OSCILLATOR PHASE NOISE AND SAMPLING CLOCK JITTER

RETHNAKARAN PULIKKOONATTU

ABSTRACT. This technical note presents and overview of oscillator phase noise and its characteristics, which are paramount to the understanding of digital receiver and modems. Oscillators used in practical systems deviate from ideal spectral characteristics. Due to this, there arise unpleasant performance issues. A/D and D/A converters used in digital receivers use sampling clocks (for sampling or sample and hold) which are derived from sinusoidal oscillators. Phase noise present in the source oscillator could translate to jitter in clock waveform. There is an established relationship between the mean square jitter and phase noise spectrum. The mathematical formula to compute the rms phase jitter from a given oscillator phase noise spectrum is derived and furnished with examples. The result obtained thus matches with those obtained using a web based tool [1]

Date: 2007 June 12.
Key words and phrases. Phase noise, Oscillator phase noise, clock jitter.
ST Microelectronics, (Genesis Microchip) Bangalore, India (www.st.com).

1. PHASE NOISE

Ideal sinusoidal oscillators of frequency f_0 have a mathematically strict spectrum in the form of delta functions, centered at $-f_0$ and f_0. However, practical oscillators seldom exhibit this kind of clean spectrum. They tend to have spreading of spectral energy around the carrier frequency, as shown in Figure-1 . The spreading or spillage of energy to neighbouring points around f_0 can cause unwanted behaviour at both transmitter and receiver mixers. This spillage of spectral energy behave like unwanted phase modulation and is called phase noise. Besides mixers, the sampling clock used to trigger A/D and DAC are also influenced because the clocks used there are derived from non-ideal sinusoidal oscillators, with phase noise.

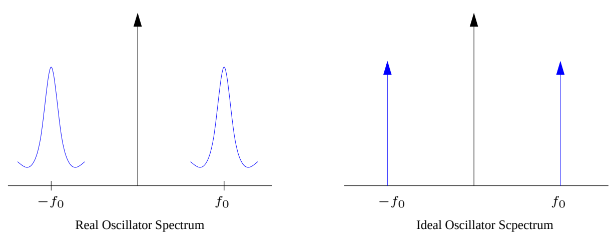

Figure 1. Oscillator Spectrum; Ideal sinusoidal oscillator will have a Delta function spectrum. Practical oscillators will have energy spread around the oscillator frequency. Figure shown here is a two sided power spectrum representation of ideal and practical oscillators.

1.1. **Phase noise in Transmitter.** In the transmitter, a direct impact of phase noise is in the form of adjacent channel leakage. When the phase noise spectrum tail has wider spread, then it could leak into the adjacent channel, leading to ACI. This effect is captured in Figure-2, .

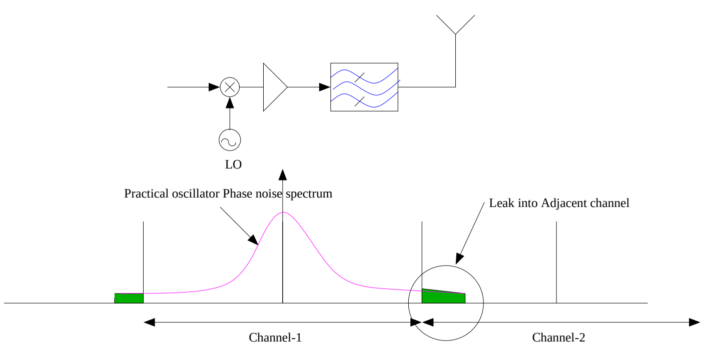

Figure 2. Phase noise in local oscillator could produce adjacent channel leakage in the transmitter

Phase noise in a transmit chain will leak power into adjacent channels. Since the power transmitted is large, say about 30dBm, an adjacent channel in a narrowband system may only reside about 200kHz away (GSM), placing a stringent specification on the transmitter spectrum.

1.2. **Phase noise in receiver.** The demodulators used at the receivers are classified as coherent or non-coherent, depending on whether they use or not a carrier signal, which ideally should have the same phase and frequency as the carrier at the transmitter, to demodulate the received signal. Typically both phase and frequency are recovered from the received signal by a phase locked loop (PLL) system, which employs a local oscillator. The recovered carrier may differ from the transmitted carrier because of the phase noise, due to short-term stability, i.e. frequency drift, of the oscillator, and because of the dynamics and transient behavior of the PLL.

In a receive chain, the fact that the LO is not a perfect delta function means that there is a continuum of Local Oscillators (LOs), that can mix with interfering signals and produce energy at the same IF. Here we observe an adjacent channel signal mixing with the skirt of the LO and falling on top of the a weak IF signal from the desired channel as reflected in Figure 3

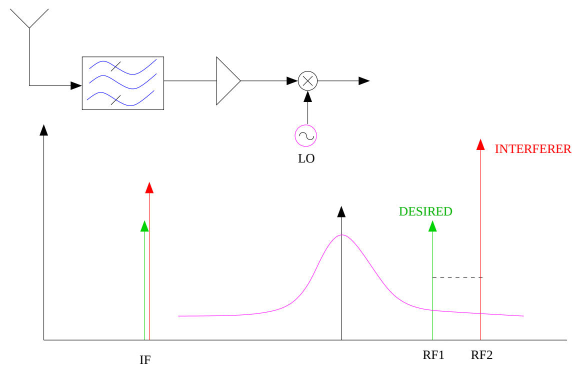

Figure 3. interference mixing effect on a receiver when the oscillator phase noise spectrum skirt is wide.

From a communication system performance point of view, significant phase noise result in irreducible error floor in bit error rate (BER) at higher Signal to noise ratio (SNR) as shown in Figure-4.

2. Representing Phase noise

The receiver carrier signal[1] is then,

$$v(t) = v_0(1 + \alpha(t)) \cos\left(\omega_0 t + \phi(t) + \frac{\beta}{2}t^2\right) \tag{1}$$

The long term drift effect of the oscillator due to ageing is reflected in β. $a(t)$ is the amplitude noise and $\phi(t)$ represents phase noise. The phase noise $\phi(t)$ will have deterministic component as well as random components. The deterministic component is attributed by physical phenomena like supply voltage, temperature change, output impedance of the oscillator etc. The random nature of the phase noise is usually represented with a power spectral density expressed in power law.

Instantaneous frequency[2] of $v(t)$ is

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}[2\pi f_0 + \phi(t)] \tag{2}$$

[1] Ideal receiver carrier should be equal to transmitter carrier $v_0 \cos(\omega_0 t)$

[2] $\omega_0 = 2\pi f_0$ and $f = 2\pi\omega$

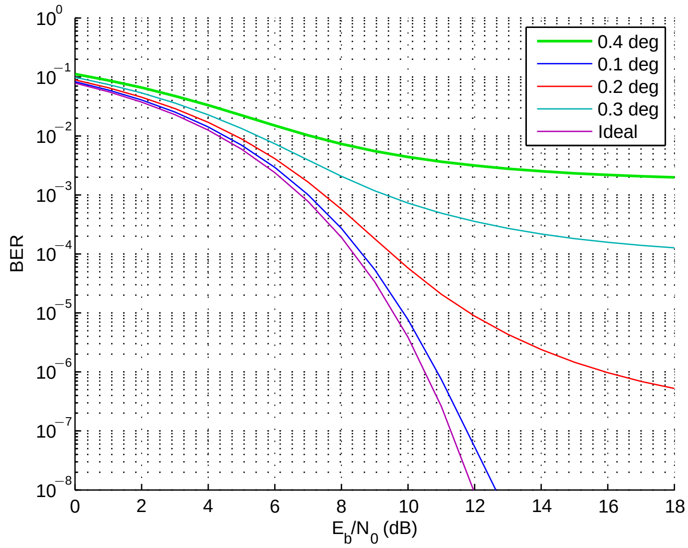

Figure 4. Phase noise cause floor effect on BER: Uncoded BPSK modulation performance with differing level of oscillator phase noise (Specified in degrees) is shown

The fractional frequency offset is then,

$$y(t) = \frac{\Delta f(t) = [f(t) - f_0]}{f_0} = \frac{1}{2\pi f_0}\frac{d\phi(t)}{dt} \tag{3}$$

Since $\phi(t)$ is assumed to be stationary, $y(t)$ is also stationary. Thus, the autocorrelation function $R_y(\tau)$ can be written as,

$$R_y(\tau) = \langle y(t), y(t-\tau)\rangle \tag{4}$$

The double sided PSD S_y^{DS} is obtained by performing fourier transform on $R_y(\tau)$.

$$S_y^{\text{DS}}(f) = \int_{-\infty}^{\infty} R_y(\tau) e^{-j2\pi f\tau} d\tau \tag{5}$$

from which, the one sided PSD S_y^f can be written as,

$$S_y(f) = \begin{cases} 2S_y^{\text{DS}}(f), & f \geq 0 \\ 0, & f < 0 \end{cases} \tag{6}$$

2.1. **Power spectral density of the Phase Noise.** Instead of representing the power spectrum of the instantaneous frequency change, it is possible to use the power spectral density (PSD) of the phase noise $\phi(t)$ itself. These two representations are equivalent[3]. Both these representations are used in literature.

First the autocorrelation of $\phi(t)$ is computed:

$$R_\phi(\tau) = \langle \phi(t), \phi(t-\tau)\rangle \tag{7}$$

[3]It is readily observed that $S_\phi(f) = \frac{f_0^2}{f^2} S_y(f)$

Upon Fourier transform, we get the double sided PSD $S_\phi^{DS}(f)$

(8) $$S_\phi^{\text{DS}}(f) = \int_{-\infty}^{\infty} R_\phi(\tau) e^{-j2\pi f \tau} d\tau$$

The commonly used single sideband representation $S_\phi(f)$ or simply[4] $S(f)$ is:

(9) $$S(f) = \begin{cases} 2S_\phi^{\text{DS}}(f), & f \geq 0 \\ 0, & f < 0 \end{cases}$$

Phase noise power spectral density are mathematically represented using the power law formula Eq. 12. Even though oscillators do not strictly adhere to the exact integer power representation, it is widely used to perform analytical calculations.

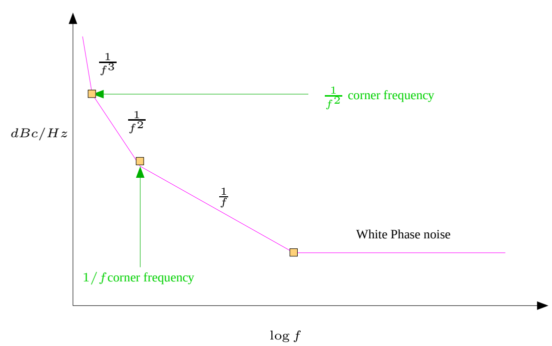

Figure 5. Phase noise PSD representation: The PSD is shown in dB scale, whereas the abscissa is on $\log(f)$ scale. The model is piecewise linear on logarithmic scale, where each segment has different slope

A simplified model of the random phase noise has a PSD of the form,

(10) $$S(f) = a + \begin{cases} b, & f_3 \leq f \\ \frac{c}{f}, & f_2 \leq f \leq f_3 \\ \frac{d}{f^2}, & f_1 \leq f \leq f_2 \end{cases}$$

Phase noise power spectral density is usually represented[5] as dBc. This expresses the statistical power of the phase noise signal with respect to the statistical power of the carrier signal in passband. A typical PSD response is shown in Figure 7. The 4 corner frequency positions (offset from the oscillator frequency) are 1 kHz, 10 kHz and 100 kHz. At these four points, the power measured over a 1 Hz band are -65 dBc, -75 dBc and -95dBc. The residual white noise floor of the phase noise is -95dBc. The sample PSD when represented in linear abscissa scale (Figure.6 has a logarithmic frequency scale) will appear like in Figure.7

[4]For simplicity the term ϕ is omitted from the PSD representation. Throughout this document, unless otherwise specified, $S(f)$ refers to $S_\phi(f)$, the phase noise PSD. The PSD $S(f)$ when specified in dB scale is denoted using the symbol $L(f)$. This notation is only to avoid confusion in dealing with linear to dB conversion later in the document, while computing average jitter from phase noise spectrum

[5]dBc stands for dB carrier

Table 1. Various constituent segment of phase noise PSD. Each noise type has a different PSD slope.

Noise type	$S_y(f)$	$S(f)$
Random run	$h_{-4}f^{-4}$	$h_{-4}f_0^2f^{-6}$
Random walk flicker FM	$h_{-3}f^{-3}$	$h_{-3}f_0^2f^{-5}$
Frequency random walk	$h_{-2}f^{-2}$	$h_{-2}f_0^2f^{-4}$
Frequency flicker noise	$h_{-1}f^{-1}$	$h_{-1}f_0^2f^{-3}$
White Frequency noise	h_0	$h_0f_0^2f^{-2}$
Flicker phase noise	h_1f	$h_1f_0^2f^{-1}$
White phase noise	h_2f^2	$h_2f_0^2$

3. Power law model

From experimental observations, the phase noise power spectrum is approximated by the well known power law model [2]. The oscillator phase noise thus follows the PSD formula,

$$S_y(f) = \sum_{n=-2}^{2} h_n f^n \tag{11}$$

This equation 11 correspond to five independent noise process listed in Table 1. Expressed in this functional form, Phase noise PSD follow a piecewise exponential relationship with the offset frequency (from oscillator center frequency). When the PSD is specified in dB scale (as is the practice) and on a $\log$ abscissa scale ($\log(f)$) this turns out to be like piecewise linear relationship. Thus frequency random walk has a slope of $-20dB/octave$ and flicker noise 10dB/octave.

Practical oscillators may not exactly have the slopes to be -20 dB/octave or -10 dB/octave. They could be arbitrary. To accommodate this change, the power law equation will have real valued powers (not necessarily integer powers).

$$S(f) = \sum_{n\geq 0, n\in\mathbb{R}} h_n f^n \tag{12}$$

Given the phase noise PSD corner frequencies and specified power (measured at 1Hz of bandwidth at these corner points) in dBc/Hz, the slope can be calculated.

4. Other ways to specify Phase noise

Phase noise manisfest as the jitter in a signal's zero crossing [4]. The variance of such (random) zero crossing jitter is related to the variance of the phase noise itself.

4.1. Time Domain characterization of Phase noise.

Even though, power spectrum of the phase noise is commonly used to specify the characteristics of phase noise, the time domain view of the same is worth a look. One could define, the variance of the fractional frequency offset (due to phase noise) $y(t)$ as follows.

$$\begin{aligned} \sigma^2(\tau) &= \langle \bar{y}^2 \rangle \\ &= \frac{1}{\tau^2}\left\langle \left[\int_{t_k-\tau}^{t_k} y(t)dt\right]^2 \right\rangle \end{aligned} \tag{13}$$

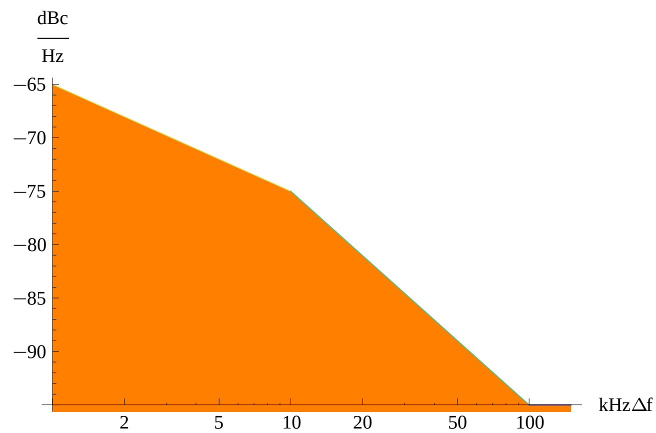

Figure 6. Example:Local oscillator Phase Noise spectrum expressed in conventional $log(f)$ scale.This is equivalent to the one shown in Figure.7

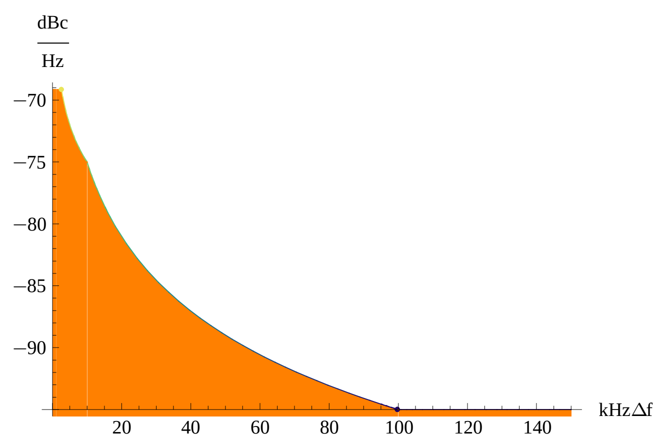

Figure 7. Example:Local oscillator Phase Noise spectrum expressed in linear f scale. This is equivalent to the one shown in Figure.6

Here, the variance is computed over a small time scale $[-\tau, 0]$. Usually, the value of τ is an integer multiple of $1/f_0$.

The averaging over τ is equivalent to filtering with a rectangular window function $h(t) \in [-\tau, 0]$. Thus we can write,

$$\sigma^2(\tau) = \left\langle \left[\int_{-\infty}^{\infty} y(u)h(t-u)du \right]^2 \right\rangle \tag{14}$$

It can be computed easier in Frequency domain as,

$$\sigma^2(\tau) = \int_0^{\infty} S_y(f) \|H(f)\|^2 df \tag{15}$$

where,

$$\|H(f)\|^2 = \left(\frac{\sin \pi\tau f}{\pi\tau f}\right)^2 \tag{16}$$

It is worth noting that, the variance computed in this section is for the fractional frequency offset, as defined in Eq.(2). A similar variance can be obtained for the timing variation (jitter) as well, which is used for A/D specifications.

5. Clock jitter

The clocks used in ADC and DAC for sampling and holding are not perfect in practice. Sources typically are square waves locked to highly stable reference oscillators (sinusoidal waveforms). Sampling and holding occur when the falling edge or trailing edge (one of them) of clock signal amplitude reaches a pre specified value. Natural phenomena, including bandwidth limitations, noise, skew prevent these clocks from being perfect. These effects cause the time interval between successive clock trigger to vary or jitter. Often these variations, with respect to ideal clocks are random[6].

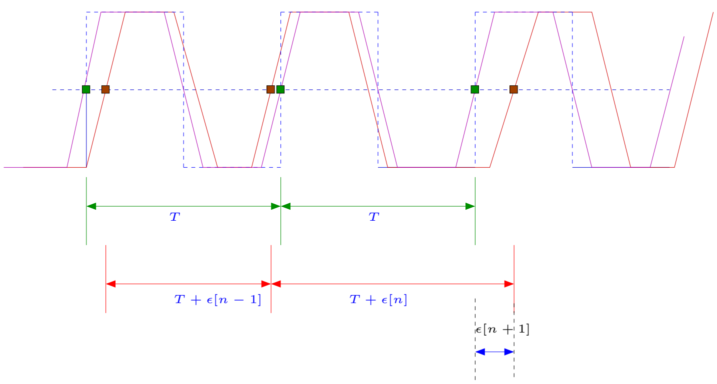

Figure 8. Sampling Clock Jitter: The time duration (period) between successive triggers vary as a result of the phase noise. Ideal clocks preserve constant period T, whereas, practical clocks vary the value randomly, leding to jitter

5.1. **RMS Phase Jitter.** Instead of computing the time variance of the instantaneous fractional frequency offset $y(t)$, it is possible to compute the phase variance and thereby the rms phase jitter. The method is identical to the previous section.

$$\begin{aligned} \sigma_\phi^2(\tau) &= \langle \bar{\phi}^2 \rangle \\ &= \frac{1}{\tau^2}\left\langle \left[\int_{t_k-\tau}^{t_k} \phi(t)dt\right]^2 \right\rangle \end{aligned} \tag{17}$$

This is equivalent to,

$$\sigma_\phi^2(\tau) = \left\langle \left[\int_{-\infty}^{\infty} \phi(u)h(t-u)du\right]^2 \right\rangle \tag{18}$$

[6]Generally, there will be a deterministic component and a random component to the jitter

and as in Eq.(15)

$$\sigma_\phi^2(\tau) = \int_0^\infty S(f)\left(\frac{\sin \pi\tau f}{\pi\tau f}\right)^2 df \tag{19}$$

We can define the rms phase jitter J as

$$J = \frac{\sigma_\phi}{2\pi f_0} \tag{20}$$

From Eq.(21) and Eq.(19) we get,

$$J = \frac{1}{2\pi f_0}\sqrt{\int_0^\infty S(f)\left(\frac{\sin \pi\tau f}{\pi\tau f}\right)^2 df} \tag{21}$$

The term $\left(\frac{\sin \pi\tau f}{\pi\tau f}\right)^2$ is usually ignored (it is almost unity for $f\tau \approx 0$) in practice. Using this approximation, we can write,

$$J = \frac{1}{2\pi f_0}\sqrt{\int_0^\infty S(f)df} \tag{22}$$

6. Impact on Signal to noise ratio (SNR)

We have seen that phase noise and jitter can be thought of as equivalent. Loosely speaking, phase noise in oscillator (reference source) translate in the form of clock jitter in A/D converter. The corresponding equivalence is discussed already in earlier sections. Since jitter cause a random variation in the sampling instant of A/D converter, the SNR of the A/D output is impacted. Here, we derive the SNR as a function of the rms jitter J

For sinusoidal input signal,

$$\begin{aligned} v(t) &= v_0 \sin 2\pi f t \\ \frac{dv(t)}{dt} &= v_0 2\pi f \cos 2\pi f t \\ \frac{dv(t)}{dt}\|_{max} &= v_0 2\pi f \\ v_{err} &= 2\pi v_0 \Delta t \end{aligned} \tag{23}$$

Thus, the voltage error v_{err} due to a timing error Δt is proportional[7] to the latter itself. When the time variation considered is the rms[8] J, the signal amplitude variation is also the rms of voltage variance.

The SNR degradation (worst case figure in dB for the model described above) caused by this time variation (jitter) can thus be expressed as,

$$\begin{aligned} \text{SNR} &= 20\log_{10}\left(\frac{v_0}{v_{err}}\right) \\ &= 20\log_{10}\left(\frac{v_0}{2\pi f v_0 J}\right) \\ &= -20\log_{10}(2\pi f J) \end{aligned} \tag{24}$$

[7]The slope is normally known as the slew rate. $S_R = \left(\frac{dv}{dt}\right)_{\max}$

[8]Normally this being the practice

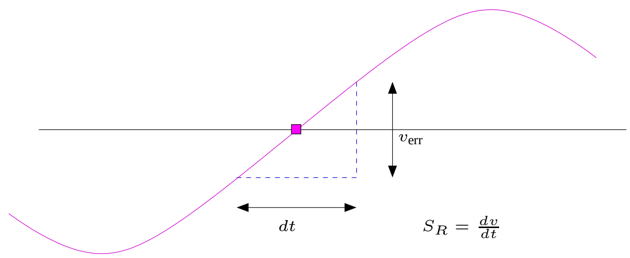

Figure 9. Clock Jitter produces an amplitude variation.The average mean squared error (rms) amplitude fluctation depend on the slew rate and average value (rms) of jitter

7. RMS Phase Jitter and Phase noise power spectrum

Usually, the phase noise PSD is specified at certain corner frequency offset Δf from the oscillator frequency f_c. As per definition, the power measured over $1Hz$ at $f_c + \Delta f$ provide the phase noise PSD value at the corner. Set of such discrete offset corners are used to characterize the phase noise.

When only, these corner points are provided (without explicitly defined PSD curve), it is assumed that the PSD response is like a piecewise logarithmic function of frequency offset. In other words, $L_i(f)$, PSD expressed in dB scale during the corner intervals $[f_i, f_{i+1})$ can be expressed mathematically as,

$$L_i(f) = b_i + \frac{b_i - b_{i+1}}{\log(f_i) - \log(f_{i+1})} \log(f) - \log(f_i) \tag{25}$$

where f is the arbitrary offset in the interval $f \in [f_i, f_{i+1})$

Using the slope[9] value a_i this can be simply expressed as,

$$L_i(f) = b_i + a_i \left(\log(f) - \log(f_i)\right) \tag{26}$$

where the $i-$th slope is,

$$a_i = \frac{b_i - b_{i+1}}{\log(f_i) - \log(f_{i+1})} \tag{27}$$

$$J = \frac{1}{2\pi f_c} \sqrt{2 \sum_{i=1}^{K} \int_{f_i}^{f_{i+1}} 10^{\frac{L_i(f)}{10}} df} \tag{28}$$

$$L(f) = \sum_{i=0}^{K-1} \left[a_i \left(\log(f) - \log(f_i) + b_i\right] \left[\Pi\left(f - f_i\right) \Pi\left(f - f_{i+1}\right)\right] \tag{29}$$

where $\log$ is the logarithm to base 10

The jitter (RMS phase jitter) can be calculated as,

[9] Using a logarithmic scale (nonlinear) on the frequency axis, the PSD can be viewed as linear piecewise affine functions.

$$J = \frac{1}{2\pi f_c}\sqrt{2\int_0^{\infty} 10^{\frac{L(f)}{10}}\, df} \tag{30}$$

For example, for a 3 point phase noise PSD spec, the rms phase jitter can be calculated as,

$$J = \frac{1}{2\pi f_c}\sqrt{2\left(\int_{f_1}^{f_2} 10^{\frac{L_1(f)}{10}}\, df + \int_{f_2}^{f_3} 10^{\frac{L_2(f)}{10}}\, df\right)} \tag{31}$$

A more accurate expression would be,

$$J = \frac{1}{2\pi f_c}\sqrt{2\left(\int_{f_1}^{f_2} 10^{\frac{L_1(f)}{10}}\left(\frac{\sin \pi\tau f}{\pi\tau f}\right)^2 df + \int_{f_2}^{f_3} 10^{\frac{L_2(f)}{10}}\left(\frac{\sin \pi\tau f}{\pi\tau f}\right)^2 df\right)} \tag{32}$$

From the corner specifications, the power (inner integrals) can be calculated explicitly.

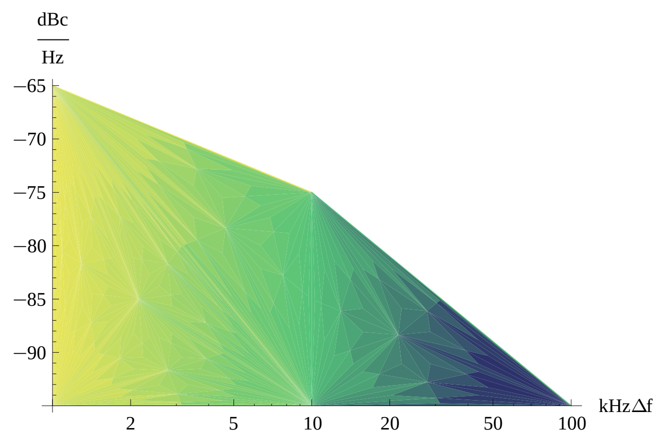

Figure 10. Phase Noise spectrum for the Analog devices ADC used in DVB-T: The $x-$axis is in $\log(f)$ scale. This specification is equivalent to Figure.11 with a different abscissa scale.

$$\begin{aligned}\int_{f_i}^{f_{i+1}} S_i(f)df &= \int_{f_i}^{f_{i+1}} 10^{\frac{L(f)}{10}}\, df \\ &= 10^{\frac{b_i}{10}} f_i^{\frac{-a_i}{10}} \int_{f_i}^{f_{i+1}} f^{\frac{a_i}{10}}\, df \\ &= 10^{\frac{b_i}{10}} f_i^{\frac{-a_i}{10}} \times \begin{cases} \log\left(f_{i+1}\right) - \log\left(f_i\right), & a_i = -10 \\ \left(f_{i+1}^{\left(1+\frac{a_i}{10}\right)} - f_i^{\left(1+\frac{a_i}{10}\right)}\right)\left(1+\frac{a_i}{10}\right), & a_i \neq -10 \end{cases}\end{aligned} \tag{33}$$

Here $L(f) = 10\log\left(S_i(f)\right)$ is the PSD specified in dBc/Hz scale
Figure.12 shows Phase noise power spectral density.

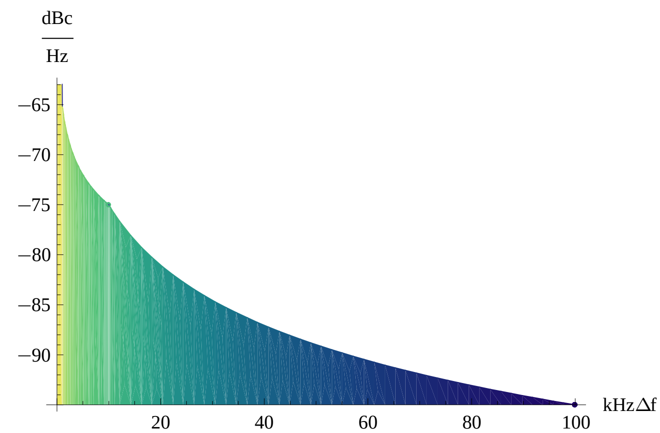

Figure 11. Phase Noise spectrum for the Analog devices ADC used in DVB-T: abscissa in linear scale: This is same as shown in Figure.10 with linear abscissa scale

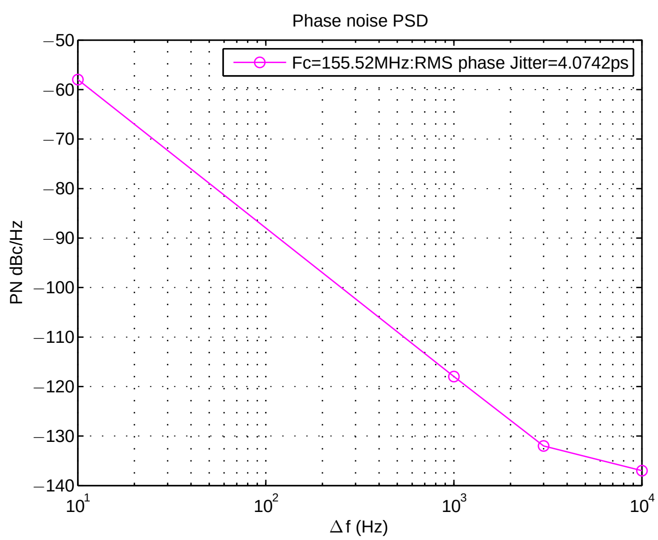

Figure 12. Phase Noise spectrum: Numerical Example-1

7.1. **Numerical Examples.** Using the mathematical formula described above, the average clock jitter (rms) can be calculated from a given phase noise power spectral density specification. Three example PSD specifications (See Figure 12, Figure 13,and Figure 14) are used. The average jitter obtained using the forumula is compared against that computed by the software (web based). The results are closely matching. The comparative figures are displayed in Table 7.1.

8. APPENDIX

The following matlab codes are used to compute rms phase noise jitter

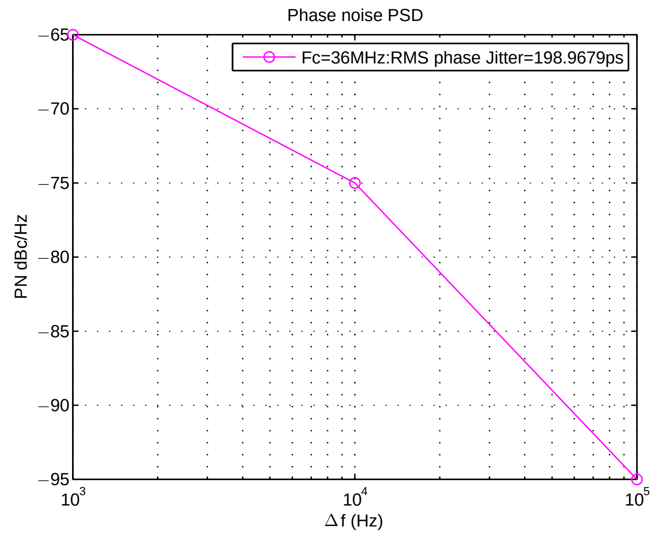

Figure 13. Phase Noise spectrum: Numerical Example-2

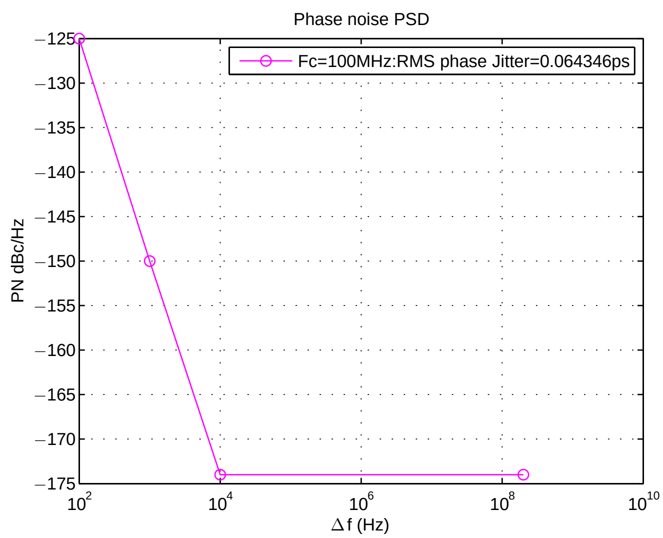

Figure 14. Phase Noise spectrum: Numerical Example-3

Table 2. Comparison of the jitter computed manually (mathematical formula) versus that obtained by the online software tool

PSD Spec	Jitter (Software)	Jitter(Computed)
Figure:-12	4.0774189 pico seconds	4.0742 pico seconds
Figure:-13	0.064288 pico seconds	0.064346 pico seconds
Figure:-14	189 pico seconds	198.9679 pico seconds

8.1. **Matlab Program to compute rms phase jitter from PSD.**

```
% This script compute the rms jitter from phase noise spec;
% Rethnakaran Pulikkoonattu
% ST Microelectronics (Genesis Microchip) www.st.com http://www.gnss.com

clear all;close all;
FigIndex=0;
disp('==============Example 1=============');
fc=44e6;
fc=155.52e6;
Scale=1/(2*pi*fc);
tau_rms=Scale;

f1=10;
f2=1000;
f3=3000;
f4=10000;
b1=-58;
b2=-118;
b3=-132;
b4= -137;

a1= (b2-b1)/(log10(f2)-log10(f1));
a2= (b3-b2)/(log10(f3)-log10(f2));
a3= (b4-b3)/(log10(f4)-log10(f3));

J1= (10^(b1/10))*( f1^(-a1/10) )* ...
    ( ( 1+ a1/10 )^-1 ) * ...
    ( f2^(1+a1/10)-f1^(1+a1/10));
J2= (10^(b2/10))*( f2^(-a2/10) )* ...
    ( ( 1+ a2/10 )^-1 ) * ...
    ( f3^(1+a2/10)-f2^(1+a2/10));
J3= (10^(b3/10))*( f3^(-a3/10) )* ...
    ( ( 1+ a3/10 )^-1 ) * ...
    ( f4^(1+a3/10)-f3^(1+a3/10));
J=Scale*sqrt(2*J1+2*J2+2*J3)

a=[a1 a2 a3];
f=[f1 f2 f3 f4];
b=[b1 b2 b3 b4];

FigIndex=FigIndex+1;
figure(FigIndex)
semilogx(f,b,'-mo');grid on;
title('Phase noise PSD');
xlabel('\Delta f (Hz)');
ylabel('PN dBc/Hz')
% legend(['Oscillator freq is ',num2str(fc/1e6),'MHz'])
```

```
legend(['Fc=',num2str(fc/1e6),'MHz:',...
    'RMS phase Jitter=',num2str(J*1e12),'ps'])

%====================New setup
disp('===============Example 2==============');
clear all;
fc=36e6;
Scale=1/(2*pi*fc);
tau_rms=Scale;
b1=-65;
b2=-75;
b3=-95;
f1=1e3;
f2=10e3;
f3=100e3;
a1= (b2-b1)/(log10(f2)-log10(f1));
a2= (b3-b2)/(log10(f3)-log10(f2));
if (1+ a1/10 ==0)
    %     J1= (10^(b1/10))*( f1^(-a1/10) );
    J1=(10^(b1/10))*( f1^(-a1/10) ) *(log(f2)-log(f1))
else
    J1= (10^(b1/10))*( f1^(-a1/10) )* ...
        ( ( 1+ a1/10 )^-1 ) * ...
        ( f2^(1+a1/10)-f1^(1+a1/10));
end
J2= (10^(b2/10))*( f2^(-a2/10) )*...
    ( ( 1+ a2/10 )^-1 ) *...
    ( f3^(1+a2/10)-f2^(1+a2/10))

rms_phase_jitter_deg=sqrt(2*J1+2*J2)*180/pi
J=Scale*sqrt(2*J1+2*J2)
a=[a1 a2 ];
f=[f1 f2 f3];
b=[b1 b2 b3];
jitter =[J1 J2]
figure(2)
semilogx(f,b,'-mo');grid on;
title('Phase noise PSD');
xlabel('\Delta f (Hz)');
ylabel('PN dBc/Hz')
legend(['Fc=',num2str(fc/1e6),'MHz:','RMS phase Jitter=',...
    num2str(J*1e12),'ps'])

disp('===============Example 3==============');
fc=100e6;
Scale=1/(2*pi*fc);
tau_rms=Scale;
```

```
f1=100;
f2=1000;
f3=10000;
f4=200e6;
b1=-125;
b2=-150;
b3=-174;
b4= -174;

a1= (b2-b1)/(log10(f2)-log10(f1));
a2= (b3-b2)/(log10(f3)-log10(f2));
a3= (b4-b3)/(log10(f4)-log10(f3));

J1= (10^(b1/10))*( f1^(-a1/10) )*...
    ( ( 1+ a1/10 )^-1 ) * ...
    ( f2^(1+a1/10)-f1^(1+a1/10));
J2= (10^(b2/10))*( f2^(-a2/10) )* ...
    ( ( 1+ a2/10 )^-1 ) *...
    ( f3^(1+a2/10)-f2^(1+a2/10));
J3= (10^(b3/10))*( f3^(-a3/10) )* ...
    ( ( 1+ a3/10 )^-1 ) * ...
    ( f4^(1+a3/10)-f3^(1+a3/10));
J=Scale*sqrt(2*J1+2*J2+2*J3)

a=[a1 a2 a3];
f=[f1 f2 f3 f4];
b=[b1 b2 b3 b4];


figure(3)
semilogx(f,b,'-mo');grid on;
title('Phase noise PSD');
xlabel('\Delta f (Hz)');
ylabel('PN dBc/Hz')
% legend(['Oscillator freq is ',num2str(fc/1e6),'MHz'])
legend(['Fc=',num2str(fc/1e6),'MHz:','RMS phase Jitter=',num2str(J*1e12),'ps'])
```

9. Acknowledgements

The author like to thank his colleagues Srevats Laxman, Min Wu, Harish Krishnan and Chin Chen for useful reviews and helpful discussions on this topic.

References

[1] Online tool to compute RMS phase jitter from phase noise PSD specification: www.raltron.com/cust/tools/osc.asp

[2] Noise, Oscillators, and Algebraic Randomness : From Noise in Communication Systems to Number Theory. Lectures in Physics Vol 550 , ed. M Planat (Springer-Verlag Heidelberg)

[3] Study report on "Characterisation of frequency and phase noise",Report 580, International Radio Consultative committee (CCIR), 1974,1978,1986 PP-142-150: Online PDF repreint available at http://tf.nist.gov/general/tn1337/Tn162.pdf

[4] Egan, William F. Practical RF system design. New York : Wiley-Interscience, 2003

E-mail address: ratnuu@gmail.com
E-mail address: rethnakaran.pulikkoonattu@gnss.com