A large neighborhood search algorithm for the vehicle routing problem with time windows

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Given a fleet of vehicles assigned to a single depot, the vehicle routing problem with time windows (VRPTW) consists of determining a set of feasible routes to deliver goods to a set of customers while minimizing, first, the number of vehicles used and, second, total mileage. Each customer must be visited exactly once by a vehicle within a prescribed time interval. A route starts from the depot and visits in order a sequence of customers before returning to the depot. It is feasible if the total amount of goods delivered to the visited customers does not exceed the vehicle capacity and if it respects the time window of each visited customer. The cost of a route is given by the sum of the traveling costs between the consecutive locations visited along the route.

On the one hand, several exact approaches for the VRPTW have been developed in the past fifteen years. Among them, branch-and-price approaches (Desrochers *et al.*, 1992, Kohl *et al.*, 1999, Irnich and Villeneuve, 2003, Feillet *et al.*, 2004, Chabrier, 2006, Jepsen *et al.*, 2006, Desaulniers *et al.*, 2006) have produced the best results, mostly because of the quality of the lower bounds yielded by the column generation method. On the other hand, a very large number of heuristic approaches have also been proposed (see the survey papers of Bräysy and Gendreau, 2005a,b), including a wide variety of metaheuristics such as tabu search (Cordeau *et al.*, 2001), evolutionary and genetic algorithms (Berger *et al.*, 2003), large neighborhood search (Shaw, 1998), variable neighborhood search (Rousseau *et al.*, 2002, Bräysy, 2003), and multi-phase approaches (Homberger and Gehring, 2005). However, to our knowledge, no heuristic approaches taking advantage of the power of branch-and-price have been proposed in the literature.

In this paper, we present such an approach, namely, a large neighborhood search algorithm

Algorithm 1 Large neighborhood search for the VRPTW

1: Build an initial solution s of value $z(s)$
2: $s^* \leftarrow s$, $z^* \leftarrow z(s)$
3: **repeat**
4: Choose a neighborhood definition strategy
5: Apply this strategy for defining the neighborhood to explore
6: Explore this neighborhood using a branch-and-price heuristic
7: Update the current solution s
8: **if** $z(s) < z^*$ **then**
9: $s^* \leftarrow s$, $z^* \leftarrow z(s)$
10: **until** at least one stopping criterion is met

that relies on a heuristic branch-and-price method for neighborhood exploration. To ensure diversification during the search, this approach uses different procedures for defining the neighborhood to explore at each iteration. The framework of this approach is presented in Section 1, the neighborhood definition procedures are exposed in Section 2, and the branch-and-price heuristic is described in Section 3. Finally, in Section 4, we discuss the computational experiments conducted.

1 Large neighborhood search framework

A large neighborhood search algorithm is an iterative process that destroys at each iteration a part of the current solution using a chosen neighborhood definition procedure and reoptimizes it in hope of finding a better solution. The pseudo-code of the proposed algorithm is given in Algorithm 1. In Step 1, an initial solution is built using a very fast greedy heuristic. In Step 4, a procedure used to define the neighborhood to explore at the current iteration is randomly chosen as in Ropke and Pisinger (2004) according to weights that assign a high probability of being selected to the most efficient procedures. A neighborhood is defined by a subset of customers that are disconnected from their current routes. The resulting reoptimization problem thus corresponds to the original VRPTW where parts of the routes in the current solution are fixed. This restricted VRPTW is solved in Step 6 using a branch-and-price heuristic. The iterative process is stopped in Step 10 when a maximum number of iterations is reached or when a time limit is exceeded.

2 Neighborhood definition procedures

Three different procedures can be used to define the neighborhood to explore at each iteration. Each procedure stops when a given number of customers is selected. Starting with an initial customer selected at random, the first procedure (similar to the one proposed by Shaw, 1998) selects at each iteration an additional customer based on a measure of spatio-temporal proximity from the current set of selected customers. The second procedure was introduced in Rousseau *et al.* (2002). It starts by choosing the customer yielding the largest detour (supplementary distance traveled to cover a customer along its route) and a few of its preceding and succeeding customers on its route. Then, at each iteration, the customer closest (according again to a proximity measure) to the set of already selected customers is chosen and a few preceding and succeeding customers. Compared to the previous procedure, this one ensures that more than isolated customers are disconnected from their routes. Finally, a third procedure simply selects all customers that are currently visited within a given time slice. This procedure corresponds to the first one except that the spatial dimension is omitted from the proximity measure.

To emphasize the search of solutions with a minimal number of vehicles, we also use a strategy that imposes an upper bound on the number of vehicles that can be used at each iteration of the large neighborhood search algorithm. To ensure the respect of this bound, it is possible (bearing a large penalty) to omit visiting some customers.

Finally, to avoid long computational times at each iteration, the time window of each selected customer is reduced according to its time of visit in the current solution and the time windows of the other selected customers.

3 Column generation heuristic

At each iteration of the large neighborhood search, the VRPTW restricted to the selected neighborhood can be modeled as a set partitioning problem. We propose to solve this problem using a heuristic branch-and-price approach, that is, a heuristic column generation approach embedded in a heuristic branch-and-cut search tree, where the column generation subproblem is an ESPPRC defined on a restricted network which guarantees that the fixed parts of the routes in the current solution remain untouched. This branch-and-price heuristic is an adaptation of the exact approach proposed by Desaulniers *et al.* (2006) who succeeded to solve to optimality 5 of the remaining 10 open Solomon's (1987) benchmark instances with 100 customers. In their approach, a sequence of column generators with varying solution times is used at each iteration to generate negative reduced cost columns. At each iteration, the first generator invoked is a tabu search algorithm that rapidly finds negative reduced cost columns in most iterations. When it fails to do so, an attempt is made to generate such columns using a dynamic programming algorithm with an aggressive dominance procedure and/or applied to a network of reduced size. If failure occurs again, then the same dynamic programming algorithm, this time with an exact dominance procedure and applied to the full network, is called upon to ensure the optimality of the solution process. Note that this last option should be avoided as much as possible because it is very time consuming. Hence, in the heuristic version of the algorithm, we simply omit it, yielding a premature halt of the column generation process at each node of the branch-and-bound tree. Furthermore, this process is also stopped when an insufficient decrease of the objective function is observed over a predefined number of iterations.

The approach of Desaulniers *et al.* (2006) also includes the generation of cutting planes, namely, the subset row inequalities for three customers introduced in Jepsen *et al.* (2006). A very limited number of these cuts are added in the branch-and-price heuristic. Finally, as opposed to Desaulniers *et al.* (2006) who applied branching decisions on arc flow variables, we impose decisions on the path flow variables and explore the search tree using a depth-first strategy without backtracking.

4 Computational experiments

To evaluate the efficiency of the proposed methodology, we will perform computational tests on the well-known Solomon's (1987) benchmark problems which involve 100 customers, and also on the Homberger's instances which contain up to 1000 customers. These tests are in progress and their results will be presented at the conference.

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