Rigid body collision response problem

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Consider the situation (illustrated in figure 1) where two spheres collide. The two spheres have radius $R = 1m$ and mass $m_1 = m_2 = 1kg$. The sphere 1 is moving in the x direction with a linear velocity of 1m/s. The sphere 2 is not moving. I will use exactly the same notations as in *Iterative Dynamics with Temporal Coherence* (from Erin Catto). The goal is to find the velocity vector V^2 (this is not V square) after the collision using the computations from the paper of Erin Catto).

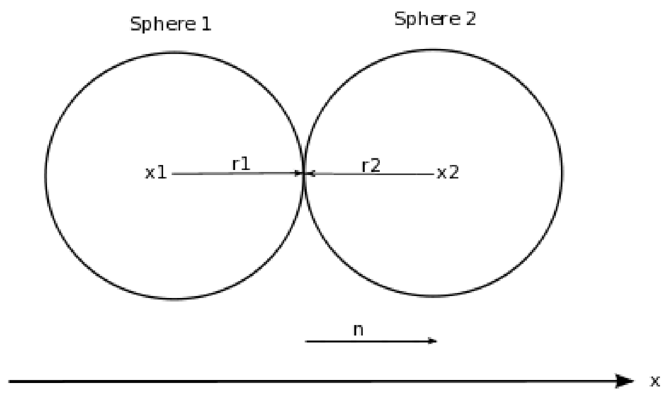

Figure 1: *A single contact between two spheres.*

According to our situation, the linear and angular velocities v and ω of both spheres are represented in the V^1 vector. The linear velocity vectors of our two spheres are :

$$\vec{v_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \vec{v_2} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

and the two spheres have no angular velocities. Therefore, the initial velocity vector V^1 is :

$$V^1 = \begin{pmatrix} \vec{v_1} \\ \vec{\omega_1} \\ \vec{v_2} \\ \vec{\omega_2} \end{pmatrix} = (1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)^T$$

The vectors $\vec{r_1}$ and $\vec{r_2}$ are vectors from the center of a sphere and the collision contact (see the figure at page 10 of Catto's paper) and $\vec{n}$ is the contact normal (from sphere 1 to sphere 2). Thus, we have :

$$\vec{r_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \vec{r_2} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \quad \vec{n} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \vec{r_1} \times \vec{n} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \vec{r_2} \times \vec{n} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Now we can compute the J matrix for the only constraint we have. According to equation (18) of the page 9 of Catto's paper, we have :

$$J = (-n^T \ \ -(r_1 \times n)^T \ n^T \ (r_2 \times n)^T) = (-1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0)$$

The M matrix (see page 13 of Catto's paper), contains the mass m and inertia tensor I (in world coordinates) of each body. Our matrix M will be exactly the same as in the paper with the two inertia tensors :

$$I_1 = I_2 = \begin{pmatrix} \frac{2}{5}mR^2 & 0 & 0 \\ 0 & \frac{2}{5}mR^2 & 0 \\ 0 & 0 & \frac{2}{5}mR^2 \end{pmatrix} = \begin{pmatrix} 0.4 & 0 & 0 \\ 0 & 0.4 & 0 \\ 0 & 0 & 0.4 \end{pmatrix}$$

and with the mass $m_1 = m_2 = 1$. According to page 14, we have the matrix B :

$$B = M^{-1}J^T = (-1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0)^T$$

Now if I assume that the penetration depth of my contact is zero (see page 10 of Catto's paper). It means that $\zeta = 0$. I also assume that the time step is $\Delta t = 0.5s$. There are no external forces and torques on both spheres, thus F_{ext} is the zero vector (see equation (29) page 13 of the paper). From the equation (35) page 14 of the paper, we have :

$$\eta = \frac{1}{\Delta t}\zeta - J(\frac{1}{\Delta t}V^1 + M^{-1}F_{ext}) = -\frac{1}{\Delta t}JV^1 = -2 \cdot (-1) = 2$$

Now, we are ready to solve the LCP problem. We want to compute λ such that the following equation holds :

$$JB\lambda = \eta$$

and because $JB = 2$ and $\eta = 2$, we have that $\lambda = 1$.

Now we will use the equation (33) page 14 of Catto's paper to find the final velocity vector V^2. According to this equation, we have :

$$M(V^2 - V^1) = \Delta t(J^T \lambda + F_{ext})$$

Therefore, we have :

$$V^2 = \Delta t(M^{-1} J^T \lambda + M^{-1} F_{ext}) + V^1 = \Delta t\ B\lambda + V^1$$

using the fact that $M^{-1}J^T = B$ and that $F_{ext} = 0$:

We can now compute V^2 :

$$V^2 \quad = \quad 0.5 \cdot \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot 1 + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (1)$$

Therefore after the contact both spheres will have the same linear velocity of 0.5m/s (which is half of the initial velocity of sphere 1) in the x axis direction. But I think that this is wrong because according to me, if one moving body b_1 (at velocity v) collides with another body b_2 that is not moving, then after the collision, the body b_1 will stop but the body b_2 will move with velocity v in the same initial direction of body b_1. We assume that both bodies have the same mass and that the collision is elastic.

So what is wrong ?