A Bi-level Network Flow Model for Planning HazMat Shipments[♣]

Lucio Bianco, *Massimiliano Caramia*, *Stefano Giordani*

Dipartimento di Ingegneria dell'Impresa
Università di Roma "Tor Vergata"
Via del Politecnico 1, I-00133 Roma, Italy
{bianco,caramia, giordani}@disp.uniroma2.it

1. Introduction

The transportation of hazardous materials (hazmats), though may be classified among the most general freight transport issues, is an activity that presents extremely typical characteristics which make its planning, management and control a particularly complex task. What differentiates hazmat shipments from the transportation of other materials is the risk associated with an accidental release of hazardous materials during transportation. To reduce the occurrence of dangerous events it is necessary to provide appropriate answers to safety management associated with dangerous goods shipments.

Risk assessment and hazmat shipments planning are two of the main research fields in hazmat transportation. Risk is the primary ingredient that distinguishes hazmat transportation problems from other transportation problems. In the literature, a lot of work has already been done in risk assessment, by modeling risk probability distribution over given areas, for example, taking into account the risk related to the carried object and the transport modality (Abkovitz et al. 1984) and the environmental conditions (Patel and Horowits, 1994). There are several excellent review articles addressing the literature related to modeling of risk for hazmat transportation; however, there is no universally definition of risk. In this work we refer to the traditional definition of risk over a link, that is the societal risk defined as the product of the population along the link within the neighborhood and the probability of an accident (Erkut and Verter, 1998).

The main issue of hazmat shipments planning is routing hazmat shipments, that involves a selection among the alternative paths between origin-destination pairs. From a carrier's perspective, shipment contracts can be considered independently and a routing decision needs to be made for each shipment, which we call the local route planning problem. At the macro level, hazmat routing is a "many to many" routing problem with multiple origins and an even greater number of destinations. In the sequel, we refer to this problem as global route planning.

The local routing problem is to select routes between a given origin-destination pair for a given hazmat, transport mode, and vehicle type. Thus, for each shipment order, this problem focuses on a single-commodity and a single origin-destination route plan. Since these plans are often made without taking into consideration the general context, certain links of the transport network tend to be overloaded with hazmat traffic. This could result in a considerable increase of accident probabilities on some road links as well as leading to inequity in the spatial distribution of risk.

Transport costs remain as the carriers' main focus. In contrast, the government has to consider the global problem by taking into account all shipments in its jurisdiction. This leads to a harder class of problems that involve multi-commodity and multiple origin-destination routing decisions.

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Moreover, besides the minimization of the total risk imposed on the public and environment, a government agency may need to consider promoting equity in the spatial distribution of risk. This becomes crucial in the case in which certain population zones are exposed to intolerable levels of risk as a result of the carriers' routing decisions.

Therefore in the global route planning for hazmat shipments, the main problem is that of finding minimum risk routes, while limiting and equitably spreading the risk in any zone in which the transportation network is embedded. As a matter of fact, risk equity has to be taken into account also whenever it is necessary to carry out several hazmat shipments from a given origin to a given destination. In this situation, the planning effort has to be devoted to distribute risk uniformly among all the zones of the geographical crossed region. This concept is well defined in (Keeney, 1980), where a measure of the collective risk is determined with explicit reference to the equity.

Hazmat local route planning has attracted the attention of many OR researchers, while the global route planning problem has attained relatively little attention in the literature. The results in this latter area include the works of Gopalan et al. (1990), Lindner-Dutton et al. (1991) and Marianov and ReVelle (1998). The works of Akgün et al. (2000), Dell'Olmo et al. (2005) and Carotenuto et al. (2007) on the problem of finding a number of spatially dissimilar paths between an origin and a destination can also be considered in this area. For a complete survey on hazmat logistics the reader is referred to (Erkut et al., 2007).

2. Contribution of the paper

In this paper, we give a more general formulation than those present in the literature for the hazmat shipment global route planning. The main advances presented are:

- admittance of more than one commodity; this gives a more realistic and general framework to study where different kinds of hazmats can be considered;
- flow based formulation as opposed to path based formulations in the literature; this helps in solving the problem without iteratively computing paths and avoiding each route search to be biased by the previously found paths;
- use of *k*-splittability constraints on the flow, acting as the number of paths to be used at most to route each commodity from its origin to its destination;
- proposal of a bi-level optimization problem, with total risk and equity as objective functions.

The problem we consider is defined as follows. A set of hazmat shipments has to be routed over a transportation network in order to transport a given amount of hazardous materials from specific origin points to specific destination points with the aim of minimizing the total risk of the shipments and spreading the risk equitably over the geographical region in which the transportation network is embedded. Let the transportation network be represented as a directed network $G = (N, A)$, with N and A being the set of n nodes, and the set of m links (arcs) of the network, respectively. Let s_i and t_i be respectively the source node (origin point) and the sink node (destination point), and let d_i be the amount of hazmat to be shipped from s_i to t_i. Representing the risk induced over the population of a link as a (linear) function of the hazmat flow on that link, the problem can be modeled as multi-commodity network flow problem, where a specific commodity i is associated with each couple $(s_i; t_i)$ of source-sink nodes.

Before illustrating the model formulation we introduce some definitions and notations. We start by considering the maximum concurrent flow problem. We are given a network $G = (N, A)$, directed or undirected, with arc capacities $u_a > 0$, $a \in A$, and a set C of (hazmat) commodities, with $|C| = c$. Each commodity $i \in C$ is associated with a certain amount of demand d_i, a source node $s_i \in V$, and a destination node $t_i \in N \backslash \{s_i\}$. The goal is to maximize the fraction α of demands that it is possible to route concurrently over the network, without violating arc capacity constraints.

Define an integer value k_i for each commodity $i \in C$ as a limit for the number of paths that can be used at most to bring flow from origin node s_i to destination node t_i. We will refer to this as the *maximum concurrent k-splittable flow problem*.

Being $FS(v)$ and $BS(v)$ respectively the forward and backward stars of each node $v \in N$, and being x_{ai} a non-negative variable used to express the flow, for commodity $i \in C$, on arc $a \in A$, a formulation of the *splittable* maximum concurrent problem is

$$\alpha^* = \max \alpha$$

$$\alpha \leq 1 \tag{1}$$

$$\sum_{i \in C} x_{ai} \leq u_a, \quad \forall a \in A \tag{2}$$

$$\sum_{a \in FS(v)} x_{ai} - \sum_{a \in BS(v)} x_{ai} = \begin{cases} \alpha d_i, & v = s_i, \forall i \in C \\ 0, & \forall v \in N \setminus \{s_i, t_i\}, \forall i \in C \\ -\alpha d_i, & v = t_i, \forall i \in C \end{cases} \tag{3}$$

$$x_{ai} \geq 0, \quad \forall a \in A, \forall i \in C \tag{4}$$

Constraint (1) says that α is a fractional number. Constraints (2) ensure that the whole flow routed on each arc does not exceed the capacity, while constraints (3) require the satisfaction of a fraction α of demands and impose the conservation of flow at nodes for each commodity. The objective function, therefore, maximize the common fraction α of flow for each commodity.

For the *k-splittable* variant (e.g. see Caramia and Sgalambro (2006, 2007)), we have to limit the number of paths used by each commodity in order to send flow from the source to the sink: we refer to those paths as *active* paths. To this aim in the arc-flow formulation we have to consider the number of active paths. We can do this by considering separately the arc-flow variables referring to each one of the k_i paths that can be used at most by each commodity $i \in C$, and by associating an arc-flow support binary variable to each one of the arc-flow variables. We will denote flow variables with x_{aih} and support variables with δ_{aih}, $\forall a \in A, i \in C, h \in \{1, ..., k_i\}$.

Given a commodity $i' \in C$, for a fixed $h' \in \{1, ..., k_i\}$, we can refer to the subset of flows associated with i' and h', say $fl_{i'h'}$, as a *flow layer* of the given network, whose state is identified by the values assigned to variables $x_{ai'h'}$, $\forall a \in A$. In this way, any assignment of flows on the network will be the outcome of the superimposition of all the flows on layers fl_{ih}, $\forall i \in C, h \in \{1, ..., k_i\}$. Within a certain layer $fl_{i'h'}$, an arc a is said to be *active* if the related variable $\delta_{ai'h'}$ has value equal to 1. The flow support binary variables δ are then used to guarantee that the flow assigned to any flow layer goes along exactly one directed path from the source to the sink of the related commodity, imposing constraints on δ variables that forbid the flow bifurcation. Here follows the complete arc-flow formulation for the k-splittable flow:

$$\alpha^* = \max \alpha$$

$$\alpha \le 1 \quad (5)$$

$$\sum_{i\in C}\sum_{h=1}^{k_i} x_{aih} \le u_a, \quad \forall a\in A \quad (6)$$

$$\sum_{h=1}^{k_i}\left(\sum_{a\in FS(v)} x_{aih} - \sum_{a\in BS(v)} x_{aih}\right) = \begin{cases} \alpha d_i, & v = s_i, \forall i\in C \\ 0, & \forall v\in N\setminus\{s_i,t_i\}, \forall i\in C \\ -\alpha d_i, & v = t_i, \forall i\in C \end{cases} \quad (7)$$

$$x_{aih} \le \delta_{aih} u_a, \quad \forall a\in A, \forall i\in C, \forall h\in\{1,...,k_i\} \quad (8)$$

$$\sum_{a\in FS(s_i)} \delta_{aih} \le 1, \quad \forall i\in C, \forall h\in\{1,...,k_i\} \quad (9)$$

$$\sum_{a\in FS(v)} \delta_{aih} - \sum_{a\in BS(v)} \delta_{aih} = 0, \quad \forall v\in N\setminus\{s_i,t_i\}, \forall h\in\{1,...,k_i\} \quad (10)$$

$$x_{aih} \ge 0, \quad \forall a\in A, \forall i\in C, \forall h\in\{1,...,k_i\} \quad (11)$$

$$\delta_{aih} \in \{0,1\}, \quad \forall a\in A, \forall i\in C, \forall h\in\{1,...,k_i\} \quad (12)$$

As in the splittable model, constraints (6) ensure that the total flow crossing each arc *a* is less than or equal to the arc capacity. Constraints (7) guarantee that the same fraction of demand is served for all the commodities. With respect to the limited splittability of flows, and the presence of at most a single path bringing flow from s_i to t_i on each flow layer, forcing constraints (8) are used to guarantee the activation of δ_{aih}, whenever any quantity of flow is assigned to the corresponding arc-flow variable x_{aih}, and constraints (9) and (10) forbid the bifurcation of the flow from the origin to the destination. The latter two set of constraints guarantee that, on each layer, starting from the source node of the related commodity, it is possible to decompose flows in a flow along a single simple path from the source to the sink, plus possibly some circulations, whose presence does not affect the value of the objective function.

Once defined the arc-flow formulation for the *k*-splittable flow, we discuss the bi-level formulation proposed for the studied hazmat transportation problem. Note that *k* in such a context models the (maximum) number of different paths related to a commodity (i.e., an origin-destination couple). Let $s(x)$ be a function that models the risk on each arc: the latter for simplicity can be assumed linearly dependent on the flow x_{ai} assigned to arc *a* related to commodity *i*. With the following *upper level formulation* we look for risk equity, while with the *lower level formulation* we look for total risk minimization using at most k_i different routes for each hazmat commodity *i*.

Upper level formulation

$$\begin{cases} \lambda^* = \min \lambda \\ \sum_{i\in C} s(x_{ai}) \le \lambda, \quad \forall a\in A & (13) \\ \sum_{i\in C}\sum_{a\in A} s(x_{ai}) \le q R_{tot}^* & (14) \\ \sum_{a\in FS(v)} x_{ai} - \sum_{a\in BS(v)} x_{ai} = \begin{cases} d_i, & v = s_i, \forall i\in C \\ 0, & \forall v\in N\setminus\{s_i,t_i\}, \forall i\in C \\ -d_i, & v = t_i, \forall i\in C \end{cases} & (15) \\ x_{ai} \ge 0, \quad \forall a\in A, \forall i\in C & (16) \end{cases}$$

Lower level formulation $\begin{cases} R^*_{tot} = \min \sum_{i \in C} \sum_{a \in A} \sum_{h=1}^{k_i} s(y_{aih}) \\ (\mathbf{y}, \boldsymbol{\delta}) \in \Omega & (17) \\ y_{aih} \le l\, x^*_{aih}, \quad \forall a \in A, \forall i \in C, \forall h \in \{1,....,k_i\} & (18) \\ y_{aih} \ge 0, \quad \forall a \in A, \forall i \in C, \forall h \in \{1,....,k_i\} & (19) \end{cases}$

The *upper level formulation* models the problem of assuring an equitable distribution of the risk over the network by finding a multi-commodity hazmat flow of limited total risk that minimizes the maximum risk λ allowed on each arc of the network. Constraints (13) say that the risk induced over the population of each arc cannot be greater than λ. Constraint (14) states that the total risk over the network for all the commodities cannot be grater than $q \cdot R^*_{tot}$, where R^*_{tot} is a parameter whose value is offered by the optimal value of the lower level formulation. Finally, constraints (15) impose the conservation of flow at nodes for each commodity.

The *low level formulation* models the problem of minimizing the total risk over the network induced by a multi-commodity hazmat *k*-splittable flow. Expression (17) is the implicitly representation of the *k*-splittability flow constraints (Ω is the set of feasible solutions for the *k*-splittability flow problem with α fixed to 1), while constraints (18) say that the flow variable of this model, namely y_{aih}, should not exceed the capacity value $l \cdot x^*_{ai}$, where x^*_{ai} is the optimal flow value of the upper level program. This shows how the latter problem has no limitations on flow values, since it can be interpreted as a way of dimensioning of the network. Note that both in the upper level and in the lower level formulations two scale parameters appear, i.e., q and l, respectively. They play an important role in the tuning of the model, since they act as stretching parameters to allow a sensitivity analysis related to solution existence and diversification. In the following, we illustrate how we have designed the overall system functionality.

We start with $R_{tot} = \infty$ and $\lambda = \infty$ and solve the upper level program. The model returns a solution that corresponds to arc network capacities for the lower level program. At this point we fix $l = 1 + \beta$ and find the *k*-splittable flow of minimum total risk R^*_{tot}. This latter value is passed to the upper level program, where we fix $h = 1 + \beta$ and find a new splittable flow; this solution is again used to feed the lower level program where l is decreased by ε. The process keeps iterating (at each iteration $h = h - \varepsilon$, and $l = l - \varepsilon$) until the overall risk of the network can be decreased. The values of β and β are made experimentally and in general are in between 0 and 1, and they depend on the topology and on the size of the network. The intensity of the parameter ε also depends on the scenario, but the smaller is, the higher the probability of a convergence to high quality solutions.

Computational results on a case study with real data belonging to a region in the centre of Italy, and on a set of synthetic instances seem promising, also when compared with known models based on finding dissimilar paths.

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