Reanalysis and Correction of Bed-Load Relation of Meyer-Peter and Müller Using Their Own Database

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Abstract: The pioneering predictor of fluvial bed-load transport rate proposed by Meyer-Peter and Müller in 1948 is still extensively used in basic research and engineering applications. A review of the basis for its formulation reveals, however, that an unnecessary bed roughness correction was applied to cases of plane-bed morphodynamic equilibrium. Its inclusion followed a flow resistance parameterization in terms of the Nikuradse roughness height, which has been shown (well after the publication of their work) to be inappropriate for the characterization of mobile bed rough conditions in rivers. Removing the unnecessary correction and incorporating an improved correction of the boundary shear stress due to sidewall effects allow elucidation of the most parsimonious form of the bed-load relation of Meyer-Peter and Müller that is dictated by their own data set. The new predictor is presented in terms of two alternative power law forms. These amended forms show that, in the case of lower-regime plane-bed equilibrium transport of uniform bed sediment, the new estimates of volume bed-load transport rates are less than or equal to half the values that would be obtained with the original relation of Meyer-Peter and Müller in the absence of the unnecessary bed roughness correction. The meticulous database and clear analysis of the original work of Meyer-Peter and Müller greatly aided the present writers in their reanalysis, which liberally uses the hindsight offered by 58 years of subsequent research.

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Introduction

A focus of current research on sediment transport in rivers, particularly in the case of gravel-bed streams, is on developing more accurate predictors of the bed-load transport rate. Estimating this rate usually involves relating it to mean characteristics of the driving force of the flow and the corresponding reach-averaged resistance properties of the bed. More than a century has passed since the introduction of the first mechanistic relation of this type by Du Boys in 1879 (Ettema and Mutel 2004), but still no formulation can be claimed to be of universal applicability. There is a lively debate, for instance, concerning the effects of turbulence and bed heterogeneities at micro- and macroscales on bed-load transport rates, but no agreement has been reached [see e.g., Ashworth et al. (1996); Cao and Carling (2002); Wilcock (2004)].

One of the formulas most widely used in laboratory and field investigations as well as in numerical simulations of bed-load transport is the empirical relation proposed by Meyer-Peter and Müller (1948); abbreviated to "MPM" below. This relation was derived from experiments carried out during a period of 16 years in the flume facilities of the Laboratory for Hydraulic Research and Soil Mechanics of the Swiss Federal Institute of Technology (ETH) at Zürich, Switzerland. It allows estimation of the bed-load transport rate in an open channel, as a function of the excess bed shear stress applied by the flowing water. MPM worked out their relation from a comprehensive experimental data set for equilibrium bed-load transport under steady, uniform flow that included 135 runs from ETH and 116 more runs from the set due to Gilbert (1914); abbreviated as "GIL" below. The data pertain to both sediment of uniform size and size mixtures, various values of sediment specific gravity, and cases both with and without the presence of bed forms. The original data of MPM were included in an internal report of ETH, but were not published until Smart and Jäggi's paper (1983) on bed-load transport on steep slopes. Review of the experimental techniques and data analysis carried out by MPM reveals a meticulous attention to detail.

The bed-load transport relation of MPM has been used extensively for almost 6 decades. With only very few exceptions, this usage has not been accompanied by detailed reanalysis of the formula itself, the data on which it is based, and its range of validity. Reanalysis to date has concentrated on extension of the formula to: (1) channels that are steeper than those of the experiments by MPM (Smart and Jäggi 1983; Smart 1984); and (2) poorly sorted sediment mixtures (Hunziker 1995; Hunziker and Jäggi 2002). The simplest and most common case to which the relation of MPM is typically applied, i.e., the transport of uniform sediment over a flat bed with a slope not exceeding 0.02, however, does not appear to have been revisited in the sole context of the original data used by MPM. Hence the question arises: Does the relation of MPM in fact fit the data used in its derivation? Answering this is of significance not only because the MPM bed-load transport predictor is commonly used for comparative purposes in basic research [see e.g., Abdel-Fattah et al. (2004); Barry

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et al. (2004); Bettess and Frangipane (2003); Bolla Pittaluga et al. (2003); Bravo-Espinosa et al. (2003); Defina (2003); Gaudio and Marion (2003); Knappen and Hulscher (2003); Martin (2003); Mikoš et al. (2003); Nielsen and Callaghan (2003); Ota and Nalluri (2003); Singh et al. (2004); Yang (2005)], but also because it is frequently used in engineering applications (it is one of the relationships available in HEC-6, a computer software used for sediment transport calculations that is very popular in the United States).

This paper is devoted to a thorough review of the basis for the formulation of the MPM relation, with special attention to the procedures by which bed-form and sidewall corrections were applied by MPM. Hindsight offered by the results of more current research on sediment transport and flow resistance in rivers reveals that (1) the form drag correction used by MPM in analyzing their data for plane-bed conditions is unnecessary; and (2) if this unnecessary correction is omitted, then the MPM bed-load transport relation needs to be modified in order to accurately reproduce the experimental observations used to derive it. The analysis presented here culminates in an amended form of the MPM relation for the case of lower-regime plane-bed equilibrium transport conditions. It is worth clarifying here that the goal of this paper is not to propose a new or improved universal predictor of the bed-load transport rate, but to correct the data analysis and results of MPM for the case of plane-bed bed-load transport in light of research results that have become available since the publication of their work.

It is equally important to point out that the present work is not the first one to conclude that the relation of MPM overpredicts bed-load transport under plane-bed conditions in the absence of a form drag correction. Credit must go to Hunziker, Jäggi, and Smart for first recognizing this (Hunziker 1995; Hunziker and Jäggi 2002; Smart and Jäggi 1983; Smart 1984). The database used in their analysis is, however, larger than that used by MPM alone. As a result, it is not readily apparent that the relation of MPM significantly overpredicts (in the absence of a form drag correction) when applied solely to the data (for plane-bed transport) originally used in its derivation. Nor is it apparent that the form drag correction is not necessary for the plane-bed data of MPM. In this paper: (1) the overprediction in the absence of a form drag correction is demonstrated in the narrow context of the plane-bed data used by MPM; (2) the fact that a form drag correction is unnecessary for the plane-bed data used by MPM is made evident; and (3) simpler modified forms of MPM which are faithful to the original data set are presented.

Fourteen references which use the original form of the MPM bed-load transport equation *without the form drag correction* of MPM are presented in the third paragraph of this "Introduction." All of these references were published subsequently to the works of Hunziker, Jäggi, and Smart aforementioned. It is the hope of the writers that the present work, combined with the contributions cited above, will finally lead to the recognition that (1) the MPM bed-load equation without the MPM form drag correction overpredicts plane-bed bed-load transport by a factor of about two; (2) the form drag correction of MPM is unnecessary for plane-bed conditions; and (3) a modified form of the MPM bed-load relation (with no form drag correction) that predicts significantly lower transport rates than the original MPM bed-load relation (when used with no form drag correction) should be used in the future for plane-bed conditions.

Empirical Bed-Load Transport Relation

Three data sets were used by MPM for the derivation of their bed-load transport relation; they are named here ETH-up, GIL-up, and ETH-nup. ETH-up ("up" is an abbreviation for "uniform sediment, plane bed") consists of the results for 52 runs, all pertaining to plane-bed transport of material of uniform size (32 runs with D_m=28.65 mm, and 20 runs with D_m=5.21 mm, where D_m=arithmetic mean diameter of the sediment) and with a constant value of submerged specific gravity of the sediment, R, of 1.68 ($R=\rho_s/\rho-1$, where ρ_s=density of the sediment and ρ=density of water). GIL-up similarly includes the results of 116 runs, all corresponding to plane-bed transport of material of uniform size (27 runs with D_m=7.01 mm, 69 runs with D_m=4.94 mm, and 20 runs with D_m=3.17 mm), with a constant value of R of 1.65. ETH-nup ("nup" is an abbreviation for "not necessarily both uniform material and plane bed") comprises the results of 83 runs, in many of which bed forms were present (channel aspect ratio, B/H, varied between 1.7 and 72.2, where B=channel width and H=water depth), the sediment consisted of size mixtures (D_m ranged between 0.38 and 5.21 mm, and D_{90}/D_m varied between 1.00 and 2.52, where D_{90}=particle size for which 90% of the sediment is finer by weight), and the value of R differed from 1.68 (R ranged between 0.25 and 3.22). The descriptor "not necessarily" is motivated by the observation that although Meyer-Peter and Müller (1948) indicated that bed forms were present in many of the runs in this set ETH-nup, they do not specify which of the runs had bed forms and which did not. Since the focus of the present work is on the transport of uniform material over a plane bed, most of the data analysis herein is performed using the sets ETH-up and GIL-up.

The relation proposed by MPM estimates the rate of bed-load transport in a river as a function of the shear force exerted by the flowing water over the sediment bed. This relation evolved over time. A first form of the relation was presented by Meyer-Peter et al. (1934); it was based solely on the sets ETH-up and GIL-up, for which bed forms were not observed and the sediment could be approximated as uniform in size. An empirical analysis of these data resulted in the following equation:

$$\frac{(\rho q_w')^{2/3} S}{D_m} = 17 + 0.40 \frac{[\rho(R+1) q_b]^{2/3}}{D_m} \tag{1}$$

where q_w'=volume discharge of water per unit channel width after including a correction for sidewall effects (details about which follow in the next section); S=slope of the energy grade line; and q_b=volume bed-load transport rate per unit channel width. Eq. (1) is not dimensionally homogeneous, and requires the use of SI units.

The data from ETH-up and GIL-up are presented in Fig. 1. The line of best fit shown in this figure was determined from a standard regression analysis; it is marginally different from Eq. (1), which Meyer-Peter et al. (1934) determined by eye. The slope of the line of best fit is thus 0.37 instead of the value of 0.40 proposed by MPM.

This first relation Eq. (1) was later modified (Meyer-Peter and Müller 1948) to include different values of specific gravity, as well as to extend the applicability to sediment mixtures. The modified expression is dimensionally homogeneous, and reduces to a functional relation between the bed-load transport rate and the shear force exerted by the water in terms of two well-known dimensionless numbers on sediment transport in rivers: the Einstein and the Shields numbers. It is presented here as Eq. (2), in

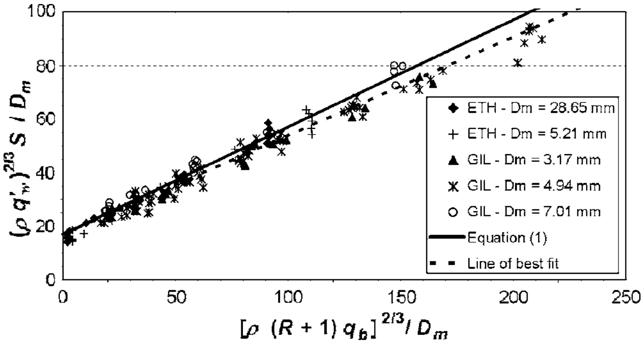

Fig. 1. Reproduction of data sets ETH-up and GIL-up in form of Eq. (1), for uniform material and lower-regime plane-bed equilibrium transport conditions

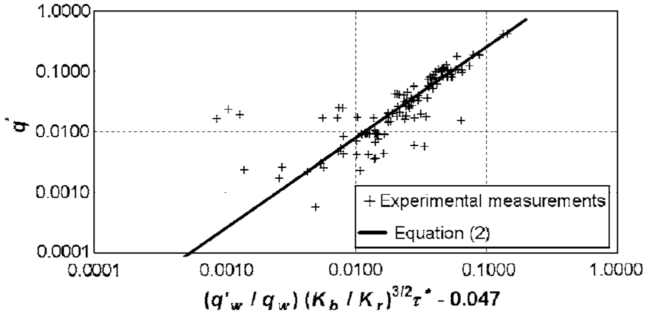

Fig. 2. Reproduction of data sets ETH-up and ETH-nup in form of Eq. (2), for normal flow conditions, with and without bed forms, and including sidewall and bed-form corrections of MPM

the dimensionless form first suggested by Chien (1954) that is completely equivalent to the original dimensioned Eq. (26) of Meyer-Peter and Müller (1948):

$$q^* = 8\left[\frac{q'_w}{q_w}\left(\frac{K_b}{K_r}\right)^{3/2}\tau^* - 0.047\right]^{3/2} \tag{2}$$

$$q^* = \frac{q_b}{\sqrt{RgD_m}D_m} \tag{3}$$

$$\tau^* = \frac{\tau_0}{\rho RgD_m} = \frac{HS}{RD_m} \tag{4}$$

In the above relations q^*=dimensionless volume bed-load transport rate per unit channel width (Einstein number); g=acceleration due to gravity; q_w=volume discharge of water per unit channel width (without any sidewall correction; recall that q'_w=volume discharge of water per unit channel width after correcting for sidewall effects); K_b=Manning–Strickler coefficient of roughness for the bed region; K_r=Manning–Strickler coefficient of bed roughness associated with skin friction only (i.e., after form drag due to bed forms has been excluded); τ^*=dimensionless boundary shear stress (Shields number); and τ_0=boundary shear stress applied by the water over the sediment bed under normal flow conditions for a wide open channel (without any sidewall correction). Separate experiments by MPM showed that the critical Shields number for the onset of sediment motion was about 0.030 (Meyer-Peter and Müller 1948), but an "effective" critical value of 0.047 was adopted instead in the formulation of Eq. (2) because it allowed a better fit of the data. The ratio of q'_w to q_w (where $q'_w \leq q_w$) represents the sidewall correction of MPM, while the ratio of K_b to K_r (where $K_b \leq K_r$) to the power of 3/2 represents the bed-form correction of MPM. Details about the sidewall and bed-form corrections of MPM follow in the next sections. These two correction factors are rarely included in recent publications referring the MPM bed-load transport relation, for reasons that are not necessarily clear.

All the results from the runs at ETH, i.e., ETH-up and ETH-nup, are presented in Fig. 2 in a plot of Einstein number, q^* versus Shields number, τ^*. In this figure the data have been reduced using the aforementioned bed form and sidewall corrections. One issue to highlight here is that MPM applied a bed-form correction even in cases for which bed forms were absent. The justification for doing that appears questionable [for more details, see discussion on p. 57 and Fig. 12 of Meyer-Peter and Müller (1948)]. MPM indicated that for a bed-load transport rate substantially greater than the threshold value for particle incipient motion, part of the shear stress applied by the flowing water over the bed is absorbed by the particles in bed-load transport. As a result, the effective force causing the movement of sediment is reduced, and a "bed-form correction" needs to be included even if no bed forms are observed. However, the meaning of this ratio K_b/K_r is interpreted in a different way in some classical books on sediment transport (Bogárdi 1974; Chang 1992; Garde and Ranga Raju 1985; Graf 1971; Raudkivi 1976; Simons and Şentürk 1977; Yang 1971). Four of these authors explicitly propose using a ratio K_b/K_r=0.5 when strong bed forms are present, and a ratio K_b/K_r=1.0 when bed forms are absent. Their interpretation of K_b/K_r is therefore strictly related to the effects of bed forms and not at all on the effects of particles moving in bed-load transport on the effective shear stress. It will be demonstrated in this paper that such a bed-form correction, or in a more general context a correction accounting for bed resistance in addition to that due to skin friction, is actually not required for the data pertaining to lower-regime plane-bed equilibrium transport of uniform sediment used by MPM.

Flow Resistance—Sidewall Correction

To determine the actual shear force exerted by the flowing water on the sediment bed, a separation of the effects due to the difference in roughness of the channel bed and sidewalls is needed. The sidewall correction of MPM cited above is based on the assumptions of uniform flow velocity throughout the cross section of the flow, equal energy slope on the bed and wall regions of the flow, and partitioning of the overall Manning–Strickler coefficient of roughness for a composite cross section, K, into a bed and a wall component, K_b and K_w, respectively. In other words, partitioning is carried out in terms of hydraulic radii. It is worth mentioning at this stage that the different Manning–Strickler coefficients of roughness used here, all denoted by the capital letter K, are equal to the inverse of the corresponding Manning's n in SI units, and are thus not at all the same parameters as the (Nikuradse or Kamphuis) bed roughness height, for which the notation k_s is used here. In addition, it should be recalled from the empirical relation proposed by Manning–Strickler for normal flow (Manning 1891; Strickler 1923) that a larger value of K should be interpreted as a smaller boundary resistance to the flow; hence, for a channel configuration like the one used by MPM in their flume experiments, $K_b < K$ for the gravel bed (rough boundary) while $K_w > K$ for the glass walls (smooth boundaries).

From the water continuity equation and the Manning–Strickler

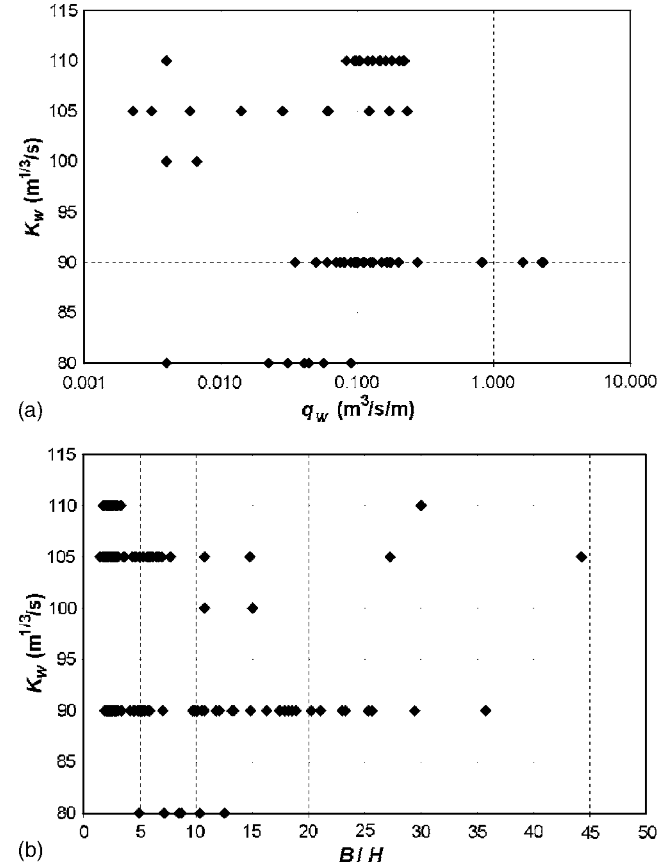

Fig. 3. Plots of Manning–Strickler coefficient of roughness for wall region, K_w, as function of: (a) water discharge per unit channel width, q_w; (b) channel aspect ratio, B/H. Data used for plot include results from ETH-up and ETH-nup. No correlation is apparent in either case.

relation (used to estimate K from the measured values of q_w, B, H, and S), applied in the case of a rectangular cross section with wetted perimeters B and $2H$ for the bed and wall regions, respectively, the coefficient of roughness for the bed region, K_b, may be computed as

$$K_b = \frac{KK_w B^{2/3}}{[BK_w^{3/2} + 2H(K_w^{3/2} - K^{3/2})]^{2/3}} \tag{5}$$

Accordingly, the value of the sidewall-corrected water discharge per unit channel width, q_w', can be obtained from

$$q_w' = q_w \frac{BK_w^{3/2}}{2HK_b^{3/2} + BK_w^{3/2}} \tag{6}$$

The computed value of q_w' and the measured value of q_w are then used in Eq. (2) to account for sidewall effects, such that the input value for the boundary shear stress represents the one effectively acting on the bed region alone. A major concern with this sidewall-correction methodology however, is that the coefficient of roughness for the wall region, K_w, is set in advance of the actual experiments on bed-load transport. Its value is thus independent of q_w and B/H (see Fig. 3), a result that does not seem to have a solid physical background. In this regard Hey (1979) and Yen (2002), among other researchers, state that the Darcy–Weisbach equation (Rouse 1946) for flow resistance has a stronger theoretical foundation, and its original formulation for pipe flow can be applied to open-channel flow in the following form:

$$f = \frac{8grS}{u^2} \tag{7}$$

where f=overall Darcy–Weisbach roughness coefficient for a composite cross section; r=hydraulic radius; and u=mean flow velocity. In such a formulation the value of f depends on the Reynolds number, R, which in the case of open-channel flow is computed by using a characteristic length of $4r$ ($\mathsf{R}=4ur/\nu$, where ν=kinematic viscosity of water).

The sidewall correction proposed by Vanoni and Brooks (Vanoni 1975) makes use of the Darcy–Weisbach formulation to estimate flow resistance, and is thus adopted here. The procedure again consists of partitioning the cross-section of the flow into two noninteracting parts, i.e., the bed and wall regions. As before, equal mean flow velocity and energy gradient in the bed and wall regions are assumed, so partitioning is in terms of hydraulic radii. All this translates into the relation given in Eq. (8) for iteratively calculating the roughness coefficient for the wall region, f_w, in the case of a smooth hydraulic boundary (Chien and Wan 1999)

$$\frac{\mathsf{R}}{f} = \frac{10^{\{[1/(2\sqrt{f_w})]+0.40\}}}{f_w^{1.5}} \tag{8}$$

From the water continuity equation and the Darcy–Weisbach relation applied to the case of a rectangular cross section with wetted perimeters B and $2H$ for the bed and wall regions, respectively, the roughness coefficient for the bed region, f_b, may be computed as

$$f_b = f + \frac{2H}{B}(f - f_w) \tag{9}$$

By combining Eqs. (7) and (9), one can estimate the value of the hydraulic radius on the bed region, r_b, and thus compute the corresponding shear velocity u_b^*

$$u_b^* = \sqrt{gr_bS} \tag{10}$$

as well as the sidewall-corrected shear stress τ_b

$$\tau_b = \rho(u_b^*)^2 \tag{11}$$

The bed shear stress computed following this procedure replaces the value of $(q_w'/q_w)\tau_0$ that represented the original sidewall correction of MPM in Eq. (2). This results in the following improvements: not only is the expected dependence of flow resistance on q_w and B/H taken into account (see Fig. 4), but also the dependence on the Reynolds number, R.

Flow Resistance—Bed-Form Correction

The other correction factor included in Eq. (2) allows removal of the form drag component of the channel bed resistance, i.e., the component due to bed forms, so quantifying that component of the shear force of the water that actually causes bed-load transport. In a more general framework, this bed-form correction can be interpreted as that accounting for all other factors besides skin friction acting on immobile grains (e.g., grain shape, bed structure, moving bed-load particles, bed forms, channel-scale morphology, surface waves, etc.) that contribute to flow resistance.

As opposed to the procedure for extracting sidewall effects, the bed-form correction proposed by MPM consists of a partitioning of the slope of the energy grade line, S, into one related to form resistance and another associated with skin friction only, the latter being denoted as S_r. The implicit assumption is that the

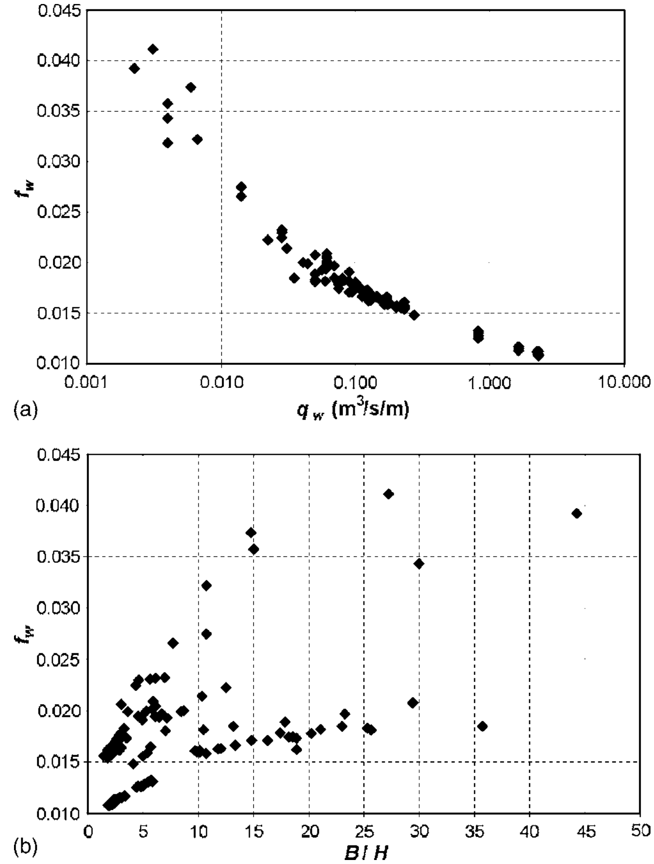

Fig. 4. Variation of Darcy–Weisbach roughness coefficient for wall region, f_w, as function of: (a) water discharge per unit channel width, q_w; (b) channel aspect ratio, B/H. Data used for plot include results from ETH-up and ETH-nup.

hydraulic radius used in the Manning–Strickler relation has the same value for both form resistance and skin friction. As a result, the bed-form correction could be represented as

$$\frac{S}{S_r} = \left(\frac{K_b}{K_r}\right)^2 \tag{12}$$

However, in the evolution of the empirical approach from Eq. (1) to Eq. (2), based on separate tests carried out to determine the criterion for particle incipient motion, MPM determined an alternative correction of the form

$$\left(\frac{S}{S_r}\right)^{2/3} = \left(\frac{K_b}{K_r}\right)^{4/3} \tag{13}$$

Hence one is left with the ambiguous conclusion that the bed-form correction of MPM is expressed in terms of the ratio K_b/K_r, raised to a power of either 4/3 or 2. Meyer-Peter and Müller (1948) resolved this ambiguity with recourse to the data, finding that an exponent of 3/2, i.e., a value in between 4/3 and 2, gave the best fit of the experimental data as per Eq. (2). This choice of exponent has the added advantage of allowing a Froude similarity collapse that was successfully used to derive a universal bed-load transport law based on runs that included uniform material and size mixtures, different values of specific gravity of the sediment, and cases both with and without the presence of bed forms.

It is not the purpose of this paper to question the validity of this procedure for bed-form correction, but to show that such a correction is not required when bed forms are absent. More specifically, it is shown that the use of what is in hindsight an inappropriate parameterization for flow resistance in these cases led Meyer-Peter and Müller to the erroneous inclusion of an additional correction (reduction) of the effective bed shear stress in the form of $(K_b/K_r)^{3/2}$ for the plane-bed transport conditions. The parameterization for flow resistance used here became available only well after the work of MPM was published (Meyer-Peter and Müller 1948).

Meyer-Peter and Müller (1948) stated that the "evaluation of the measurements of the Laboratory confirms for all tests the fully developed turbulence, so that the coefficient of particle roughness may be calculated with sufficient accuracy from"

$$K_r = \frac{26}{D_{90}^{1/6}} \tag{14}$$

Eq. (14) is based on the results of the famous set of experiments on pipe flow by Nikuradse [NACA (1950), which is an English translation of a document dating from 1933], who found that the bed roughness height, k_s, can be estimated as follows:

$$k_s = D_{90} \tag{15}$$

Van Rijn (1984) and Millar (1999), for example, have used results from several tests carried out for a range of grain sizes including both gravel and sand particles to demonstrate that this one-to-one scaling is misleading. Chien and Wan (1999) reinforce this claim by arguing that the procedure by which Nikuradse glued uniform sand particles to the pipe walls resulted in an effective roughness smaller than the actual size of the sand particles. An alternative, better tested parameterization is given by Kamphuis (1974), who concluded that for large values of the ratio of H to D_{90} [$H/D_{90} \geqslant 10$; see Kironoto and Graf (1994)] and fully developed turbulent flow ($u_b^* k_s/\nu \geqslant 70$; see Schlichting 1979), a better approximation for k_s is

$$k_s = 2D_{90} \tag{16}$$

124 out of the 135 runs comprising data sets ETH-up and ETH-nup complied with the criterion of $H/D_{90} \geqslant 10$, and all 135 tests corresponded to completely rough turbulent flow. As a result, the evaluation of the bed roughness height in the form of Eq. (16) is used from here onward, unless explicitly stated in a different form. It should be noted here that the justification for amending Eq. (15) to Eq. (16) was not yet available to Meyer-Peter and Müller when they developed their relations of 1934 and 1948.

It is of interest to note as well that Jäggi (1984) reached a conclusion that is nearly equivalent to that of Kamphuis (1974). In particular, he modified the constant in Eq. (14) from 26 to a value between 20 and 22. If the form of Eq. (14) is retained but the transformation $D_{90} \rightarrow k_s$ is made, Eq. (14) now takes the form

$$K_r = \frac{26}{k_s^{1/6}} \tag{17}$$

Introducing Eq. (16) into Eq. (7) yields

$$K_r = \frac{23.2}{D_{90}^{1/6}} \tag{18}$$

i.e., a form very close to that obtained by Jäggi (1984).

It is well known the vertical profile of flow velocity for turbulent open-channel flow may be represented by a logarithmic law, leading to the Keulegan relation (Keulegan 1938) for depth-averaged flow velocity, u, in the hydraulically rough regime. Making the transformations $H \rightarrow r_b$ and $u^* \rightarrow u_b^*$ (where u^*=shear

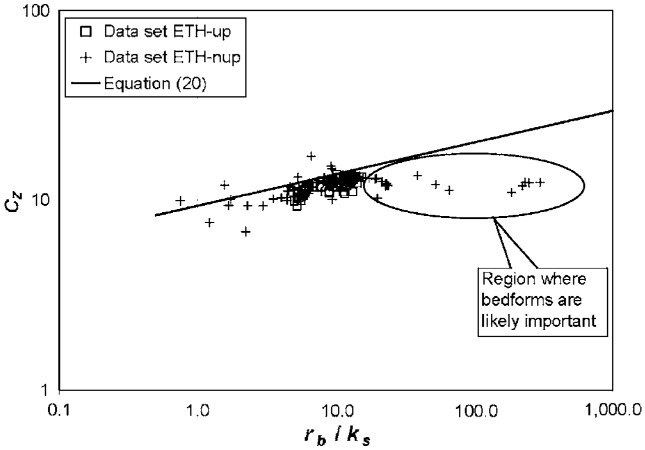

Fig. 5. Comparison of flow resistance measurements for data sets ETH-up and ETH-nup against Eq. (20)

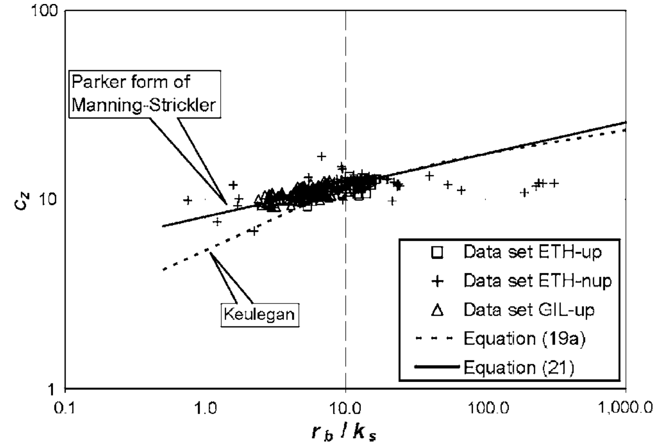

Fig. 6. Comparison of flow resistance measurements for data sets ETH-up, GIL-up, and ETH-nup against logarithmic law Eq. (19a) of Keulegan and Parker's form Eq. (21) of Manning–Strickler

velocity), in order to incorporate the sidewall correction of Vanoni and Brooks presented in the previous section, the Keulegan relation can be expressed as

$$C_Z = \frac{1}{\kappa} \ln\left(11 \frac{r_b}{k_s}\right) \tag{19a}$$

where C_Z=dimensionless Chezy resistance coefficient, is defined in the following way:

$$C_Z = \frac{u}{u_b^*} \tag{19b}$$

and κ=0.4 denotes the Karman constant. It should be noted that in analogy to the Manning–Strickler coefficient of roughness, a larger value of C_Z implies a smaller resistance to the flow. It is also important to note that the original flow resistance relation proposed by Keulegan (1938) was based on the definition of k_s given in Eq. (15), whereas here it is based on the improved definition of k_s given in Eq. (16).

The relation (14) used by MPM to evaluate flow resistance can be reduced to the following form with the aid of Eq. (16) for the bed roughness height, and of Eq. (19b) for the definition of C_Z

$$C_Z = 9.32\left(\frac{r_b}{k_s}\right)^{1/6} \tag{20}$$

It has long been known that the Manning–Strickler power form of bed resistance given in Eq. (20) yields results that are very similar to those of the Keulegan form in Eq. (19a). One must ask, however, if Eq. (20) really does provide an accurate evaluation of bed resistance in the absence of bed forms. In order to test this, Fig. 5 presents a comparison of the estimates provided by Eq. (20) against the measured values of the data set ETH-up for which bed forms were absent, as well as the data set ETH-nup for which bed forms were present in some cases. It is seen in this figure that nearly all the data for C_Z plot below Eq. (20), implying that the boundary resistance is higher than predicted by this Eq. (20). In other words, with sidewall effects removed through $H \rightarrow r_b$ and $u^* \rightarrow u_b^*$, an observed value of C_Z that is smaller than the one estimated via Eq. (20) would mean that a component of flow resistance due to bed forms should be present in addition to the one associated with skin friction alone. Fig. 5 thus indicates a measurable effect of bed forms not only in the results of data set ETH-nup for which bed forms may have been present, but also for data set ETH-up for which bed forms were verifiably absent. The implication is a contradiction; that is, correcting the boundary shear stress in Eq. (2) to extract only that portion obeying Eq. (20) results in a measurable correction for bed forms that are not actually present.

MPM argue somewhat unconvincingly that even in the absence of bed forms, the resistance offered by a mobile plane bed may be larger than that of a static plane bed, so accounting for the discrepancy in Fig. 5. While this might in fact be true under certain regimes of bed-load transport, it is not true in the case of the data sets without bed forms used by MPM to derive their relation. The fact that Eq. (20) does indeed underpredict plane-bed resistance (i.e., overpredict C_Z) can be demonstrated with reference to the fit of Manning–Strickler type to the Keulegan relation developed by Parker (1991):

$$C_Z = 8.10\left(\frac{r_b}{k_s}\right)^{1/6} \tag{21}$$

where k_s is given by Eq. (16) of Kamphuis (1974) rather than by Eq. (15) of Nikuradse (NACA 1950). A comparison of the estimates obtained with the power law Eq. (21) versus: (1) the data set ETH-up for which bed forms were absent; (2) the data set ETH-nup for which bed forms were present in some runs; and (3) the data set GIL-up for which bed forms were absent, is given in Fig. 6. Also included in this figure is the logarithmic law Eq. (19a) of Keulegan in which k_s is evaluated from Eq. (16). It is seen that Eq. (21) provides an excellent fit of the data from the cases for which bed forms were not observed, while the experimental values of C_Z remain, as expected, smaller than those predicted by the power law Eq. (21) for the cases in which bed forms may have been present. The direct inference from this is that a resistance relation based on skin friction only, i.e., Eq. (21), is adequate and sufficient to explain the data for which bed forms were absent. That is, no bed-form correction is needed to characterize the results of sets ETH-up and GIL-up. Given that $K_b \leq K_r$ in the original formulation of MPM, dropping the bed-form correction means the whole curve presented in Fig. 2 is shifted to the right. As a result, the parameters used to fit an equation to the data of Fig. 2 must necessarily differ from those of Eq. (2), i.e., the form due to MPM. The resulting reanalysis is presented in the next section.

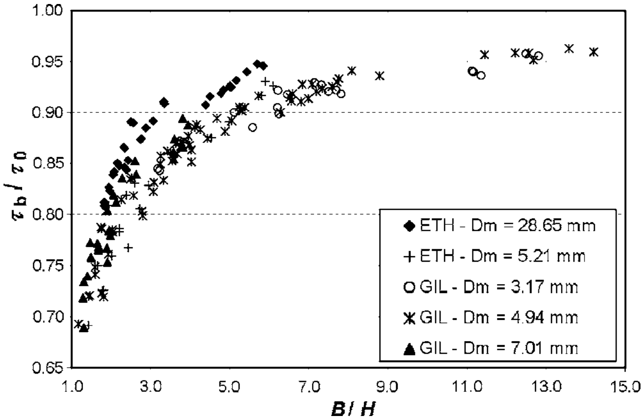

Fig. 7. Results of Vanoni–Brooks sidewall correction for MPM and GIL data sets in which bedforms were not observed, as function of channel aspect ratio, B/H

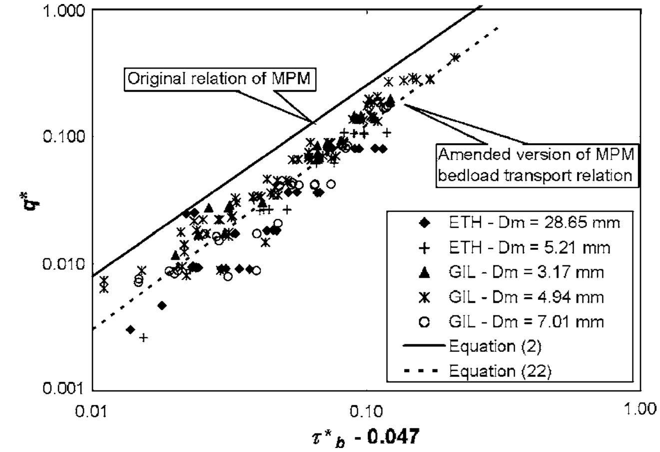

Fig. 8. Comparison of original empirical relation Eq. (2) proposed by MPM, and its amended version Eq. (22) recommended for case of lower-regime plane-bed normal flow equilibrium transport conditions

Summarizing, the use by MPM of a parameterization of skin friction that does not properly account for flow resistance, i.e., Eq. (14) or Eq. (15), was the source of their need to include a bed-form correction for cases in which bed forms were verifiably absent. In this regard, Fig. 6 shows that by using an appropriate flow resistance relation, i.e., Eq. (21) with the aid of Eq. (16), it is found that for the sets ETH-up and GIL-up: (1) bed forms were not observed; and (2) there is no discernible difference on the bed resistance of a mobile bed versus a static bed.

Reanalysis of Data and Amended MPM Bed-Load Transport Relation

Data sets ETH-up and GIL-up for which bed forms were absent are now reanalyzed in the form of Eq. (2), but with: (1) the Vanoni–Brooks sidewall correction; and (2) no correction for bed forms. All the experimental information used subsequently corresponds to lower-regime plane-bed equilibrium transport conditions, and uniform material in the gravel-size range only.

Fig. 7 presents the results of implementing the sidewall correction as per Vanoni and Brooks (Vanoni 1975). It can be easily seen that no correction may be necessary for channel aspect ratios, B/H, larger than 5.0. In many if not most experiments carried out in flume facilities, however, smaller values of B/H prevail. In such cases, omitting the sidewall correction might result in a significant overprediction of the bed-load transport rate. Based on the data from MPM for instance, it is seen in Fig. 8 that most of the tests correspond to a dimensionless excess shear stress (after applying the Vanoni–Brooks sidewall correction) ranging between 0.02 and 0.10. If no sidewall correction is applied for this range of shear stress, the estimate of the bed-load transport rate would be larger by a factor of 1.27–1.66 for the 65% of the experiments in which the effect of wall drag is at least 10% of the boundary shear stress (according to Fig. 7). Moreover, an overprediction factor of 1.45–2.10 would be obtained for the 39% of the experiments in which sidewall effects account for at least 15% of the boundary shear stress (according to Fig. 7). Hence, a proper channel-averaged evaluation of bed-load transport under controlled laboratory conditions should consider, as a minimum requirement, splitting the estimate of the boundary shear stress into bed and wall components, with only the first one used in the analysis of bed-load transport.

Working with values of the boundary shear stress acting on the bed region, τ_b, that have been corrected for sidewall effects with the Vanoni–Brooks method but not corrected for nonexistent form drag, two new power laws of the form of Eq. (2) have been developed. The first is presented as Eq. (22) below

$$q^* = 4.93(\tau_b^* - 0.0470)^{1.60} \tag{22}$$

where

$$\tau_b^* = \frac{\tau_b}{\rho R g D_m} \tag{23}$$

It keeps the same value of 0.0470 used by MPM for the critical Shields number, while a new value of 1.60 for the exponent is obtained from a statistical fitting of the experimental results. The second is presented as Eq. (24) below

$$q^* = 3.97(\tau_b^* - 0.0495)^{1.50} \tag{24}$$

It keeps the same exponent of 1.50 used by MPM, whereas the “effective” critical Shields number of 0.0495 is obtained from a statistical fitting of the data. In either case, the final outcome of the reanalysis is a line of best fit that gives estimates of the bed-load transport rate that are smaller than the ones predicted with Eq. (2) by a factor of 2.0–2.5. This is shown in Fig. 8, where the famous relation of MPM appears as an upper envelope of a data set that is much better fit by either of the alternatives Eqs. (22) or (24).

It is again emphasized here that the goal of the present paper is not to propose a new or improved universal predictor of the bed-load transport rate. Rather, the goal is to (1) highlight the fact that the form drag correction of MPM is unnecessary in the context of the plane-bed transport data used to derive the MPM bed-load transport relation; and (2) modify the MPM bed-load transport relation into a simpler form that uses no form drag correction for plane-bed conditions.

Hunziker and Jäggi (2002) have previously indicated that bed-load transport rates are overpredicted when the MPM relation is applied to sediment mixtures. They attributed this to the fact that the condition of equal mobility for all grain sizes does not always hold, and as a result mobile-bed armor can develop in gravel-bed streams. A similar remark about the lack of accuracy of the MPM relation was given by Smart and Jäggi (1983) and Smart (1984), both in reference to the prediction of bed-load transport rates for bed slopes steeper than 0.02.

In the specific case of uniform bed sediment, Hunziker (1995) stated that the procedure used by MPM to correct for bed forms resulted in a form drag that is too large. The reason for this is the inaccurate parameterization of skin friction used by MPM, i.e., Eq. (14). This result is consistent with the earlier results of Jäggi (1984), Smart and Jäggi (1983), and also with the reanalysis presented here in the context of Eq. (20). So Hunziker (1995) decided to use the alternative bed-form correction proposed by Yalin and Scheuerlein (1988). This correction was intended to account not only for bed-form effects, but to consider also the influence of particles moving in bed-load transport. The boundary shear stress is thus "bed-form corrected" even for cases in which bed forms are absent. Hunziker (1995) then proceeded to perform a new regression analysis of data from ETH for experiments on bed-load transport of uniform gravel. This set included: (1) only the part of the data from ETH used by Meyer-Peter and Müller (1948) to develop their original relation (i.e., ETH-up, but not GIL-up); and (2) the data from Smart and Jäggi (1983). As a result, Hunziker (1995) derived the following improved MPM relation, i.e., Eq. (12) quoted in Hunziker and Hunziker and Jäggi (2002)

$$q^* = 5.00(\tau^*_{H-\mathrm{YS}} - 0.05)^{1.50} \tag{25}$$

where $\tau^*_{H-\mathrm{YS}}$=dimensionless boundary shear stress, after including the same sidewall correction procedure used by MPM (Meyer-Peter and Müller 1948) but now implementing the bed-form correction of Yalin and Scheuerlein (1988) instead of the original bed-form correction used by MPM. This modified form of the MPM relation, i.e., Eq. (25), predicts transport rates that are 0.42–0.57 times that of the original relation (2) when applied to the range of boundary shear stresses covering the data sets ETH-up and GIL-up reanalyzed here.

Eq. (25) is similar to Eqs. (22) and (24), and the main reason for the downward adjustment in transport rate offered by Hunziker (1995) is similar to that offered in the present work: MPM overcorrected for form drag. The valuable conclusion of Hunziker (1995) is, nevertheless, incomplete and clouded by extraneous factors. It is clouded by the implication that the reevaluation of the MPM relation depends in some way on the data of Smart and Jäggi (1983), so the problem may not necessarily be with MPM's analysis itself. It is incomplete in the sense that it indicates that some form of correction for form drag, the one according to Yalin and Scheuerlein (1988), is still required even for data without bed forms. The essential points of the present work are as follows: (1) in contradistinction to MPM, Hunziker (1995) and Hunziker and Jäggi (2002), no form drag correction whatsoever is needed or should be used when analyzing the data without bed forms used by MPM; and (2) when the original data for plane beds used by MPM and only that data, i.e. without inclusion of other data sets such as Smart and Jäggi (1983), are reanalyzed without the unnecessary form drag correction, the result is a regression relation that predicts not more than half of the load that would be predicted by the original MPM bed load Eq. (2) in the absence of the MPM form drag correction.

When applied to data sets ETH-up and GIL-up that correspond to lower-regime plane-bed transport of uniform bed material, the error norm of the bed-load transport rates predicted with any of the two amended forms of the MPM relation presented in this paper is smaller than the one obtained with the modified relation of Hunziker (1995). The improvement on the prediction is particularly important for dimensionless transport rates ranging between 10^{-3} and 10^{-1}. In this range, the error norm associated to Eq. (22) for instance, is one third smaller than that related to Eq. (25).

Conclusions

The bed-load equation of Meyer-Peter and Müller (1948) includes a form drag correction that is intended to account for flow resistance of the stream bed due to factors other than skin friction (e.g., bed forms or moving bed-load particles). MPM introduced this correction after finding that a flow resistance relation for skin friction, based on Strickler (1923) and Nikuradse (NACA 1950), underpredicted the overall bed resistance measured in their flume experiments. It is demonstrated here with recourse to only the data pertaining to plane-bed equilibrium transport of uniform sediment used by MPM, however, that this form drag correction is unnecessary for plane-bed conditions and should be dropped. Since the form of the MPM bed-load transport equation itself that resulted from their data analysis is critically dependent upon this unnecessary form drag correction, a reanalysis of the data indicates a substantially revised form of MPM.

Put simply, this revision can be described as follows in the context of lower-regime plane-bed transport. The original MPM formulation makes no error in obtaining a curve fit for the bed-load transport rate. Instead, their curve fitting is forced to correct one error (including nonexistent form drag) by adding another (increasing the coefficient in the bed-load transport relation). So if the practitioner is to use the original MPM bed-load relation (with a coefficient that is about twice as high as it should be), then the (unnecessary and incorrect) form drag correction of MPM must be used in order to obtain predictions that are faithful to the plane-bed data used by MPM. The practitioner will find it much easier to instead use one of the two modified MPM bed-load relations proposed here (which for the most part involve a significantly lowered coefficient) in the absence of any form drag correction. The modified formulation is equally faithful to the original data, is simpler to use, and has a firmer scientific basis.

Hunziker (1995) had previously realized the existence of a problem with the analysis done by MPM. He attempted to "fix" (i.e. improve the validity of) the MPM equation by: (1) adding a new set of data to the regression analysis [the one due to Smart and Jäggi (1983)]; and (2) muting, but not eliminating the MPM correction for form drag. The reanalysis presented in this paper is different, in the sense that it does not attempt to "fix" the MPM equation, but instead results in its rederivation. By using the same data used by MPM for runs without bed forms, and only that data, it is shown here that no correction for form drag is needed for these runs. More specifically, it is shown that the resistance relation satisfied for the mobile-bed experiments without bed forms used by MPM is identical to that observed for a static bed composed of the same material.

In 1948 Meyer-Peter and Müller would not have had access to the revised resistance relations that demonstrate that no form drag correction was needed in their analysis of data on plane-bed transport. Here their data have been reanalyzed in precisely the way that Meyer-Peter and Müller would have done it had they had access to the relevant information.

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Notation

The following symbols are used in this paper:

B = channel width (m);
C_Z = dimensionless Chezy resistance coefficient;
D_m = arithmetic mean diameter of sediment (m);
D_{90} = particle size for which 90% of sediment is finer by weight (m);
f = overall Darcy–Weisbach roughness coefficient for composite channel cross section;
f_b = Darcy–Weisbach roughness coefficient for bed region;
f_w = Darcy–Weisbach roughness coefficient for wall region;
g = acceleration due to gravity (m/s^2);
H = water depth (m);
K = overall Manning–Strickler coefficient of roughness for composite channel cross section ($m^{1/3}/s$);
K_b = Manning–Strickler coefficient of roughness for bed region ($m^{1/3}/s$);
K_r = Manning–Strickler coefficient of bed roughness associated with skin friction only ($m^{1/3}/s$);
K_w = Manning–Strickler coefficient of roughness for wall region ($m^{1/3}/s$);
k_s = (Nikuradse or Kamphuis) bed roughness height (m);
n = Manning's coefficient of roughness ($s/m^{1/3}$);
q_b = volume bed-load transport rate of sediment per unit width ($m^3/s/m$);
q_w = volume discharge of water per unit width ($m^3/s/m$);
q_w' = volume discharge of water per unit width, including correction for sidewall effects ($m^3/s/m$);
q^* = dimensionless volume bed-load transport rate per unit width (Einstein number);
R = submerged specific gravity of sediment;
R = Reynolds number;
r = hydraulic radius (m);
r_b = hydraulic radius for bed region (m);
S = slope of energy grade line;
S_r = component of energy slope related to skin friction;
$t^*_{H\text{–YS}}$ = dimensionless boundary shear stress corrected according to Hunziker (1995);
u = mean flow velocity (m/s);
u^* = shear velocity (m/s);
u_b^* = shear velocity for bed region (m/s);
κ = Karman constant;
ν = kinematic viscosity of water (m^2/s);
ρ = density of water (kg/m^3);
ρ_s = density of sediment (kg/m^3);
τ_b = boundary shear stress for bed region (Pa);
τ_0 = boundary shear stress for hydraulically wide-open channel flow (Pa);
τ^* = dimensionless boundary shear stress (Shields number); and
τ_b^* = sidewall-corrected dimensionless boundary shear stress.

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