Scaling analysis of the DSD variability at small spatial scales

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1 Introduction

Given the large number of raindrops in a given volume of rainfall, the rain drop size distribution (DSD hereinafter) is a convenient statistical way to summarize the variety of drop sizes encountered. The DSD reflects the microphysical processes at work in the clouds and during the fall of raindrops. From a remote sensing point of view, DSD is crucial to understand what a weather radar actually measures, and how to convert these measurements into values of rain rate, the variable of interest for many applications.

Because of the complex interactions between these microphysical processes and turbulence in the atmosphere, DSD (like precipitation in general) is strongly variable in space and time. This variability has an influence on radar measurements and their quantitative interpretation as rain rate estimates. There is however a lack of understanding of the DSD variability at small spatial scales, mainly because of a lack of adequate measurements. Some experiments (Miriovsky et al., 2004; Lee et al., 2009) have investigated this issue, but because of limited number of instruments involved and/or because of their different types, they were not conclusive in terms of the quantification of the spatial variability of the DSD at small scales.

A network of 16 optical disdrometers (Parsivel) has been deployed over a typical radar pixel (1x1 km$^2$) on EPFL campus in Spring 2009. In the present work, we use the scaling approach proposed by Sempere-Torres et al. (1994) and further generalized by Lee et al. (2004) in order to investigate the DSD variability at the radar pixel scale. Section 2 presents the methodology employed, while Section 3 describes the data used in the present analysis. Section 4 shows the main results and the conclusions are given in Section 5.

2 Methodology

2.1 Single-moment normalization

Sempere-Torres et al. (1994) proposed a general scaling formalism based on the normalization of the DSD by a single moment (of the DSD). Let $D$ [mm] be the equivolumetric diameter of a rain drop, and $N(D)$ [mm$^{-1}$m$^{-3}$] the DSD. Within the proposed scaling formalism, $N(D)$ can be expressed as

$$N(D, M_i) = M_i^\alpha g(x_1)$$

(1)

where $x_1 = DM_i^\beta$. $\alpha$ and $\beta$ are the (single-moment) scaling parameters. $g$ is called the single-moment generalized DSD. From Eq.(1), we have

$$M_n = C_{1,n}M_i^{\gamma_n}$$

(2)
with \( C_{1,n} = \int x_1^n g(x_1) \, dx_1 \) and \( \gamma_n = \alpha + \beta(n+1) \). The index 1 in \( C_{1,n} \) indicates that the single-moment normalization is considered. As indicated by the latter equation, \( \beta \) can be estimated as the slope of the linear regression of \( \gamma_n \) as a function of the moment order \((n+1)\) indeed. Using \( n = i \), we obtain the self-consistency constraints:

\[
\begin{align*}
C_{1,i} &= 1 = \int x_1^i g(x_1) \, dx_1 \\
\alpha + \beta(i+1) &= 1
\end{align*}
\]

According to Eq.(1), any moment of the DSD can be expressed as a power law of the reference moment \( M_i \), with prefactor and exponent being function of the scaling parameters \( \alpha \) and \( \beta \) as well as of the order \( n \) of the considered moment.

### 2.2 Double-moment normalization

Lee et al. (2004) have extended the DSD scaling approach and proposed a double-moment normalization. The DSD is then expressed as

\[
N(D, M_i, M_j) = M_j^{(j+1)/(j-i)} M_i^{(i+1)/(i-j)} h(x_2)
\]

where \( x_2 = D h_{i,j}(x_2) \). \( h \) is called the double-moment generalized DSD. From Eq.(5), we have

\[
M_n = C_{2,n} M_i^{(j-n)/(j-i)} M_j^{(n-i)/(j-i)}
\]

with \( C_{2,n} = \int x_2^n h(x_2) \, dx_2 \). The index 2 in \( C_{2,n} \) indicates that the double-moment normalization is considered. Using \( n = i \) and \( n = j \), we obtain the self-consistency constraints \( C_{2,i} = 1 \) and \( C_{2,j} = 1 \). According to Eq.(6), any moment of the DSD can be expressed as a double power law of the reference moments \( M_i \) and \( M_j \), with prefactor and exponent being function of the orders \( i \) and \( j \) of the reference moments as well as of the order \( n \) of the considered moment.

### 2.3 Application

The single/double-moment normalization of the DSD presented in the previous sections is used to investigate the variability of the DSD at small scales, in particular by analyzing the differences between the scaling parameters values and the generalized DSD functions \((g \text{ and } h)\) at different spatial scales. The reference moments used are the rain rate \( R \) (order about 3.67) for the single-moment normalization, and the third and fourth moment \((i = 3 \text{ and } j = 4)\) for the double-moment normalization.

### 3 Data

A network of 16 optical disdrometers (Parsivel) has been deployed over EFPL campus, covering a typical operational weather radar pixel (about \( 1 \times 1 \) km\(^2\)). Identical instruments have been deployed to avoid issues related to the difference in sampling uncertainty when using different types of disdrometers. The collected measurements can be affected by a possible bias, but we are interested in the relative variability in this work, so this would have no influence on the results of our analyses. Two contrasted rain events have been selected: a rather convective one which occurred on 1-2 September 2009 (rain rate peaks above 100 mm.h\(^{-1}\)), and a rather stratiform one which occurred on 1-2 May 2010 (rain rate peaks below 20 mm.h\(^{-1}\)).

The raw measured DSD spectra are collected every 30 s. To have enough drops collected for reliable estimation of the DSD, we will consider DSD data at a 60-s temporal resolution. The
Figure 1: Mean rain rate values (black) as well as min and max (grey) of the 16 stations, for the 2 studied events.

Figure 2: $\alpha$ and $\beta$ values for the convective (left) and stratiform (right) event at the point scale. The couple of values corresponding to the pixel size is marked by the red cross.

scaling analysis will be conducted for the entire events. The 2 following spatial scales will be considered: (1) the point scale (i.e., individual disdrometer measurements), and (2) the pixel scale (average of all 16 point measurements).

4 Results

4.1 Single-moment normalization

4.1.1 Scaling parameters

For the 2 studied rain events, Figure 2 presents the $\alpha$ and $\beta$ values derived from the collected data. Because of Eq.(4), $\alpha$ and $\beta$ values follow a line, and the upper left corner corresponds to a drop concentration control of the DSD, while the lower right corner corresponds to a size control of the DSD. Points from the convective event are more to the upper left than the ones from the stratiform event. The variability of the scaling parameters appears significant and is stronger for the convective event. The pixel values are relatively consistent (within the cloud of points) with the point values.
4.1.2 Single-moment generalized DSD

Figure 3 shows the generalized DSD \( g \) as a function of \( x_1 \) estimated at a given disdrometer (station 12) and at the pixel scale, for the convective event (the behaviour is similar for the stratiform event). The points do not superimpose on a single well defined curve, indicating that the single-moment scaling does not capture all the DSD variability. In order to have a functional form for \( g \), a gamma function can be fitted on these points using a moment method (e.g., Chapon et al., 2008):

\[
g(x_1) = \kappa_g x_1^\mu_g - 1 \exp (-\lambda_g x_1).
\]

Because of the self-consistency constraints, \( \kappa_g \) is a function of \( \mu_g \) and \( \lambda_g \), the shape and scale parameters. So only the last two will be considered in the following analyses.

The values of these 2 gamma parameters at the point and pixel scales are plotted in Figure 4. The pixel values are smaller than the point ones, with an average deviation of about 15 % for \( \mu_g \) and about 5 % for \( \lambda_g \), for both events. The sampling uncertainty associated with individual disdrometer measurements could explain some of this variability, but this issue is beyond the scope of this paper.

4.1.3 Double-moment generalized DSD

As in the previous section, the values of the gamma shape and scale parameters fitted to the double-moment normalization DSD \( h \) at the point and pixel scales are plotted in Figure 5. The average deviation between values at point and pixel scales is larger than for the single-moment normalization, with the pixel values being lower of about 20-30 % for \( \mu_h \) and about 15-20 % for \( \lambda_h \).

5 Conclusion

Using the scaling approach to analyze the DSD variability at the radar pixel scale, it has been shown that the scaling parameters and the gamma parameters fitted to the single- or double-normalization generalized DSD can be significant within 1×1 km². This variability should be taken into account when dealing with radar data which are integrated at such a scale.

The effect of the sampling uncertainty in the data collected, as well as the influence of the temporal scale considered (e.g., year, event, intra-event) will be investigated in future work.
Figure 4: Fitted $\mu_g$ (left column) and $\lambda_g$ (right column) values for $g$ at point and pixel scales, for the convective (top line) and stratiform (bottom line) events.

Figure 5: Fitted $\mu_h$ (left column) and $\lambda_h$ (right column) values for $h$ at point and pixel scales, for the convective (top line) and stratiform (bottom line) events.
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References


