even} ,
\end{aligned} \tag{B16}$$

with $F^s(\tau, l) = F_v^s(\tau, l)|_{v_1 = v_2 = 0}$ both in the even and the odd cases. We can use Wick's theorem evaluating matrix elements by using the propagators defined in eq. (B16) (which for the odd spin structure refers to the four fermions case) and eq. (B14) (two fermions case). Notice that we have also $< \psi^1\psi^{Ti} > = < \bar{\psi}^1\bar{\psi}^{Ti} > = < \psi^1\bar{\psi}^{Ti} > = 0$.

In the NSNS$\pm$ sectors only fermionic oscillator modes appear, whereas in the RR$\pm$ sectors we have also the fermionic zero modes. Their contributions are

$$\begin{aligned}
F_v^{oR+} &= -\frac{1}{2}\frac{\cosh(v_1+v_2)}{\cosh(v_1-v_2)} , \quad F_v^{oR-} = -\frac{1}{2}\frac{\sinh(v_1+v_2)}{\sinh(v_1-v_2)} , \\
G_v^{oR+} &= -\frac{1}{2}\frac{\sinh(v_1+v_2)}{\cosh(v_1-v_2)} , \quad G_v^{oR-} = -\frac{1}{2}\frac{\cosh(v_1+v_2)}{\sinh(v_1-v_2)} , \\
U_v^{oR+} &= \frac{1}{2}\tanh(v_1-v_2) , \quad U_v^{oR-} = \frac{1}{2}\coth(v_1-v_2) .
\end{aligned} \tag{B17}$$

The full correlators are then obtained as

$$F_v^{R\pm} = F_v^{oR\pm} + \tilde{F}_v^{R\pm} , \quad G_v^{R\pm} = G_v^{oR\pm} + \tilde{G}_v^{R\pm} , \quad U_v^{R\pm} = U_v^{oR\pm} + \tilde{U}_v^{R\pm} , \tag{B18}$$

where

$$\begin{aligned}
\tilde{F}_v^{R\pm} &= -\sum_{n=0}^{\infty}(\mp)^n \left\{ \cosh 2[(v_1-v_2)n - v_2]\frac{q^{2n}e^{-4\pi\tau}}{1-q^{2n}e^{-4\pi\tau}} \right. \\
&\qquad\qquad \left. \pm \cosh 2[(v_2-v_1)n - v_1]\frac{q^{2n}e^{-4\pi l'}}{1-q^{2n}e^{-4\pi l'}} \right\} , \\
\tilde{G}_v^{R\pm} &= \sum_{n=0}^{\infty}(\mp)^n \left\{ \sinh 2[(v_1-v_2)n - v_2]\frac{q^{2n}e^{-4\pi\tau}}{1-q^{2n}e^{-4\pi\tau}} \right. \\
&\qquad\qquad \left. \pm \sinh 2[(v_2-v_1)n - v_1]\frac{q^{2n}e^{-4\pi l'}}{1-q^{2n}e^{-4\pi l'}} \right\} , \\
\tilde{U}_v^{R\pm} &= -\frac{(v_1-v_2)}{2\pi l} - 2\sum_{n=1}^{\infty}(\mp)^n \sinh 2[(v_1-v_2)n]\frac{q^{2n}}{1-q^{2n}} ,
\end{aligned} \tag{B19}$$

in the RR sector and

$$F_v^{NS\pm} = -\sum_{n=0}^{\infty}(\mp)^n \left\{\cosh 2[(v_1-v_2)n - v_2]\frac{q^n e^{-2\pi\tau}}{1-q^{2n}e^{-4\pi\tau}}\right.$$
$$\left. \pm \cosh 2[(v_2-v_1)n - v_1]\frac{q^n e^{-2\pi l'}}{1-q^{2n}e^{-4\pi l'}}\right\} ,$$
$$G_v^{NS\pm} = \sum_{n=0}^{\infty}(\mp)^n \left\{\sinh 2[(v_1-v_2)n - v_2]\frac{q^n e^{-2\pi\tau}}{1-q^{2n}e^{-4\pi\tau}}\right.$$
$$\left. \pm \sinh 2[(v_2-v_1)n - v_1]\frac{q^n e^{-2\pi l'}}{1-q^{2n}e^{-4\pi l'}}\right\} ,$$
$$U_v^{NS\pm} = -\frac{(v_1-v_2)}{2\pi l} - 2\sum_{n=1}^{\infty}(\mp)^n \sinh 2[(v_1-v_2)n]\frac{q^n}{1-q^{2n}} , \tag{B20}$$

in the NSNS sector. The equal-point correlators $U^{R\pm}$ and $U^{NS\pm}$ can be deduced from the other correlators by using the eq. (A25) to reflect left and right movers at the boundaries.

Notice that world sheet supersymmetry is enforced between the bosons and the RR odd spin structure fermions. Since $K_v = \tilde{F}_v^{R-}$, $L_v = \tilde{G}_v^{R-}$ and $W_v = \tilde{F}_v^{R-}$, we explicitly check the relations

$$< \partial X^\mu(z)\bar{X}^\nu(\bar{z}) >_{osc} = \frac{1}{2} < \psi^\mu(z)\bar{\psi}^\nu(\bar{z}) >_{osc}^{R-} ,$$
$$< \partial X^\mu(z)X^\nu(z) >_{osc} = \frac{i}{2} < \psi^\mu(z)\psi^\nu(z) >_{osc}^{R-} ,$$
$$< \bar{\partial}\bar{X}^\mu(\bar{z})X^\nu(z) >_{osc} = \frac{\eta}{2} < \bar{\psi}^\mu(\bar{z})\psi^\nu(z) >_{osc}^{R-} ,$$
$$< \bar{\partial}\bar{X}^\mu(z)\bar{X}^\nu(\bar{z}) >_{osc} = -\frac{i}{2} < \bar{\psi}^\mu(\bar{z})\bar{\psi}^\nu(\bar{z}) >_{osc}^{R-} . \tag{B21}$$

The periodicities of the fermionic propagators in the four spin structures, which should follow from an involution from the torus to the cylinder, can be seen considering the light-cone combinations $\psi^\pm = \psi^0 \pm \psi^1$ and in particular their propagators $< \psi^\pm(z)\bar{\psi}^\pm(\bar{z}) >_s = P^s_{v(\pm)}$, which are given by

$$P^s_{v(\pm)} = \frac{i}{2}(F_v^s \pm G_v^s) . \tag{B22}$$

One can then explicitly check the transformation around the two cycles of the covering torus, which has modulus $\nu = 2il$, $w \to w + m + \nu n$ with $w = z - \bar{z} = 2i\tau$, that is $\tau \to \tau - \frac{i}{2}m + nl$, getting

$$P^{R+}_{v(\pm)}(\tau - \frac{i}{2}m + nl, l) = e^{i\pi n \pm 2n(v_1-v_2)} P^{R+}_{v(\pm)}(\tau, l) ,$$
$$P^{R-}_{v(\pm)}(\tau - \frac{i}{2}m + nl, l) = e^{\pm 2n(v_1-v_2)} P^{R-}_{v(\pm)}(\tau, l) ,$$
$$P^{NS+}_{v(\pm)}(\tau - \frac{i}{2}m + nl, l) = e^{i\pi m + i\pi n \pm 2n(v_1-v_2)} P^{NS+}_{v(\pm)}(\tau, l) ,$$
$$P^{NS-}_{v(\pm)}(\tau - \frac{i}{2}m + nl, l) = e^{i\pi m \pm 2n(v_1-v_2)} P^{NS-}_{v(\pm)}(\tau, l) . \tag{B23}$$

These transformation rules for $m = 0$ correspond to the boundary conditions at the two ends of the cylinder for the $\psi^{\pm}$ which are

$$
\begin{aligned}
\psi^{\pm}(z)|_{\tau=0} &= -ie^{\pm 2v_2}\bar{\psi}^{\mp}(\bar{z})|_{\tau=0} \ , \\
\psi^{\pm}(z)|_{\tau=l} &= -ie^{\pm 2v_1}\bar{\psi}^{\mp}(\bar{z})|_{\tau=l} \ .
\end{aligned}
\tag{B24}
$$

The local behavior of these functions for $\tau \to 0$ is found to be

$$
P^{s}_{v(\pm)}(\tau, l) \to \frac{1}{8\pi i\tau} e^{\pm 2v_2} \ . \tag{B25}
$$

It is convenient to rescale the fermions according to $\psi^{\pm} \to \hat{\psi}^{\pm} = e^{\mp v_2}\psi^{\pm}$, so that their propagators are $\hat{P}^{s}_{v(\pm)} = e^{\mp 2v_2} P^{s}_{v(\pm)}$. The monodromy properties do not change, but the boundary conditions now become

$$
\begin{aligned}
\hat{\psi}^{\pm}(z) &= -i\hat{\bar{\psi}}^{\mp}(\bar{z}) \ , \quad z = \bar{z} \ , \\
\hat{\psi}^{\pm}(z) &= -ie^{\pm 2(v_1 - v_2)}\hat{\bar{\psi}}^{\mp}(\bar{z}) \ , \quad z = \bar{z} + \nu \ ,
\end{aligned}
\tag{B26}
$$

and the local behavior for $\tau \to 0$ simplifies to the conventional one

$$
\hat{P}^{s}_{v(\pm)}(\tau, l) \to \frac{1}{4\pi w} \ . \tag{B27}
$$

It has become now clear how to do the twisted involution to pass from the covering torus to the cylinder: the twisted boundary conditions on the cylinder are obtained from a non-trivial phase transformation around the long cycle of the torus with imaginary angle $\epsilon = \frac{v_1 - v_2}{\pi}$. Actually, the monodromy properties of the functions $\hat{P}^{s}_{v(\pm)}$, together with their local behavior, imply them to be combinations of twisted θ-functions, with argument $w = 2i\tau$, modulus $\nu = 2il$ and imaginary twist $\epsilon = \frac{v_1 - v_2}{\pi}$. In fact, one can check that

$$
\begin{aligned}
\hat{P}^{R+}_{v(\pm)}(w, \nu) &= \frac{1}{4\pi} \frac{\theta_2(w \pm i\epsilon|\nu)\theta_1'(0|\nu)}{\theta_1(w|\nu)\theta_2(\pm i\epsilon|\nu)} \ , \\
\hat{P}^{NS+}_{v(\pm)}(w, \nu) &= \frac{1}{4\pi} \frac{\theta_3(w \pm i\epsilon|\nu)\theta_1'(0|\nu)}{\theta_1(w|\nu)\theta_3(\pm i\epsilon|\nu)} \ , \\
\hat{P}^{NS-}_{v(\pm)}(w, \nu) &= \frac{1}{4\pi} \frac{\theta_4(w \pm i\epsilon|\nu)\theta_1'(0|\nu)}{\theta_1(w|\nu)\theta_4(\pm i\epsilon|\nu)} \ .
\end{aligned}
\tag{B28}
$$

In order to study the amplitudes in the large distance limit, we will need the $l \to +\infty$ asymptotics of the correlations. For the bosonic exponential one gets

$$
< e^{ip\cdot X} >_{osc} \to \left[1 - e^{-4\pi\tau}\right]^{-\frac{p_B^{(2)2}}{2\pi}} \left[1 - e^{-4\pi l'}\right]^{-\frac{p_B^{(1)2}}{2\pi}} \ , \tag{B29}
$$

whereas the fermionic propagators in the four spin structures, in this limit, reduce to

$$
\begin{aligned}
\tilde{F}_v^{R\pm} &\to -\cosh 2v_2 \frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}} \mp \cosh 2v_1 \frac{e^{-4\pi l'}}{1-e^{-4\pi l'}} \ , \\
\tilde{G}_v^{R\pm} &\to -\sinh 2v_2 \frac{e^{-4\pi\tau}}{1-e^{-4\pi\tau}} \mp \sinh 2v_1 \frac{e^{-4\pi l'}}{1-e^{-4\pi l'}} \ , \\
\tilde{U}_v^{R\pm} &\to -\frac{(v_1-v_2)}{2\pi l} \pm 2\sinh 2(v_1-v_2) e^{-4\pi l} \ ,
\end{aligned} \tag{B30}
$$

and

$$
\begin{aligned}
F_v^{NS\pm} \to & -\cosh 2v_2 \frac{e^{-2\pi\tau}}{1-e^{-4\pi\tau}} \mp \cosh 2v_1 \frac{e^{-2\pi l'}}{1-e^{-4\pi l'}} \\
& + e^{-2\pi l}(\pm \cosh 2(v_1-2v_2) e^{-2\pi\tau} + \cosh 2(v_2-2v_1) e^{-2\pi l'}) \ , \\
G_v^{NS\pm} \to & -\sinh 2v_2 \frac{e^{-2\pi\tau}}{1-e^{-4\pi\tau}} \mp \sinh 2v_1 \frac{e^{-2\pi l'}}{1-e^{-4\pi l'}} \\
& + e^{-2\pi l}(\mp \sinh 2(v_1-2v_2) e^{-2\pi\tau} - \sinh 2(v_2-2v_1) e^{2\pi l'}) \ , \\
U_v^{NS\pm} \to & -\frac{(v_1-v_2)}{2\pi l} \pm 2\sinh 2(v_1-v_2) e^{-2\pi l} \ .
\end{aligned} \tag{B31}
$$

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