

Simple metastable de Sitter vacua in N=2 gauged supergravity

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Abstract

We construct a simple class of N=2 gauged supergravity theories that admit metastable de Sitter vacua, generalizing the recent work done in the context of rigid supersymmetry. The setup involves one hypermultiplet and one vector multiplet spanning suitably curved quaternionic-Kähler and special-Kähler geometries, with an Abelian gauging based on a single triholomorphic isometry, but neither Fayet-Iliopoulos terms nor non-Abelian gauge symmetries. We construct the most general model of this type and show that in such a situation the possibility of achieving metastable supersymmetry breaking vacua crucially depends on the value of the cosmological constant V relative to the gravitino mass squared $m_{3/2}^2$ in Planck units. In particular, focusing on de Sitter vacua with positive V , we show that metastability is only possible when $V \gtrsim 2.17 m_{3/2}^2$. We also derive an upper bound on the lightest scalar mass in this kind of model relative to the gravitino mass $m_{3/2}$ as a function of the cosmological constant V , and discuss its physical implications.

1 Introduction

Supersymmetry, if it is realized in Nature, must be spontaneously broken on a ground state that is at least metastable, in such a way to comply with the negative experimental searches performed so far. In the context of supergravity theories, this has motivated several systematic studies of the circumstances under which metastable non-supersymmetric vacua may arise. Finding such vacua turns out to be surprisingly difficult, even when allowing the freedom of adjusting all the parameters of the theory, because the scalar potential has a restricted functional form in supersymmetric theories. This difficulty moreover increases with the number of supersymmetries.

One general strategy that can be pursued to uncover possible obstructions against metastable supersymmetry breaking in generic supergravity theories consists of studying the general structure of the masses of the sGoldstini, which represent an endemic danger of instability. For $N=1$ theories, this study has been performed exhaustively in [1–3] (see also [4–6]). The outcome is that metastable de Sitter vacua may exist only if the scalar manifold has a suitable curvature. For $N=2$ theories, the same study has only been carried out in certain special classes of theories. In particular, it has been shown that metastable de Sitter vacua are ruled out in theories with only hypermultiplets [9] or only Abelian vector multiplets [10]. On the other hand, for the more general classes of theories with charged hypermultiplets or non-Abelian vector multiplets a few examples of metastable de Sitter vacua are known [7, 8], but no constraint on the existence of such vacua has been worked out so far. For $N=4$ and $N=8$ theories, a similar study has been done in [11, 12], where it was proven that metastable de Sitter vacua could only arise in a very constrained region of parameter space. No genuine examples of such vacua are known in this context, but their existence has not yet been excluded (examples of unstable de Sitter vacua [13–17] and marginally stable Minkowski vacua possessing flat directions [18] are known). Finally, the same kind of study has also been applied to theories defined through a truncation that reduces the amount of supersymmetry, where an unstable de Sitter vacuum of the original theory may lead to a metastable de Sitter vacuum in the truncated theory. The simplest case of $N=2$ to $N=1$ truncations with only scalar multiplets has been studied in [19], and several non-trivial examples in the context of $N=8$ to $N=4$ and $N=4$ to $N=2$ were described in [20].

The same issue of vacuum metastability when supersymmetry is spontaneously broken already arises in the simpler context of theories with global supersymmetry. One may then also study the problem in this simpler context, without losing any of its essential features, at least for $N=1$ and $N=2$ theories where, contrary to $N=4$ and $N=8$ theories, one can have a generic scalar geometry even in the rigid limit. Moreover, for simplicity one may discard constant Fayet-Iliopoulos terms, since these are essentially an accidental feature of rigid supersymmetry and are only

compatible with gravity under quite restrictive circumstances. This rigid version of the metastability problem was studied in some detail in [21, 22], again using the strategy of looking at the masses of the sGoldstini. The rigid limit of all the results known from the supergravity analyses for generic N=1 and special N=2 theories were recovered in a simpler and more transparent way. Furthermore, it was argued that in N=2 theories with non-Abelian vector multiplets there is no obstruction against metastability from the sGoldstini. On the other hand, no constraint has been derived so far with this approach for the general case of N=2 theories with charged hypermultiplets.

In the context of rigid supersymmetry, one can also use other types of analyses that better exploit the control that one has over the off-shell theory. For instance, assuming that metastable supersymmetry breaking is possible and that the only massless states are the Goldstini, one can try to explicitly construct their low-energy effective theory in which supersymmetry is realized non-linearly. Any obstruction showing up in this attempt can then be interpreted as signaling the impossibility of realizing the assumed metastable supersymmetry breaking vacuum. Proceeding along these lines, it was shown in [23] that N=2 theories possessing an $SU(2)_R$ global symmetry cannot admit a genuinely metastable supersymmetry breaking vacuum, at least under the further, more technical assumption that they admit a well-defined supercurrent superfield satisfying a conservation law that involves at most a superconformal linear anomaly multiplet. This result strongly suggests that a crucial requirement for a generic N=2 theory to admit a viable metastable non-supersymmetric vacuum is that it should not possess any global $SU(2)_R$ symmetry. Remarkably, the presence of such an $SU(2)_R$ symmetry rests on radically different features in the hypermultiplet and vector multiplet sectors, and this introduces an important distinction between them. In the vector multiplet sector, an $SU(2)_R$ symmetry automatically emerges whenever constant Fayet-Iliopoulos terms are absent, even if the scalar manifold is arbitrarily curved. In the hypermultiplet sector, on the other hand, no $SU(2)_R$ symmetry can arise if one considers a sufficiently generic curved scalar manifold. As a consequence, the simplest candidates for N=2 theories admitting metastable non-supersymmetric vacua are those involving charged hypermultiplets. Following this expectation, it has been shown in [24] that the simplest class of such theories based on just one hypermultiplet and one Abelian vector multiplet with suitably curved scalar manifolds does indeed admit metastable non-supersymmetric vacua.

The aim of this work is to construct a simple, minimal class of N=2 supergravity theories that admit metastable de Sitter vacua, without involving either constant Fayet-Iliopoulos terms or non-Abelian gaugings, by generalizing the construction presented in [24] from rigid to local supersymmetry. The setup involves one hypermultiplet and one vector multiplet spanning suitably curved quaternionic-Kähler and special-Kähler geometries, with an Abelian gauging based on a single triholo-

morphic isometry. Our aim is to construct the most general model of this type and study the circumstances under which this admits a metastable supersymmetry breaking vacuum. We shall see how this can be achieved by fixing a point in the scalar manifold and then tuning the geometry in the neighborhood of that point such that it corresponds to a metastable de Sitter vacuum. As already argued in [24], this possibility could a priori depend on the value of the cosmological constant V relative to the gravitino mass squared $m_{3/2}^2$ in Planck units, and the relevant dimensionless parameter is therefore expected to be given by the following expression, which we restrict for simplicity to be positive:

$$\epsilon = \frac{V}{m_{3/2}^2}. \quad (1.1)$$

More precisely, when $\epsilon \gg 1$ the influence of gravitational effects on scalar masses is negligible. Therefore, by virtue of the results derived in [24] for the rigid limit, it should be possible to achieve a viable metastable vacuum by adjusting the form of the scalar manifolds. On the other hand, when $\epsilon \ll 1$ the influence of gravitational effects on scalar masses is a priori significant and the possibility to achieve a viable metastable vacuum by adjusting the form of the scalar manifolds must be reexamined within supergravity.

The rest of this paper is structured as follows. In section 2 we briefly review the main features of N=2 gauged supergravity theories that will be relevant for our purposes. In particular, we describe useful explicit parametrizations of the most general quaternionic-Kähler and special-Kähler manifolds of minimal dimensions four and two in terms of Toda and Laplace potentials, respectively, which are the main building blocks of our model. In sections 3 and 4 we then review the known properties of the models with only one hypermultiplet and only one vector multiplet that can be constructed with these spaces, and recall specifically the structure of their scalar mass matrices and the sum rules forbidding the existence of metastable vacua. In section 5 we then explicitly construct the class of models involving one hypermultiplet and one vector multiplet that we are interested in. Next we compute the structure of the scalar mass matrix and study the constraints that can be put on its eigenvalues. Finally, we derive a sharp upper bound on the smallest scalar mass as a function of the parameter ϵ , and deduce from this the range of values for ϵ for which metastable vacua are allowed. In section 6, we briefly discuss the rigid limit of our supergravity analysis and show how it matches the rigid supersymmetry analysis of [24]. In section 7, we then present a class of explicit examples of models admitting metastable de Sitter vacua and compute for these the full spectrum of scalar masses. In section 8 we present our conclusions.

2 N=2 gauged supergravity

Let us briefly review the general structure of N=2 gauged supergravity theories, restricting to Abelian symmetries and using Planck units. In general, there can be n_H hypermultiplets and n_V vector multiplets. The $4n_H$ real scalars q^u from the hypermultiplets span a quaternionic-Kähler manifold with metric g_{uv} while the n_V complex scalars z^i from the vector multiplets span a special-Kähler manifold with metric $g_{i\bar{j}}$. The n_V vectors A_μ^a from the vector multiplets and the graviphoton A_μ^0 , denoted altogether by A_μ^A , have kinetic metric $\gamma_{AB} = -\text{Im}\mathcal{N}_{AB}$ and topological angles $\theta_{AB} = \text{Re}\mathcal{N}_{AB}$ in terms of the so-called period matrix \mathcal{N}_{AB} associated with the special-Kähler manifold. They can be used to gauge a maximum of $n_V + 1$ of isometries, which are described by triholomorphic Killing vectors k_A^u on the quaternionic-Kähler manifold. The scalar and vector kinetic energy is given by [25–32]:

$$T = -\frac{1}{4}\gamma_{AB}F_{\mu\nu}^A F^{B\mu\nu} + \frac{1}{4}\theta_{AB}F_{\mu\nu}^A \tilde{F}^{B\mu\nu} - \frac{1}{2}g_{uv}D_\mu q^u D^\mu q^v - g_{i\bar{j}}\partial_\mu z^i \partial^\mu \bar{z}^{\bar{j}}. \quad (2.1)$$

In this expression $F_{\mu\nu}^A = \partial_\mu A_\nu^A - \partial_\nu A_\mu^A$, $\tilde{F}_{\mu\nu}^A = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{A\rho\sigma}$ and $D_\mu q^u = \partial_\mu q^u + k_A^u A_\mu^A$. The scalar potential is instead given by

$$V = 2g_{uv}k_A^u k_B^v L^A \bar{L}^B + g^{i\bar{j}}f_i^A \bar{f}_{\bar{j}}^B P_A^x P_B^x - 3P_A^x P_B^x L^A \bar{L}^B. \quad (2.2)$$

Here L^A denotes the covariantly holomorphic symplectic section of the special-Kähler manifold and $f_i^A = \nabla_i L^A$, while P_A^x denote the three Killing potentials admitted by each of the triholomorphic Killing vector k_A^u . Finally, let us also recall that the gravitino mass is given by

$$m_{3/2}^2 = P_A^x P_B^x L^A \bar{L}^B. \quad (2.3)$$

In the following, we will concentrate on the simplest cases where n_H is 0 or 1 and n_V is 0 or 1. To construct the most general theories of this type, we will need to consider the most general quaternionic-Kähler manifold of dimension four with at least one triholomorphic isometry and the most general special-Kähler manifold of dimension two. Fortunately, there exist general local parametrizations for these two kinds of manifolds, in terms of potentials h and l of three and two real variables satisfying the Toda and the Laplace equations, respectively. It is then possible to construct general theories based on generic choices for these two functions.

2.1 Quaternionic-Kähler manifold

In the hypermultiplet sector, we will consider a generic four-dimensional quaternionic-Kähler space admitting at least one triholomorphic isometry. With the canonical normalization adopted in (2.1) for the scalar kinetic term (corresponding to $\lambda = -\frac{1}{2}$ in the notation of [33]), the scalar curvature must be $R = -12$. It turns out that

the line element of such a manifold can always be locally brought into the following Przanowski-Tod form [34, 35], using four real coordinates $q^u = \rho, \varphi, \chi, \tau$:

$$ds^2 = g_{uv} dq^u dq^v = \frac{1}{2\rho^2} \left(f d\rho^2 + f e^h (d\varphi^2 + d\chi^2) + f^{-1} (d\tau + \Theta)^2 \right). \quad (2.4)$$

This depends on a single function h of the three variables $q^{\hat{u}} = \rho, \varphi, \chi$, which must satisfy the three-dimensional Toda equation:

$$(e^h)_{\rho\rho} + h_{\varphi\varphi} + h_{\chi\chi} = 0. \quad (2.5)$$

The function f is then related to the function h by

$$f = 2 - \rho h_\rho. \quad (2.6)$$

The 1-form Θ is instead determined, modulo an irrelevant exact form, by the following equation, whose integrability is guaranteed by the Toda equation:

$$d\Theta = (f_\varphi d\chi - f_\chi d\varphi) \wedge d\rho + (f e^h)_\rho d\varphi \wedge d\chi. \quad (2.7)$$

Further details about the geometric properties of this space can be found for example in [19, 36]. The $SU(2)$ connection ω^x is found to be:

$$\omega^1 = -\rho^{-1} e^{h/2} d\chi, \quad (2.8)$$

$$\omega^2 = -\rho^{-1} e^{h/2} d\varphi, \quad (2.9)$$

$$\omega^3 = -\frac{1}{2} \rho^{-1} (d\tau + \Theta) + \frac{1}{2} h_\chi d\varphi - \frac{1}{2} h_\varphi d\chi. \quad (2.10)$$

The three hyper-Kähler forms J^x , which satisfy $\nabla J^x = 0$ as a consequence of the equation defining Θ and are thus only covariantly closed, are given by:

$$J^1 = \frac{1}{2} \rho^{-2} e^{h/2} d\varphi \wedge (d\tau + \Theta) - \frac{1}{2} \rho^{-2} f e^{h/2} d\rho \wedge d\chi, \quad (2.11)$$

$$J^2 = -\frac{1}{2} \rho^{-2} e^{h/2} d\chi \wedge (d\tau + \Theta) - \frac{1}{2} \rho^{-2} f e^{h/2} d\rho \wedge d\varphi, \quad (2.12)$$

$$J^3 = \frac{1}{2} \rho^{-2} d\rho \wedge (d\tau + \Theta) + \frac{1}{2} \rho^{-2} f e^h d\varphi \wedge d\chi. \quad (2.13)$$

Finally, the manifest isometry amounts to a shift in the variable t and the corresponding Killing vector takes the following expression, involving an arbitrary parameter ξ of dimension one:

$$k = \xi \partial_\tau. \quad (2.14)$$

In the above parametrization in terms of the four real coordinates $q^u = \rho, \varphi, \chi, \tau$, the components of the metric are:

$$g_{uv} = \frac{1}{2\rho^2} \begin{pmatrix} f + f^{-1} \Theta_\rho^2 & f^{-1} \Theta_\rho \Theta_\varphi & f^{-1} \Theta_\rho \Theta_\chi & f^{-1} \Theta_\rho \\ f^{-1} \Theta_\varphi \Theta_\rho & f e^h + f^{-1} \Theta_\varphi^2 & f^{-1} \Theta_\varphi \Theta_\chi & f^{-1} \Theta_\varphi \\ f^{-1} \Theta_\chi \Theta_\rho & f^{-1} \Theta_\chi \Theta_\varphi & f e^h + f^{-1} \Theta_\chi^2 & f^{-1} \Theta_\chi \\ f^{-1} \Theta_\rho & f^{-1} \Theta_\varphi & f^{-1} \Theta_\chi & f^{-1} \end{pmatrix}. \quad (2.15)$$

Positivity of this metric requires $f > 0$. A simple choice for the vielbein $e_u{}^p$ that can be used to locally trivialize this metric as $g_{uv} = e_u{}^p \delta_{pq} (e^T)^q{}_v$ is given by:

$$e_u{}^p = \frac{1}{\sqrt{2}\rho} \begin{pmatrix} f^{1/2} & 0 & 0 & f^{-1/2}\Theta_\rho \\ 0 & f^{1/2}e^{h/2} & 0 & f^{-1/2}\Theta_\varphi \\ 0 & 0 & f^{1/2}e^{h/2} & f^{-1/2}\Theta_\chi \\ 0 & 0 & 0 & f^{-1/2} \end{pmatrix}. \quad (2.16)$$

The components of the three hyper-Kähler forms are also easily worked out and seen to satisfy the algebra $(J^x)^u{}_w (J^y)^w{}_v = -\delta^{xy} \delta_v^u + \epsilon^{xyz} (J^z)^u{}_v$. One can then easily verify that the Killing vector with components k^u is triholomorphic and compute the corresponding Killing potentials P^x from their defining relation $\nabla_u P^x = -(J^x)_{uv} k^v$. The result is:

$$k^u = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \xi \end{pmatrix}, \quad |\vec{P}| = \frac{1}{2} \xi \rho^{-1}. \quad (2.17)$$

It will be useful for the analysis of the forthcoming sections to define the following three-dimensional matrix of dimensionless parameters associated to the second derivatives of the function f with respect to its variables $q^{\hat{u}} = \rho, \varphi, \chi$:

$$\alpha_{\hat{u}\hat{v}} = \frac{1}{2} (e^{-1})_{\hat{u}}{}^{\hat{p}} f_{\hat{p}\hat{q}} (e^{-1T})^{\hat{q}}{}_{\hat{v}}. \quad (2.18)$$

A simple computation shows that the entries of this matrix are given by

$$\alpha_{\rho\rho} = f^{-1} \rho^2 f_{\rho\rho}, \quad (2.19)$$

$$\alpha_{\rho\varphi} = f^{-1} \rho^2 f_{\rho\varphi} e^{-h/2}, \quad \alpha_{\rho\chi} = f^{-1} \rho^2 f_{\rho\chi} e^{-h/2}, \quad (2.20)$$

$$\alpha_{\varphi\varphi} = f^{-1} \rho^2 f_{\varphi\varphi} e^{-h}, \quad \alpha_{\chi\chi} = f^{-1} \rho^2 f_{\chi\chi} e^{-h}, \quad \alpha_{\varphi\chi} = f^{-1} \rho^2 f_{\varphi\chi} e^{-h}. \quad (2.21)$$

The definitions and properties of the functions h and f imply an important constraint on the second derivatives of f , and thus on the parameter $\alpha_{\hat{u}\hat{v}}$. More precisely, using the relation between f and h and the Toda equation (2.5) satisfied by h , one easily shows that $f_{\rho\rho} + e^{-h} f_{\varphi\varphi} + e^{-h} f_{\chi\chi} = -\rho^{-2} f(f-1)(f-2) + \rho^{-1}(3f-4)f_\rho$. As a consequence of this property, it follows that the trace of the matrix $\alpha_{\hat{u}\hat{v}}$ involving the second derivatives of f is completely fixed in terms of the first derivatives of f and the function f itself, and one finds:

$$\delta^{\hat{u}\hat{v}} \alpha_{\hat{u}\hat{v}} = -(f-1)(f-2) + (3f-4)f^{-1}\rho f_\rho. \quad (2.22)$$

2.2 Special-Kähler manifold

In the vector multiplet sector, we will consider a general two-dimensional special-Kähler manifold of the local type. This can be locally described by a metric of the following form, using the coordinates $q^\alpha = z, \bar{z}$:

$$ds^2 = g_{\alpha\bar{\beta}} dq^\alpha dq^{\bar{\beta}} = 2 g_{z\bar{z}} dz d\bar{z} = 2l |dz|^2. \quad (2.23)$$

The line element depends on a single real function l of z and \bar{z} . This function is restricted by the fact that it can be expressed in terms of a holomorphic prepotential. The resulting restriction on the curvature of the space can be taken into account efficiently in terms of the symplectic section $L^A(z, \bar{z})$, where $A = 0, 1$. In the local case, this section is defined to be covariantly holomorphic and thus satisfies:

$$\nabla_{\bar{z}} L^A = 0. \quad (2.24)$$

Moreover, the constrained form of the geometry implies two special properties for the two possible kinds of second covariant derivatives of the section. The first is that $\nabla_{\bar{z}} \nabla_z L^A = [\nabla_{\bar{z}}, \nabla_z] L^A = g_{z\bar{z}} L^A$. This implies that

$$\nabla_{\bar{z}} \nabla_z L^A = l L^A. \quad (2.25)$$

The second is that $\nabla_z \nabla_z L^A = C_{zzz} g^{z\bar{z}} \nabla_{\bar{z}} \bar{L}^A$, where C_{zzz} is covariantly holomorphic and thus satisfies $\nabla_{\bar{z}} C_{zzz} = 0$. Acting with a further derivative on this relation we then deduce that $\nabla_{\bar{z}} \nabla_z \nabla_z L^A = C_{zzz} g^{z\bar{z}} \nabla_{\bar{z}} \nabla_z \bar{L}^A$. Multiplying this equation with $\nabla_{\bar{z}} \bar{L}^B$ and using the previous relation, one finally finds that

$$\nabla_{\bar{z}} \nabla_z^2 L^A \nabla_{\bar{z}} \bar{L}^B = \nabla_z^2 L^B \nabla_{\bar{z}}^2 \bar{L}^A. \quad (2.26)$$

In the above parametrization in terms of the coordinates $q^\alpha = z, \bar{z}$, the components of the metric are given by:

$$g_{\alpha\bar{\beta}} = \begin{pmatrix} l & 0 \\ 0 & l \end{pmatrix}. \quad (2.27)$$

An obvious choice for the vielbein $e_\alpha{}^\gamma$ that allows the metric to be locally trivialized as $g_{\alpha\bar{\beta}} = e_\alpha{}^\gamma \delta_{\gamma\bar{\delta}} (e^\dagger)^{\bar{\delta}}{}_{\bar{\beta}}$ is given by:

$$e_\alpha{}^\gamma = \begin{pmatrix} l^{1/2} & 0 \\ 0 & l^{1/2} \end{pmatrix}. \quad (2.28)$$

It will be convenient for the analysis of the next sections to introduce the following dimensionless parameter related to the third derivatives of the section:

$$\beta = \frac{\nabla_z^3 L L^2}{(\nabla_z L)^3}. \quad (2.29)$$

It will also be useful to define the following quantity:

$$\gamma = -\arg \frac{\nabla_z L}{L}. \quad (2.30)$$

3 Models with one hyper

In models with just one hypermultiplet, the only possible source of potential comes from a gauging of the isometry with the graviphoton. This corresponds to taking $k_0^u = k^u$ and $P_0^x = P^x$, where k^u and P^x are given by eq. (2.17). Furthermore, we can choose $L^0 = 1$. The resulting potential is $V = 2g_{uv}k^uk^v - 3|\vec{P}|^2$ and only depends on the three variables $q^{\hat{u}} = \rho, \varphi, \chi$. Its explicit form reads

$$V = \xi^2 \rho^{-2} \left(f^{-1} - \frac{3}{4} \right). \quad (3.1)$$

The first derivatives $V_u \equiv \partial_u V$ are given by $V_\tau = 0$ and

$$V_{\hat{u}} = \xi^2 \rho^{-2} \left[-f^{-2} f_{\hat{u}} - 2\rho \xi^{-2} V \delta_{\hat{u}\rho} \right]. \quad (3.2)$$

The second derivatives $V_{uv} \equiv \partial_u \partial_v V$ are instead given by $V_{\tau\tau} = 0$, $V_{\tau\hat{v}} = 0$ and

$$V_{\hat{u}\hat{v}} = \xi^2 \rho^{-2} \left[-f^{-2} f_{\hat{u}\hat{v}} + 2f^{-3} f_{\hat{u}} f_{\hat{v}} + 4\rho^{-1} f^{-2} f_{(\hat{u}} \delta_{\hat{v})\rho} + 6\xi^{-2} V \delta_{\hat{u}\rho} \delta_{\hat{v}\rho} \right]. \quad (3.3)$$

The gravitino mass reads:

$$m_{3/2}^2 = \frac{1}{4} \xi^2 \rho^{-2}. \quad (3.4)$$

It follows that the parameter (1.1) takes the value

$$\epsilon = -3 + 4f^{-1}. \quad (3.5)$$

We see from these formulae that the strength of supersymmetry breaking is controlled by the value of f^{-1} .

In order to study the possible vacua and their properties, we must first find the critical points of the scalar potential. The value of the cosmological constant V fixes the value of f to be given by:

$$f = \frac{4}{3 + \epsilon}. \quad (3.6)$$

The stationarity conditions $V_u = 0$ fix instead the values of the first derivatives $f_{\hat{u}}$ to be given by $f_{\hat{u}} = -2\rho f^2 \xi^{-2} V \delta_{\hat{u}\rho}$, and since $\xi^{-2} V = \frac{1}{4} \rho^{-2} \epsilon$ this gives:

$$f_{\hat{u}} = -\frac{8\epsilon}{(3 + \epsilon)^2} \rho^{-1} \delta_{\hat{u}\rho}. \quad (3.7)$$

Using the above relations, one can then compute the Hessian matrix V_{uv} and the physical scalar mass matrix $m_{uv}^2 = (e^{-1})_u{}^p V_{pq} (e^{-1T})^q{}_v$ in terms of the values of the second derivatives $f_{\hat{u}\hat{v}}$, which are related to the parameters $\alpha_{\hat{u}\hat{v}}$ by the definition (2.18). This is found to be given by $m_{\tau\tau}^2 = 0$, $m_{\tau\hat{v}}^2 = 0$ and

$$m_{\hat{u}\hat{v}}^2 = \left[-\frac{1}{2} (3 + \epsilon)^2 \alpha_{\hat{u}\hat{v}} - 3\epsilon (1 - \epsilon) \delta_{\hat{u}\rho} \delta_{\hat{v}\rho} \right] m_{3/2}^2. \quad (3.8)$$

The parameters $\alpha_{\hat{u}\hat{v}}$ can be adjusted by suitably choosing the function f and therefore the values of its second derivatives at the point under consideration. The only constraint is that the corresponding function h should solve the Toda equation (2.5). This then results in the constraint (2.22) which implies the following sum rule, after using the stationarity condition (3.7):

$$\delta^{\hat{u}\hat{v}} \alpha_{\hat{u}\hat{v}} = \frac{2 + 6\epsilon^2}{(3 + \epsilon)^2}. \quad (3.9)$$

To check whether the scalar masses can all be positive, we may now compute the average of the three eigenvalues of the physical mass matrix:

$$m^2 \equiv \frac{1}{3} \delta^{\hat{u}\hat{v}} m_{\hat{u}\hat{v}}^2. \quad (3.10)$$

It turns out that the value of this average mass is entirely fixed by the constraint (3.9) and is found to be:

$$m^2 = \left[-\frac{1}{3} - \epsilon \right] m_{3/2}^2. \quad (3.11)$$

By construction m^2 represents an upper bound on the smallest mass eigenvalue and also a lower bound on the largest mass eigenvalue. Moreover, both of these bounds can be saturated by tuning the parameters $\alpha_{\hat{u}\hat{v}}$. We then conclude that:

$$\min \{m_i^2\} \leq -\frac{1}{3} m_{3/2}^2 - V. \quad (3.12)$$

This result implies that Minkowski and de Sitter vacua are necessarily unstable. It reproduces the result derived in [9], which looked at the average sGoldstino mass for theories with an arbitrary number of hypermultiplets.

4 Models with one vector

In models with just one vector multiplet, the only possible source of potential comes from a Fayet-Iliopoulos term associated to constant Killing potentials for an arbitrary linear combination of the graviphoton and the matter vector. Without loss of generality, we can choose $P_0^x = 0$ and $P_1^x = \frac{1}{2}\xi v^x$, where ξ is a constant and v^x is an arbitrary unit vector. The potential is then independent of L^0 and only involves $L^1 \equiv L$. It is given by $V = l^{-1}|\vec{P}|^2|\nabla_z L|^2 - 3|\vec{P}|^2|L|^2$ and depends on the two variables $q^\alpha = z, \bar{z}$. Its explicit form reads

$$V = \xi^2 \left[\frac{1}{4} l^{-1} |\nabla_z L|^2 - \frac{3}{4} |L|^2 \right]. \quad (4.1)$$

The first derivatives $V_\alpha \equiv \nabla_\alpha V$ are found to be:

$$V_z = \xi^2 \left[\frac{1}{4} l^{-1} \nabla_z^2 L \nabla_{\bar{z}} \bar{L} - \frac{1}{2} \nabla_z L \bar{L} \right]. \quad (4.2)$$

The second derivatives $V_{\alpha\bar{\beta}} \equiv \nabla_\alpha \nabla_{\bar{\beta}} V$ are instead given by

$$V_{z\bar{z}} = \xi^2 \left[\frac{1}{2} l^{-1} |\nabla_z^2 L|^2 - \frac{1}{2} (l|L|^2 + |\nabla_z L|^2) \right], \quad (4.3)$$

$$V_{zz} = \xi^2 \left[\frac{1}{4} l^{-1} \nabla_z^3 L \nabla_{\bar{z}} \bar{L} - \frac{1}{4} \nabla_z^2 L \bar{L} \right]. \quad (4.4)$$

The gravitino mass reads:

$$m_{3/2}^2 = \frac{1}{4} \xi^2 |L|^2. \quad (4.5)$$

It follows that the parameter (1.1) takes the value

$$\epsilon = -3 + l^{-1} \left| \frac{\nabla_z L}{L} \right|^2. \quad (4.6)$$

We see from these formulae that the strength of supersymmetry breaking is controlled by the value of $l^{-1} |\nabla_z L/L|$.

In order to study the possible vacua and their properties, we must again first find the critical points of the scalar potential. The value of the cosmological constant V fixes the value of $\nabla_z L$ to be given by

$$\nabla_z L = \sqrt{3 + \epsilon} l^{1/2} e^{-i\gamma} L, \quad (4.7)$$

where γ was defined in (2.30). The stationarity conditions $V_z = 0$ fix instead the value of $\nabla_z^2 L$ to be given by

$$\nabla_z^2 L = 2 l e^{-2i\gamma} L. \quad (4.8)$$

Using the above relations, one can then compute the Hessian matrix $V_{\alpha\bar{\beta}}$ and the physical scalar mass matrix $m_{\alpha\bar{\beta}}^2 = (e^{-1})_\alpha{}^\epsilon V_{\epsilon\bar{\delta}} (e^{-1\bar{\dagger}})^{\bar{\delta}}{}_{\bar{\beta}}$ in terms of the value of $\nabla_z^3 L$, which is related to the parameter β by the definition (2.29). This gives:

$$m_{z\bar{z}}^2 = [-2\epsilon] m_{3/2}^2, \quad (4.9)$$

$$m_{zz}^2 = [(3 + \epsilon)^2 \beta - 2] m_{3/2}^2 e^{-2i\gamma}. \quad (4.10)$$

The parameter β can be adjusted to any desired value by changing the function l and therefore the value of its second derivative at the point under consideration, without any constraint.

To check whether the scalar masses can all be positive, we can now compute the average of the two eigenvalues of the physical mass matrix:

$$m^2 \equiv \frac{1}{2} \delta^{\alpha\bar{\beta}} m_{\alpha\bar{\beta}}^2. \quad (4.11)$$

We see that this is completely fixed, independently of the parameter β , and reads

$$m^2 = [-2\epsilon] m_{3/2}^2. \quad (4.12)$$

By construction m^2 represents an upper bound on the smallest mass eigenvalue and also a lower bound on the largest mass eigenvalue. Moreover, both of these bounds can be saturated by tuning the parameter β . From this we deduce that

$$\min \{m_i^2\} \leq -2V. \quad (4.13)$$

This result implies that de Sitter vacua are necessarily unstable. It reproduces the result derived in [10], which looked at the average sGoldstino mass for theories involving an arbitrary number of Abelian vector multiplets.

5 Models with one hyper and one vector

In models with one hypermultiplet and one vector multiplet, the source of potential comes from a gauging of the hypermultiplet isometry with a linear combination of the graviphoton and the matter vector. Without loss of generality, we can choose $k_0^u = 0$, $k_1^u = k^u$ and $P_0^x = 0$, $P_1^x = P^x$, where k^u and P^x are given by eq. (2.17). Once again, the potential involves $L^1 \equiv L$ but not L^0 . It is given by $V = 2g_{uv}k^uk^v|L|^2 + l^{-1}|\vec{P}|^2|\nabla_z L|^2 - 3|\vec{P}|^2|L|^2$ and depends only on the five variables $q^{\hat{I}} = \rho, \varphi, \chi, z, \bar{z}$. Its explicit expression is

$$V = \xi^2 \rho^{-2} \left[\left(f^{-1} - \frac{3}{4} \right) |L|^2 + \frac{1}{4} l^{-1} |\nabla_z L|^2 \right]. \quad (5.1)$$

The first derivatives V_I are given by $V_\tau = 0$ and

$$V_{\hat{u}} = \xi^2 \rho^{-2} \left[-f^{-2} f_{\hat{u}} |L|^2 - 2\rho \xi^{-2} V \delta_{\hat{u}\rho} \right], \quad (5.2)$$

$$V_z = \xi^2 \rho^{-2} \left[\frac{1}{4} l^{-1} \nabla_z^2 L \nabla_{\bar{z}} \bar{L} - \frac{1}{2} (1 - 2f^{-1}) \nabla_z L \bar{L} \right]. \quad (5.3)$$

The second derivatives $V_{I\bar{J}}$, defined to be ordinary and covariant in the hyper and vector sectors, respectively, are instead given by $V_{\tau\tau} = 0$, $V_{\tau\hat{j}} = 0$ and

$$V_{\hat{u}\hat{v}} = \xi^2 \rho^{-2} \left[(-f^{-2} f_{\hat{u}\hat{v}} + 2f^{-3} f_{\hat{u}} f_{\hat{v}} + 4\rho^{-1} f^{-2} f_{(\hat{u}} \delta_{\hat{v})\rho}) |L|^2 + 6\xi^{-2} V \delta_{\hat{u}\rho} \delta_{\hat{v}\rho} \right], \quad (5.4)$$

$$V_{z\bar{z}} = \xi^2 \rho^{-2} \left[\frac{1}{2} l^{-1} |\nabla_z^2 L|^2 - \frac{1}{2} (1 - 2f^{-1}) (l|L|^2 + |\nabla_z L|^2) \right], \quad (5.5)$$

$$V_{zz} = \xi^2 \rho^{-2} \left[\frac{1}{4} l^{-1} \nabla_z^3 L \nabla_{\bar{z}} \bar{L} - \frac{1}{4} (1 - 4f^{-1}) \nabla_z^2 L \bar{L} \right], \quad (5.6)$$

$$V_{\hat{u}z} = \xi^2 \rho^{-2} \left[-f^{-2} f_{\hat{u}} \nabla_z L \bar{L} + \rho^{-1} ((1 - 2f^{-1}) \nabla_z L \bar{L} - \frac{1}{2} l^{-1} \nabla_z^2 L \nabla_{\bar{z}} \bar{L}) \delta_{\hat{u}\rho} \right]. \quad (5.7)$$

The gravitino mass reads:

$$m_{3/2}^2 = \frac{1}{4} \xi^2 \rho^{-2} |L|^2. \quad (5.8)$$

It follows that the parameter (1.1) is given by

$$\epsilon = -3 + 4f^{-1} + l^{-1} \left| \frac{\nabla_z L}{L} \right|. \quad (5.9)$$

We see from these formulae that the strength of supersymmetry breaking originating from the hyper and vector sectors is controlled by the expectation values of f^{-1} and $l^{-1} |\nabla_z L / L|$, respectively. The relative importance of these two contributions is most conveniently parametrized by an angle θ , defined as:

$$\tan \theta = \frac{1}{2} \sqrt{\frac{f}{l}} \left| \frac{\nabla_z L}{L} \right|. \quad (5.10)$$

In order to study the possible vacua and their properties, we must again first find the critical points of the scalar potential. The values of the cosmological constant V and the angle θ fix the values of f and $\nabla_z L$ to:

$$f = \frac{4}{3 + \epsilon} \cos^{-2} \theta, \quad (5.11)$$

$$\nabla_z L = \sqrt{3 + \epsilon} l^{1/2} \sin \theta e^{-i\gamma} L, \quad (5.12)$$

where we recall again that γ was defined in (2.30). The stationarity conditions $V_I = 0$ fix instead the values of $f_{\hat{u}}$ and $\nabla_z^2 L$ to be given by $f_{\hat{u}} = -2\rho f^2 \xi^{-2} |L|^{-2} V \delta_{\hat{u}\rho}$ and $\nabla_z^2 L = 2l(1 - 2f^{-1})e^{-2i\gamma} L$, and since $\xi^{-2} |L|^{-2} V = \frac{1}{4} \rho^{-2} \epsilon$ this gives:

$$f_{\hat{u}} = -\frac{8\epsilon}{(3 + \epsilon)^2} \cos^{-4} \theta \rho^{-1} \delta_{\hat{u}\rho}, \quad (5.13)$$

$$\nabla_z^2 L = -l((3 + \epsilon) \cos^2 \theta - 2) e^{-2i\gamma} L. \quad (5.14)$$

Using the above relations, one can then compute the Hessian matrix $V_{I\bar{J}}$ and the physical scalar mass matrix $m_{I\bar{J}}^2 = (e^{-1})_I^P V_{P\bar{Q}} (e^{-1\dagger})^{\bar{Q}}_{\bar{J}}$ in terms of the values of $f_{\hat{u}\hat{v}}$ and $\nabla_z^3 L$, which are related to the parameters $\alpha_{\hat{u}\hat{v}}$ and β by the definitions (2.18) and (2.29). This is found to be given by $m_{\tau\tau}^2 = 0$, $m_{\tau\hat{v}}^2 = 0$, $m_{\tau\alpha}^2 = 0$ and

$$m_{\hat{u}\hat{v}}^2 = \left[-\frac{1}{2} (3 + \epsilon)^2 \cos^4 \theta \alpha_{\hat{u}\hat{v}} - \epsilon ((3 + \epsilon) \cos^2 \theta - 4\epsilon) \delta_{\hat{u}\rho} \delta_{\hat{v}\rho} \right] m_{3/2}^2, \quad (5.15)$$

$$m_{z\bar{z}}^2 = \left[((3 + \epsilon) \cos^2 \theta - 2) ((3 + \epsilon) \cos^2 \theta + \epsilon) \right] m_{3/2}^2, \quad (5.16)$$

$$m_{zz}^2 = \left[(3 + \epsilon)^2 \sin^4 \theta \beta - ((3 + \epsilon) \cos^2 \theta - 1) ((3 + \epsilon) \cos^2 \theta - 2) \right] m_{3/2}^2 e^{-2i\gamma} \quad (5.17)$$

$$m_{\hat{u}z}^2 = \left[\sqrt{2} \epsilon (3 + \epsilon) \cos \theta \sin \theta \delta_{\hat{u}\rho} \right] m_{3/2}^2 e^{-i\gamma}. \quad (5.18)$$

The parameters $\alpha_{\hat{u}\hat{v}}$ can be adjusted by suitably choosing the function f and therefore the values of its second derivatives at the point under consideration. The only constraint comes from the Toda equation for h (2.5), which can be rewritten as in

(2.22). After using the stationarity condition (5.13) this in turn implies the following sum rule:

$$\delta^{\hat{u}\hat{v}} \alpha_{\hat{u}\hat{v}} = \frac{-2(3+\epsilon)^2 + 4(3+\epsilon)(3+2\epsilon) \cos^2\theta - 8(2+3\epsilon) \cos^4\theta}{(3+\epsilon)^2}. \quad (5.19)$$

The parameter β can instead be adjusted by suitably choosing the function l and the associated section, and is completely arbitrary. Finally, the angle θ can be adjusted by suitably choosing the relative overall sizes of f and l .

To check whether the scalar masses can all be positive, we can now study the two-dimensional matrix obtained by averaging over the three non-trivial directions in the hyper sector and the two directions in the vector sector. This real symmetric matrix takes the form

$$m^2 \equiv \begin{pmatrix} m_{\text{hh}}^2 & m_{\text{hv}}^2 \\ m_{\text{hv}}^2 & m_{\text{vv}}^2 \end{pmatrix}, \quad (5.20)$$

and its three independent elements are defined by

$$m_{\text{hh}}^2 \equiv \frac{1}{3} \delta^{\hat{u}\hat{v}} m_{\hat{u}\hat{v}}^2, \quad m_{\text{vv}}^2 \equiv \frac{1}{2} \delta^{\alpha\bar{\beta}} m_{\alpha\bar{\beta}}^2, \quad m_{\text{hv}}^2 \equiv \sqrt{\frac{1}{6} \delta^{\hat{u}\hat{v}} \delta^{\alpha\bar{\beta}} m_{\hat{u}\alpha}^2 m_{\hat{v}\bar{\beta}}^2}. \quad (5.21)$$

It turns out that the values of these average entries of the mass matrix are entirely fixed, as in the previous cases, and are found to be

$$m_{\text{hh}}^2 = \left[\frac{1}{3} ((3+\epsilon) \cos^2\theta - (2+\epsilon)) ((3+\epsilon) \cos^2\theta - 4(1+\epsilon)) \right] m_{3/2}^2, \quad (5.22)$$

$$m_{\text{vv}}^2 = \left[((3+\epsilon) \cos^2\theta - 2) ((3+\epsilon) \cos^2\theta + \epsilon) \right] m_{3/2}^2, \quad (5.23)$$

$$m_{\text{hv}}^2 = \left[\sqrt{\frac{2}{3}} \epsilon (3+\epsilon) \cos\theta \sin\theta \right] m_{3/2}^2. \quad (5.24)$$

Note that in the hyper sector one correctly recovers $m_{\text{hh}}^2 \rightarrow -\frac{1}{3}(1+3\epsilon) m_{3/2}^2$ when $\theta \rightarrow 0$, matching the case with only one hypermultiplet (3.11), and one finds instead $m_{\text{hh}}^2 \rightarrow \frac{4}{3}(1+\epsilon)(2+\epsilon) m_{3/2}^2$ when $\theta \rightarrow \frac{\pi}{2}$. Similarly, in the vector sector one correctly recovers $m_{\text{vv}}^2 \rightarrow -2\epsilon m_{3/2}^2$ when $\theta \rightarrow \frac{\pi}{2}$, matching the case with only one vector multiplet (4.12), and one finds instead $m_{\text{vv}}^2 \rightarrow (1+\epsilon)(3+2\epsilon) m_{3/2}^2$ when $\theta \rightarrow 0$. Finally, for the mixing between the two sectors, one finds $m_{\text{hv}}^2 \rightarrow 0$ when either $\theta \rightarrow 0$ or $\theta \rightarrow \frac{\pi}{2}$.

To get the sharpest possible bounds on the mass eigenvalues for the case we consider here, it is natural to consider the two eigenvalues of the two-dimensional averaged mass matrix (5.20). These are given by

$$m_{\pm}^2 = \frac{1}{2} (m_{\text{hh}}^2 + m_{\text{vv}}^2) \pm \sqrt{\frac{1}{4} (m_{\text{hh}}^2 - m_{\text{vv}}^2)^2 + m_{\text{hv}}^4}. \quad (5.25)$$

Using the results (5.22), (5.23) and (5.24), these two eigenvalues are found to be of the form

$$m_{\pm}^2 = \left[X \pm \sqrt{Y} \right] m_{3/2}^2, \quad (5.26)$$

where:

$$X = \frac{2}{3}(3 + \epsilon)^2 \cos^4 \theta - \frac{1}{3}(3 + \epsilon)(6 + \epsilon) \cos^2 \theta + \frac{1}{3}(4 + 3\epsilon + 2\epsilon^2), \quad (5.27)$$

$$Y = \frac{1}{9}(3 + \epsilon)^4 \cos^8 \theta + \frac{8}{9}(3 + \epsilon)^3 \epsilon \cos^6 \theta - \frac{2}{9}(3 + \epsilon)^2 (4 + 9\epsilon - 3\epsilon^2) \cos^4 \theta - \frac{2}{9}(3 + \epsilon) \epsilon (16 + 27\epsilon + 5\epsilon^2) \cos^2 \theta + \frac{1}{9}(4 + 9\epsilon + 2\epsilon^2)^2. \quad (5.28)$$

We can now study the behavior of m_{\pm}^2 as functions of θ for given value of ϵ . For m_-^2 , one finds a local maximum for some finite value of θ , with a magnitude that is negative or positive depending on whether ϵ is smaller or larger than a certain critical value, as illustrated in figure 1. More precisely, one finds the following results. For $\epsilon = 0$, m_-^2 is everywhere negative, except at $\theta = \arccos \sqrt{\frac{2}{3}} \simeq 0.62$ where it vanishes. This implies that Minkowski vacua ($V = 0$) can be at most loosely metastable. For $\epsilon \in [0, 2.17[$, m_-^2 is everywhere negative, and de Sitter vacua with $V \lesssim 2.17 m_{3/2}^2$ are thus necessarily unstable. For $\epsilon \simeq 2.17$, m_-^2 is everywhere negative except at $\theta \simeq 0.65$ where it vanishes, and de Sitter vacua with $V \simeq 2.17 m_{3/2}^2$ can thus be loosely metastable. Finally for $\epsilon \gtrsim 2.17$, m_-^2 becomes positive for a finite range of values of θ , centered around a value between 0.65 and $\frac{\pi}{4} \simeq 0.79$ where the maximum occurs. Therefore de Sitter vacua with $V > 2.17 m_{3/2}^2$ can be genuinely metastable. For m_+^2 , one finds a local minimum for some finite value of θ , with a magnitude that is always positive.

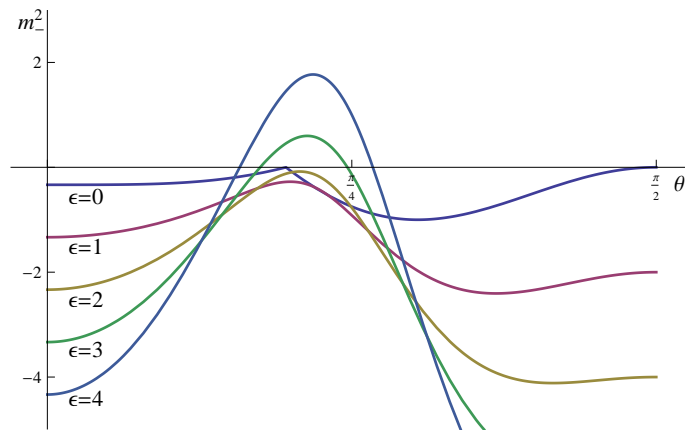


Figure 1: Plot of m_-^2 as a function of θ for various values of ϵ , in units of $m_{3/2}^2$.

By construction m_-^2 represents an upper bound on the smallest mass eigenvalue and m_+^2 a lower bound on the largest mass eigenvalue, for given values of θ and ϵ . From these quantities one can then derive two other bounds that only depend on ϵ by suitably extremizing m_-^2 and m_+^2 over θ , for fixed $m_{3/2}^2$ and V . More precisely, we can compute:

$$m_{\text{up}}^2 \equiv \max_{\theta} \{m_-^2\}, \quad (5.29)$$

$$m_{\text{low}}^2 \equiv \min_{\theta} \{m_+^2\}. \quad (5.30)$$

One can then see that $\min\{m_i^2\} \leq m_-^2 \leq m_{\text{up}}^2$ and $\max\{m_i^2\} \geq m_+^2 \geq m_{\text{low}}^2$. Moreover, any of these bounds can be saturated by tuning the parameters $\alpha_{\hat{u}\hat{v}}$, β and θ . We can now study the behavior of $m_{\text{up,low}}^2$ as functions of ϵ . For m_{up}^2 , one finds a non-monotonic function that first decreases and is negative and then increases and becomes positive, as shown in figure 2. For m_{low}^2 , one finds instead a monotonically increasing function that is always positive.

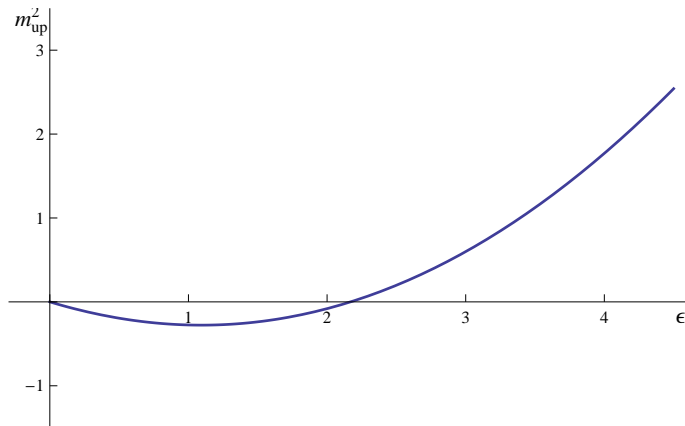


Figure 2: Plot of m_{up}^2 as a function of ϵ , in units of $m_{3/2}^2$.

To summarize, we see that when V is positive and small compared to $m_{3/2}^2$, as required for particle phenomenology, the vacuum is necessarily unstable, but when V is positive and large compared to $m_{3/2}^2$, as could be desirable for inflation, the vacuum can be metastable. In these two limits of small and large cosmological constant, one can actually derive some simpler result for the bound on the smallest mass eigenvalue. This will also allow us to describe the general case of intermediate cosmological constant in a simpler, qualitatively way.

In the limit where ϵ is small, which corresponds to a situation where the cosmological constant is made small through a tuning of parameters, the entries of the

two-dimensional averaged mass matrix are given by:

$$m_{\text{hh}}^2 \simeq \left[\left(\frac{8}{3} - 6 \cos^2 \theta + 3 \cos^4 \theta \right) + \left(4 - 7 \cos^2 \theta + 2 \cos^4 \theta \right) \epsilon \right] m_{3/2}^2, \quad (5.31)$$

$$m_{\text{vv}}^2 \simeq \left[\left(-6 \cos^2 \theta + 9 \cos^4 \theta \right) + \left(-2 + \cos^2 \theta + 6 \cos^4 \theta \right) \epsilon \right] m_{3/2}^2, \quad (5.32)$$

$$m_{\text{hv}}^2 \simeq \left[\left(0 \right) + \left(\sqrt{6} \cos \theta \sin \theta \right) \epsilon \right] m_{3/2}^2. \quad (5.33)$$

When $\epsilon = 0$, the two eigenvalues m_{\pm}^2 of (5.20) take degenerate and vanishing extremal values $m_{\text{up,low}}^2$ for the same angle $\theta = \theta_0$ given by $\theta_0 = \arccos \sqrt{\frac{2}{3}} \simeq 0.62$. When $\epsilon \neq 0$, the two eigenvalues m_{\pm}^2 of (5.20) instead take non-degenerate and non-vanishing extremal values $m_{\text{up,low}}^2$ for two slightly different angles $\theta = \theta_0 + \Delta\theta_{\pm}$. To compute the approximate values of $m_{\text{up,low}}^2$ for small ϵ , one can then further expand the entries (5.31), (5.32) and (5.33) at $\theta = \theta_0 + \Delta\theta_{\pm}$ up to first order in $\Delta\theta_{\pm} = \kappa_{\pm} \epsilon$, then compute the eigenvalues and finally extremize them with respect to the parameter κ_{\pm} . Proceeding in this way one finds:

$$m_{\text{up}}^2 \simeq -\frac{1}{2} \epsilon m_{3/2}^2, \quad (5.34)$$

$$m_{\text{low}}^2 \simeq \frac{3}{2} \epsilon m_{3/2}^2. \quad (5.35)$$

It follows that in this regime:

$$\min \{m_i^2\} \lesssim -\frac{1}{2} V. \quad (5.36)$$

This quantifies the extent to which de Sitter vacua with a small cosmological constant are unstable in this class of models.

In the limit where ϵ is large, which includes the situation where the main contribution to the vacuum energy is non-gravitational, the entries of the two-dimensional averaged mass matrix are instead given by:

$$m_{\text{hh}}^2 \simeq \left[\frac{1}{3} (\cos^2 \theta - 1) (\cos^2 \theta - 4) \right] \epsilon^2 m_{3/2}^2, \quad (5.37)$$

$$m_{\text{vv}}^2 \simeq \left[\cos^2 \theta (\cos^2 \theta + 1) \right] \epsilon^2 m_{3/2}^2, \quad (5.38)$$

$$m_{\text{hv}}^2 \simeq \left[\sqrt{\frac{2}{3}} \cos \theta \sin \theta \right] \epsilon^2 m_{3/2}^2. \quad (5.39)$$

After computing m_{\pm}^2 , one can check that m_{-}^2 develops a maximum for $\theta \simeq \frac{\pi}{4} \simeq 0.79$ and m_{+}^2 develops a minimum for $\theta \simeq 0.89$, with the values

$$m_{\text{up}}^2 \simeq \frac{1}{4} \epsilon^2 m_{3/2}^2, \quad (5.40)$$

$$m_{\text{low}}^2 \simeq 1.05 \epsilon^2 m_{3/2}^2. \quad (5.41)$$

It follows that in this regime:

$$\min \{m_i^2\} \lesssim \frac{1}{4} \frac{V^2}{m_{3/2}^2}. \quad (5.42)$$

This quantifies the extent to which de Sitter vacua with a large cosmological constant are stable in this class of models.

In more general situations where ϵ is neither very large nor very small, one has in principle to use the exact entries of the two-dimensional averaged mass matrix. The extrema m_{up}^2 and m_{low}^2 must then be computed by extremizing the corresponding complicated expressions (5.26) for m_-^2 and m_+^2 . However, it turns out that the exact results for these bounds are reasonably well approximated for any value of ϵ by just adding up the behaviors that we derived for small and large values of ϵ . One can in fact write:

$$m_{\text{up}}^2 < \left(-\frac{1}{2}\epsilon + \frac{1}{4}\epsilon^2\right)m_{3/2}^2, \quad (5.43)$$

$$m_{\text{low}}^2 > \left(\frac{3}{2}\epsilon + 1.05\epsilon^2\right)m_{3/2}^2. \quad (5.44)$$

It follows in particular that:

$$\min \{m_i^2\} < -\frac{1}{2}V + \frac{1}{4}\frac{V^2}{m_{3/2}^2}. \quad (5.45)$$

This quantifies in a simpler but weaker way the situation for de Sitter vacua with an intermediate cosmological constant in this class of models. Note that this alternative bound can no longer be strictly saturated, and in relation to this the transition between positive and negative values on the right-hand side occurs at $\epsilon = 2$ rather than $\epsilon \simeq 2.17$.

6 Rigid limit

Let us briefly study what happens in the rigid limit, when gravitational effects are negligible, in order to compare our results with those of [24]. In this decoupling limit $m_{3/2}^2 \rightarrow 0$ while V stays finite but is dominated by non-gravitational effects. One is then in the situation where $\epsilon \rightarrow +\infty$, and the entries of the mass matrix simplify. Moreover, all these entries take the form of a finite coefficient depending on the angle θ times the squared energy scale $\epsilon^2 m_{3/2}^2$, which stays finite in the limit. More precisely, one finds

$$m_{\hat{u}\hat{v}}^2 = \left[-\frac{1}{2}\cot^2\theta\alpha_{\hat{u}\hat{v}} + (7 + 3\cot^2\theta + 4\tan^2\theta)\delta_{\hat{u}\rho}\delta_{\hat{v}\rho}\right]m_A^2, \quad (6.1)$$

$$m_{\hat{z}\hat{z}}^2 = \left[1 + 2\cot^2\theta\right]m_A^2, \quad (6.2)$$

$$m_{\hat{z}\hat{z}}^2 = \left[\tan^2\theta\beta - \cot^2\theta\right]m_A^2e^{-2i\gamma}, \quad (6.3)$$

$$m_{\hat{u}\hat{z}}^2 = \left[\sqrt{2}(\cot\theta + \tan\theta)\delta_{\hat{u}\hat{z}}\right]m_A^2e^{-i\gamma}, \quad (6.4)$$

where $m_A^2 = \cos^2\theta \sin^2\theta \epsilon^2 m_{3/2}^2$, which using the relations (5.11) and (5.12) is recognized to correspond to the vector mass in the rigid limit, namely

$$m_A^2 = \xi^2 \rho^{-2} f^{-1} l^{-1} |\nabla_z L|^2. \quad (6.5)$$

Notice also that in the rigid limit the constraint that the parameters $\alpha_{\hat{u}\hat{v}}$ must satisfy simplifies to

$$\delta^{\hat{u}\hat{v}} \alpha_{\hat{u}\hat{v}} = 6 + 8 \tan^2\theta. \quad (6.6)$$

The averages of the above blocks are then found to be:

$$m_{\text{hh}}^2 = \left(1 + \frac{4}{3} \tan^2\theta\right) m_A^2, \quad (6.7)$$

$$m_{\text{vv}}^2 = \left(1 + 2 \cot^2\theta\right) m_A^2, \quad (6.8)$$

$$m_{\text{hv}}^2 = \sqrt{\frac{2}{3}} \left(\tan\theta + \cot\theta\right) m_A^2. \quad (6.9)$$

Notice finally that in the rigid limit it is more meaningful to study the extrema of m_{\pm}^2 with respect to θ at fixed m_A^2 . By doing so, one finds that the extrema of m_{\pm}^2 both occur at $\theta = \frac{\pi}{4}$, with values m_A^2 and $\frac{13}{3}m_A^2$. This implies in particular that

$$\min \{m_i^2\} \leq m_A^2. \quad (6.10)$$

At this point, we can compare the above formulae with the results derived in [24] in rigid supersymmetry, where the Przanowski-Tod quaternionic-Kähler manifold reduces to a Gibbons-Hawking hyper-Kähler manifold [37–39] and the local special-Kähler manifold reduces to a global special-Kähler manifold. We see that the averaged masses (6.7), (6.8) and (6.9) exactly match the corresponding results in [24]. This is expected, since these quantities are, by definition, independent of any parameter and any coordinate choice. On the other hand, the non-averaged masses (6.1), (6.2), (6.3) and (6.4) can be compared with those in [24] only after taking into account the differences in the coordinates used here and in [24] to describe the scalar manifold and the associated parameters.

In the hypermultiplet sector, the parameters $\alpha_{\hat{u}\hat{v}}$ defined here must map to the parameters a_{ij} defined in [24]. However, the coordinates $\rho, \varphi, \chi, \tau$ used here are related in a non-trivial way to the coordinates x_i, t used in [24]. By comparing the form of the metric, the Killing vector and the Killing potentials for the two spaces in the Ricci-flat limit where they should coincide, one can see that ρ is related to $|\vec{x}|^{-1/2}$, while φ and χ are related to the two angular variables u, v describing the orientation of the vector $\vec{x}/|\vec{x}|$, and finally τ is related to t . Moreover, the function f in the Przanowski-Tod metric is directly related to the function f in the Gibbons-Hawking metric by a simple rescaling involving two powers of the radial coordinate. The precise relation between the complete $\alpha_{\hat{u}\hat{v}}$ and the complete a_{ij} is then not

totally straightforward to determine, due to this non-trivial change of coordinates involving the Planck scale (see [40] for a related discussion in a specific example). For this reason, we will not attempt to compare more explicitly the non-averaged mass matrix in this sector with the results of [24].

In the vector multiplet sector, the parameter β defined here must map to the parameter b defined in [24]. Moreover, the coordinate z used here maps to the coordinate z used in [24]. Similarly, the function l used here maps to the function l in [24]. The precise relation between β and b is thus straightforward to determine. Using special coordinates and taking the rigid limit (in which the Kähler connection drops out from the covariant derivatives) the symplectic section simply reads $L = z$, and one finds that $\beta = -b + 3 \cot^4 \theta$. Taking into account this relation, the structure of the non-averaged mass matrix in this sector then precisely matches the results of [24] in the rigid limit, as it should.

7 Examples

In order to illustrate the general statements of the previous sections, let us now study a class of more explicit examples and compute the full spectrum of their mass eigenvalues. For concreteness and simplicity, we shall focus on a family of solutions where the eigenvalues of the mass matrix can be computed analytically. More precisely, let us keep θ and ϵ arbitrary, but take all the real parameters $\alpha_{\hat{u}\hat{v}}$ to be correlated and controlled by a single real parameter x and similarly choose the complex parameter β to be controlled by a single real parameter y :

$$\alpha_{\rho\rho} = \frac{-2(1+x)(3+\epsilon)^2 + 4(3+\epsilon)(3+2\epsilon)\cos^{-2}\theta - 8(2+3\epsilon)\cos^{-4}\theta}{(3+\epsilon)^2}, \quad (7.1)$$

$$\alpha_{\varphi\varphi} = x, \quad \alpha_{\chi\chi} = x, \quad \alpha_{\rho\varphi} = 0, \quad \alpha_{\rho\chi} = 0, \quad \alpha_{\varphi\chi} = 0, \quad (7.2)$$

$$\beta = y. \quad (7.3)$$

For simplicity we shall also set $\gamma = 0$, as this does not affect the eigenvalues. With this choice of parameters the mass matrix takes the form

$$m_{I\bar{J}}^2 = \lambda_{I\bar{J}} m_{3/2}^2, \quad (7.4)$$

where the non-trivial entries of the matrix $\lambda_{I\bar{J}}$ are given by

$$\lambda_{\rho\rho} = (3+\epsilon)^2(1+x)\cos^4\theta - (3+\epsilon)(6+5\epsilon)\cos^2\theta + 4(1+\epsilon)(2+\epsilon), \quad (7.5)$$

$$\lambda_{\varphi\varphi} = -\frac{1}{2}(3+\epsilon)^2 x \cos^4\theta, \quad \lambda_{\chi\chi} = -\frac{1}{2}(3+\epsilon)^2 x \cos^4\theta, \quad (7.6)$$

$$\lambda_{\rho\varphi} = 0, \quad \lambda_{\rho\chi} = 0, \quad \lambda_{\varphi\chi} = 0, \quad (7.7)$$

$$\lambda_{z\bar{z}} = ((3+\epsilon)\cos^2\theta - 2)((3+\epsilon)\cos^2\theta + \epsilon), \quad (7.8)$$

$$\lambda_{zz} = (3+\epsilon)^2 \sin^4\theta y - ((3+\epsilon)\cos^2\theta - 1)((3+\epsilon)\cos^2\theta - 2), \quad (7.9)$$

$$\lambda_{\rho z} = \sqrt{2}\epsilon(3+\epsilon)\cos\theta\sin\theta, \quad \lambda_{\varphi z} = 0, \quad \lambda_{\chi z} = 0. \quad (7.10)$$

The five non-trivial masses are then given in terms of the eigenvalues λ_i of the non-trivial block $\lambda_{i\hat{j}}$ of this matrix:

$$m_i^2 = \lambda_i m_{3/2}^2. \quad (7.11)$$

In this simple four-parameter special family of vacua, the eigenvalues λ_i can be computed analytically as functions of the parameters θ , ϵ , x and y characterizing the scalar geometry in the vicinity of the vacuum point. They are found to be

$$\lambda_{1,2} = -\frac{1}{2}(3 + \epsilon)^2 x \cos^4 \theta, \quad (7.12)$$

$$\lambda_3 = -(3 + \epsilon)^2 y \sin^4 \theta + ((3 + \epsilon) \cos^2 \theta - 2)(2(3 + \epsilon) \cos^2 \theta - 1 + \epsilon), \quad (7.13)$$

$$\lambda_{4,5} = P \pm \sqrt{Q}, \quad (7.14)$$

where

$$P = \frac{1}{2}(3 + \epsilon)^2 y \sin^4 \theta + \frac{1}{2}(3 + \epsilon)^2 (1 + x) \cos^4 \theta - \frac{1}{2}(3 + \epsilon)(5 + 4\epsilon) \cos^2 \theta + (1 + \epsilon)(3 + 2\epsilon), \quad (7.15)$$

$$Q = \frac{1}{4}(3 + \epsilon)^4 y^2 \sin^8 \theta - \frac{1}{2}(3 + \epsilon)^4 y(1 + x) \sin^4 \theta \cos^4 \theta + \frac{1}{2}(3 + \epsilon)^3 y(7 + 6\epsilon) \sin^4 \theta \cos^2 \theta - (3 + \epsilon)^2 y(1 + \epsilon)(5 + 2\epsilon) \sin^4 \theta + \frac{1}{4}(3 + \epsilon)^4 (1 + x)^2 \cos^8 \theta - \frac{1}{2}(3 + \epsilon)^3 (1 + x)(7 + 6\epsilon) \cos^6 \theta + \frac{1}{4}(3 + \epsilon)^2 (69 + 112\epsilon + 28\epsilon^2 + 4x(1 + \epsilon)(5 + 2\epsilon)) \cos^4 \theta - (3 + \epsilon)(35 + 79\epsilon + 44\epsilon^2 + 8\epsilon^3) \cos^2 \theta + (1 + \epsilon)^2 (5 + 2\epsilon)^2. \quad (7.16)$$

Using the same kind of notation, we also denote

$$m_{\text{up}}^2 = \lambda_{\text{up}} m_{3/2}^2, \quad (7.17)$$

$$m_{\text{low}}^2 = \lambda_{\text{low}} m_{3/2}^2. \quad (7.18)$$

In this class of models, it is straightforward to verify all the statements of the previous section concerning the range that the mass eigenvalues are allowed to take. For any given value of ϵ , one may make the vacuum as stable as possible by first adjusting θ to the optimal value that allows the upper bound λ_{up} on the smallest eigenvalue to be maximized, and then adjusting x and y to saturate this value. It turns out that this best situation occurs when $\lambda_{1,2} = \lambda_3 = \lambda_4$, while λ_5 is always bigger. In table 1 we list some sample models illustrating this point. We see that in this simple class of models where only two real parameters are retained among $\alpha_{\hat{u}\hat{v}}$ and β , there are thus always four of the five eigenvalues that become degenerate when the parameters are adjusted in such a way to saturate the bound defined by λ_{up} . In more general models where additional independent parameters are retained among $\alpha_{\hat{u}\hat{v}}$ and β , this feature is expected to disappear.

ϵ	θ	x	y	λ_{1-4}	λ_5	λ_{up}	λ_{low}
0	0.6155	0.0000	0.0000	0.0000	0.000	0.0000	0.000
1	0.6268	0.0799	1.8748	-0.2752	7.645	-0.2752	3.210
2	0.6510	0.0161	2.5531	-0.0807	19.731	-0.0807	8.553
3	0.6705	-0.0883	2.8275	0.5989	36.323	0.5989	16.058
4	0.6854	-0.2011	2.9547	1.7701	57.425	1.7701	25.730

Table 1: Some explicit examples of models with their spectrum of masses.

8 Conclusions

In this work we have shown that metastable de Sitter vacua may arise in rather simple N=2 supergravity theories with a single hypermultiplet and a single vector multiplet, without Fayet-Iliopoulos terms or non-Abelian gauge symmetries, provided that the scalar manifold is suitably curved. A first, crucial requirement for this to be possible is that both sectors should be involved in the process of supersymmetry breaking, as it is known that in theories with only one hypermultiplet or only one vector multiplet no metastable vacua can ever arise. A second, important limitation is that the cosmological constant V should be sufficiently large compared to the gravitino mass squared $m_{3/2}^2$ in Planck units. More precisely, for positive $\epsilon = V/m_{3/2}^2$ we showed that metastable de Sitter vacua are only possible if $\epsilon \gtrsim 2.17$. As a consequence, the simple de Sitter vacua that we constructed can realize slow-roll inflation with a large Hubble parameter corresponding to $\epsilon \gg 1$ but not a late-time vacuum with small cosmological constant corresponding to $\epsilon \ll 1$.

In the simple class of models that we have studied, the quaternionic-Kähler manifold describing the hypermultiplet sector is a generic four-dimensional Przanowski-Tod space possessing at least one triholomorphic isometry. The special-Kähler manifold describing the vector multiplet sector is a totally general two-dimensional special space and the potential is generated by gauging the isometry of the hypermultiplet geometry. For this class of models, we were able to derive a simple upper bound on the mass of the lightest scalar, which depends only on the angle θ defining the spread of the supersymmetry breaking direction between the two sectors and the parameter ϵ related to the cosmological constant. In the particular cases where only the hypermultiplet or only the vector multiplet is involved in supersymmetry breaking, corresponding to $\theta \rightarrow 0$ and $\theta \rightarrow \frac{\pi}{2}$ respectively, one recovers for any ϵ the

results of [9] and [10], namely the saturable bounds:

$$\min \{m_i^2\} \leq -\frac{1}{3}m_{3/2}^2 - V \quad (\text{only hyper}), \quad (8.1)$$

$$\min \{m_i^2\} \leq -2V \quad (\text{only vector}). \quad (8.2)$$

In the more general case where both the hyper- and vector multiplets are involved in supersymmetry breaking, there exists for any given ϵ an optimal choice for θ that maximizes the smallest mass. This allows us to define a bound depending only on ϵ for the situation where the two sectors are optimally mixed. The exact result for the saturable upper bound on $\min \{m_i^2\}$ as a function of ϵ is plotted in figure 2 but cannot be expressed in any simple, analytic form. However, we showed that the following simpler bound, which is slightly weaker and non-saturable, is also true and gives a good approximation to the exact bound:

$$\min \{m_i^2\} < -\frac{1}{2}V + \frac{1}{4} \frac{V^2}{m_{3/2}^2} \quad (\text{hyper and vector}). \quad (8.3)$$

Notice that on the right-hand side of this bound, the transition between positive and negative values arises at $\epsilon = 2$, but as already said the precise critical point for which metastable vacua really become possible is $\epsilon \simeq 2.17$.

It is straightforward to generalize the analysis presented in this paper to study the possibility of getting metastable supersymmetry breaking anti de Sitter vacua with a negative cosmological constant $V > -3m_{3/2}^2$ in the same setting. In fact, the equations (5.26)–(5.28) defining the bounds m_{\pm}^2 on the scalar masses hold true unchanged also in the case of negative ϵ , and one can repeat the same analysis as for positive ϵ . The maximal value m_{up}^2 that m_-^2 can take is found to be positive for $\epsilon \in]-3, -\frac{3}{2}[\cup]-1, 0[$ and negative for $\epsilon \in]-\frac{3}{2}, 1[$, but always larger than the Breitenlohner–Freedman bound $\frac{3}{4}V$ for stability [41]. This implies that metastable supersymmetry breaking anti de Sitter vacua may exist for any $\epsilon \in]-3, 0[$.

It would be interesting to investigate whether the results found in this paper for models with a single hypermultiplet and a single vector multiplet can be extended to theories involving several of these multiplets, at least in the simplest case of Abelian gaugings without constant Fayet-Iliopoulos terms. A natural way to try to do this is to look at the mass matrix in the subspace of scalar fields defined by the various sGoldstini, along the lines of [1–3, 9]. Indeed, this approach allows one to extend the results (8.1) and (8.2) to theories with an arbitrary number of hypermultiplets and an arbitrary number of vector multiplets, respectively. In this way, one might also hope to derive a result similar to (8.3) for theories with several hyper- and several vector multiplets. A first hint in favor of this comes from the fact that the known examples of metastable de Sitter vacua in such theories [7] do indeed satisfy this type of bound, albeit in a rather trivial way, thanks to the fact that they all lead to $m_{3/2} = 0$. We leave a thorough analysis of this general problem for future work.

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References

- [1] M. Gomez-Reino and C. A. Scrucca, *Locally stable non-supersymmetric Minkowski vacua in supergravity*, JHEP **0605** (2006) 015 [hep-th/0602246].
- [2] M. Gomez-Reino and C. A. Scrucca, *Constraints for the existence of flat and stable non-supersymmetric vacua in supergravity*, JHEP **0609** (2006) 008 [hep-th/0606273].
- [3] M. Gomez-Reino and C. A. Scrucca, *Metastable supergravity vacua with F and D supersymmetry breaking*, JHEP **0708** (2007) 091 [arXiv:0706.2785].
- [4] F. Denef and M. R. Douglas, *Distributions of nonsupersymmetric flux vacua*, JHEP **0503** (2005) 061 [hep-th/0411183].
- [5] L. Covi, M. Gomez-Reino, C. Gross, J. Louis, G. A. Palma and C. A. Scrucca, *De Sitter vacua in no-scale supergravities and Calabi-Yau string models*, JHEP **0806** (2008) 057 [arXiv:0804.1073].
- [6] L. Covi, M. Gomez-Reino, C. Gross, J. Louis, G. A. Palma and C. A. Scrucca, *Constraints on modular inflation in supergravity and string theory*, JHEP **0808** (2008) 055 [arXiv:0805.3290].
- [7] P. Frè, M. Trigiante and A. Van Proeyen, *Stable de Sitter vacua from $N = 2$ supergravity*, Class. Quant. Grav. **19** (2002) 4167 [arXiv:hep-th/0205119].
- [8] O. Ogetbil, *Stable de Sitter vacua in 4 dimensional supergravity originating from 5 dimensions*, Phys. Rev. D **78** (2008) 105001 [arXiv:0809.0544 [hep-th]].
- [9] M. Gomez-Reino, J. Louis and C. A. Scrucca, *No metastable de Sitter vacua in $N=2$ supergravity with only hypermultiplets*, JHEP **0902** (2009) 003 [arXiv:0812.0884].
- [10] E. Cremmer, C. Kounnas, A. Van Proeyen, J. P. Derendinger, S. Ferrara, B. de Wit and L. Girardello, *Vector multiplets coupled to $N=2$ supergravity: superhiggs effect, flat potentials and geometric structure*, Nucl. Phys. B **250** (1985) 385.
- [11] A. Borghese and D. Roest, *Metastable supersymmetry breaking in extended supergravity*, JHEP **1105** (2011) 102 [arXiv:1012.3736].
- [12] A. Borghese, R. Linares and D. Roest, *Minimal Stability in Maximal Supergravity*, [arXiv:1112.3939].

- [13] M. de Roo, D. B. Westra and S. Panda, *De Sitter solutions in $N=4$ matter coupled supergravity*, JHEP **0302** (2003) 003 [hep-th/0212216].
- [14] M. de Roo, D. B. Westra, S. Panda and M. Trigiante, *Potential and mass matrix in gauged $N=4$ supergravity*, JHEP **0311** (2003) 022 [hep-th/0310187].
- [15] C. M. Hull and N. P. Warner, *The potentials of the gauged $N=8$ supergravity theories*, Nucl. Phys. B **253** (1985) 675.
- [16] R. Kallosh, A. D. Linde, S. Prokushkin and M. Shmakova, *Gauged supergravities, de Sitter space and cosmology*, Phys. Rev. D **65** (2002) 105016 [hep-th/0110089].
- [17] G. Dall'Agata and G. Inverso, *De Sitter vacua in $N = 8$ supergravity and slow-roll conditions*, Phys. Lett. B **718** (2013) 1132 [arXiv:1211.3414].
- [18] G. Dall'Agata and G. Inverso, *On the vacua of $N = 8$ gauged supergravity in 4 dimensions*, Nucl. Phys. B **859** (2012) 70 [arXiv:1112.3345].
- [19] F. Catino, C. A. Scrucca and P. Smyth, *Metastable de Sitter vacua in $N=2$ to $N=1$ truncated supergravity*, JHEP **1210** (2012) 124 [arXiv:1209.0912].
- [20] D. Roest and J. Rosseel, *De Sitter in extended supergravity*, Phys. Lett. B **685** (2010) 201 [arXiv:0912.4440].
- [21] J.-C. Jacot and C. A. Scrucca, *Metastable supersymmetry breaking in $N=2$ non-linear sigma-models*, Nucl. Phys. B **840** (2010) 67 [arXiv:1005.2523].
- [22] L. Brizi and C. A. Scrucca, *The lightest scalar in theories with broken supersymmetry*, JHEP **1111** (2011) 013 [arXiv:1107.1596].
- [23] I. Antoniadis and M. Buican, *Goldstinos, supercurrents and metastable SUSY breaking in $N=2$ supersymmetric gauge theories*, JHEP **1104** (2011) 101 [arXiv:1005.3012].
- [24] B. L egeret, C. A. Scrucca and P. Smyth, *Metastable spontaneous breaking of $N=2$ supersymmetry*, [arXiv:1211.7364].
- [25] L. Alvarez-Gaum e and D. Z. Freedman, *Geometrical structure and ultraviolet finiteness in the supersymmetric sigma model*, Commun. Math. Phys. **80** (1981) 443.
- [26] L. Alvarez-Gaum e and D. Z. Freedman, *Potentials For The Supersymmetric Nonlinear Sigma Model*, Commun. Math. Phys. **91** (1983) 87.
- [27] J. Bagger and E. Witten, *Matter couplings in $N=2$ supergravity*, Nucl. Phys. B **222** (1983) 1.
- [28] B. de Wit and A. Van Proeyen, *Potentials and symmetries of general gauged $N=2$ supergravity: Yang-Mills models*, Nucl. Phys. B **245** (1984) 89.
- [29] B. de Wit, P. G. Lauwers, and A. Van Proeyen, *Lagrangians of $N=2$ supergravity - matter systems*, Nucl. Phys. B **255** (1985) 569.

- [30] C. M. Hull, A. Karlhede, U. Lindstrom and M. Rocek, *Nonlinear sigma models and their gauging in and out of superspace*, Nucl. Phys. B **266** (1986) 1.
- [31] A. Strominger, *Special geometry*, Commun. Math. Phys. **133** (1990) 163.
- [32] R. D'Auria, S. Ferrara and P. Frè, *Special and quaternionic isometries: general couplings in $N=2$ supergravity and the scalar potential*, Nucl. Phys. B **359** (1991) 705.
- [33] L. Andrianopoli, M. Bertolini, A. Ceresole, R. D'Auria, S. Ferrara, P. Fré and T. Magri, *$N = 2$ supergravity and $N = 2$ super Yang-Mills theory on general scalar manifolds: symplectic covariance, gaugings and the momentum map*, J. Geom. Phys. **23** (1997) 111 [hep-th/9605032].
- [34] M. Przanowski, *Killing vector fields in selfdual, Euclidean Einstein spaces with $\Lambda \neq 0$* , J. Math. Phys. **32** (1991) 1004.
- [35] K. P. Tod, *The $SU(\infty)$ -Toda field equation and special four-dimensional metrics*, Lect. Notes in Pure and Appl. Math. **184** (1997) 307.
- [36] M. Davidse, F. Saueressig, U. Theis and S. Vandoren, *Membrane instantons and de Sitter vacua*, JHEP **0509** (2005) 065 [hep-th/0506097].
- [37] S. W. Hawking, *Gravitational instantons*, Phys. Lett. A **60** (1977) 81.
- [38] G. W. Gibbons and S. W. Hawking, *Gravitational multi-instantons*, Phys. Lett. B **78** (1978) 430.
- [39] N. J. Hitchin, A. Karlhede, U. Lindstrom and M. Rocek, *Hyperkahler metrics and supersymmetry*, Commun. Math. Phys. **108** (1987) 535.
- [40] N. Ambrosetti, I. Antoniadis, J. -P. Derendinger and P. Tziveloglou, *The hypermultiplet with Heisenberg isometry in $N=2$ global and local supersymmetry*, JHEP **1106** (2011) 139 [arXiv:1005.0323].
- [41] P. Breitenlohner and D. Z. Freedman, *Stability in gauged extended supergravity*, Annals Phys. **144** (1982) 249.