3.1 General equations

The train movement is essentially a mobile of mass *m* with a single degree liberty. It's described by a scalar Newton-equation.

$$\sum_{i=1}^{n} F_{\mathrm{i}} = m^{*} a \qquad (3.1)$$

Forces produced in the train: forces from motors or from mechanical brake and the forces undergone by the trains: proper frictions and localized forces (slope, frictions in tunnels or curves).

The proper frictions have three parts: a constant, a part proportional to the speed and a part proportional to the square of speed.

$$F_{\mathrm{f}} = A + Bv + Cv^{2} \quad [\mathrm{N}] \qquad (3.5)$$

Please take care if numeric values are given for speeds in [m/s] in [mph] or in [km/h]. The curves on the figures 3.3 to 3.4 are quoted in forces relatives to train weight. Note that the hauled trains have different curves than locomotives, which open the way in the air.

1 UIC car about 1960 (SNCF, CFF, DB, FS)
2 Corail car 1975 (SNCF), Eurofima 1980 (DB, FS, ÖBB, …) or VUIV 1985 (CFF)
3 Light car 1940 (CFF)
3 adding 20 [N/t] BOB car on rack section.

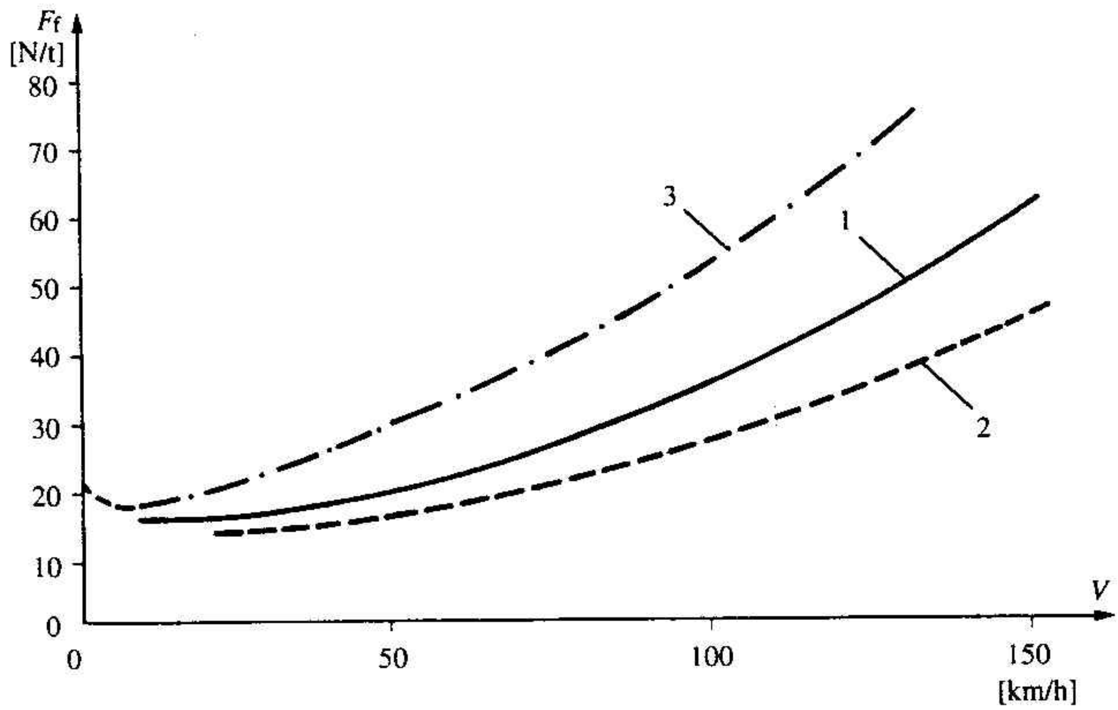

Fig. 3.4 Movement resistance for passenger trains.

1 locomotive Ae 6/6 (CFF): CoCo of 120 t
2 locomotive 9001 (SNCF): BB of 80t
3 locomotive 6001 (SNCF): CC of 120 t
4 locomotives Am 4/6 (CFF) : 1BoBo1 of 93 t, Re 460 (CFF) : BoBo of 84 t (———)
5 locomotive 2D2 (PO)
6 locomotive BBB (FS)
7 locomotive Re 4/4 II (CFF) : BoBo of 80 t
8 locomotive Re 6/6 (CFF) : BoBoBo of 120 t
9 motor-car BOB on rack
10 motor-car BOB on adherence
11 articulated motor-car TSOL or Stadtbahn B

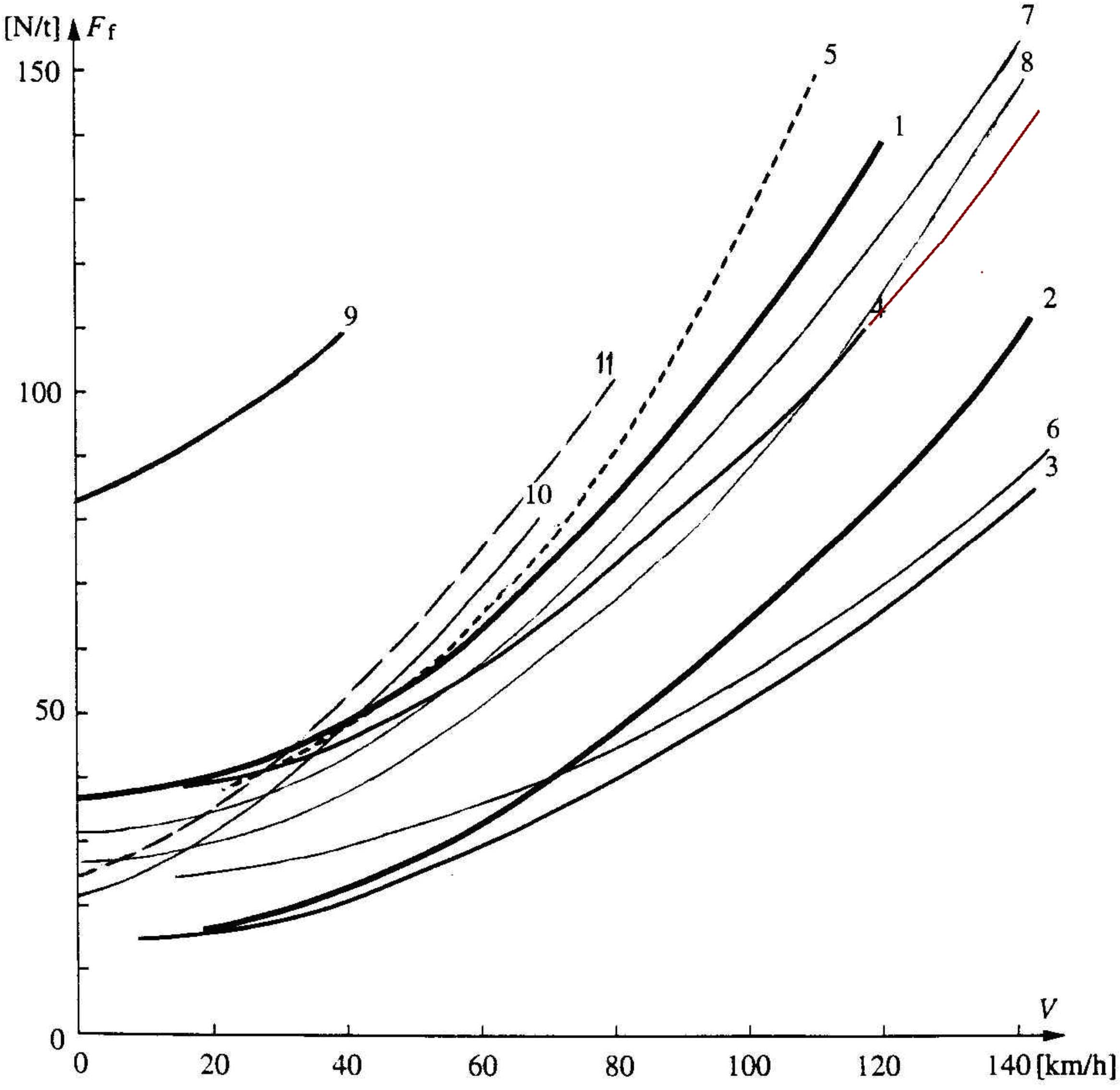

Fig. 3.3 Movement resistance of motor vehicles.

The freight trains have different characteristic, because the different forms end the non-flat wall increase the aerodynamic frictions with turbulences.

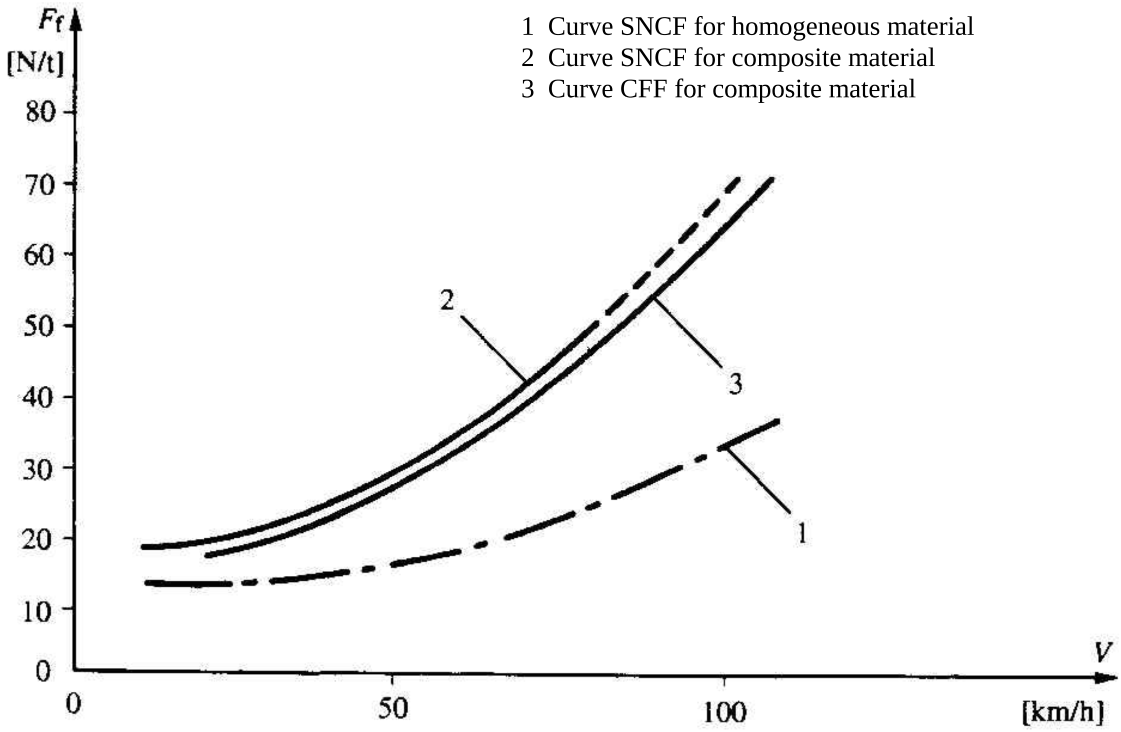

Fig. 3.5 Movement resistance for freight trains.

On a slope, the movement resistance is obtained by multiplication of the train weight and the pent *i* of the track, given en per mille in the tables. Note that angle sinus is replaced by angle tangent, which is good until about 120 ‰.

$$F_d = m\, g\, i\, 10^{-3} \quad \text{[kN]} \tag{3.12}$$

If the mass is measured in kilograms and not in tons, the force is obtained in [N].

The additional friction force F_c in curve is limited on certain segments of the way, where the curve radius is small.

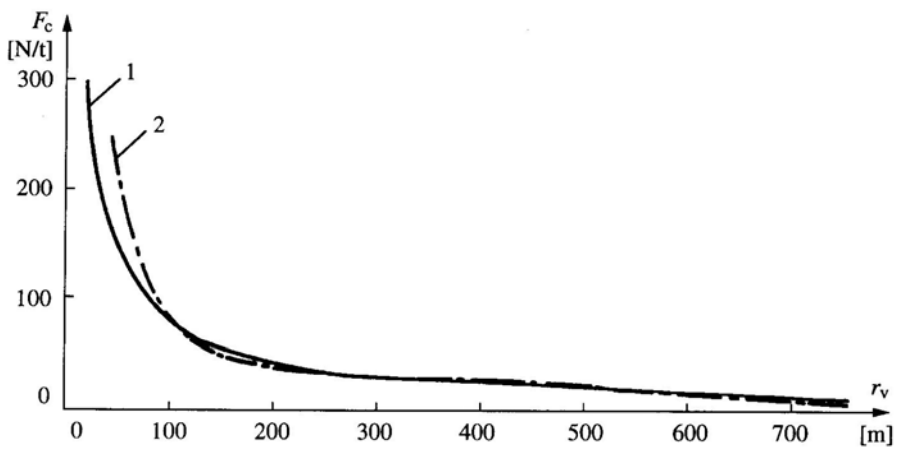

Fig. 3.10 Additional movement in curves on standard gauge.

For the additional friction force in tunnel, the value « Cv^2 » of the equation (3.5) is doubled in a double-tracks tunnel and tripled in a single-track tunnel. This takes care of the piston effect in the tube of tunnel.

For the train acceleration (3.1), note the weight mass has to be increased by the effect of rotating masses – wheels, cog wheels, motor rotors – which causes an apparent increased mass, symbolized by the factor ξ.

$$m^* = \xi\, m \qquad (3.22b)$$

Vehicles		ξ
Complete train	for adhesion	1,06 à 1,10
Cars and wagons		1,02 à 1,04
Empty cars		1,05 à 1,12
Motor-cars		1,08 à 1,14
Locomotives		1,15 à 1,30
Cars	for rack	1,05 à 1,10
Motor-cars		1,30 à 2,50
Locomotives		1,50 à 3,50

Fig. 3.13 Coefficient of rotating masses.

The traction effort $Z = F_{in}$ is computed from the torque M_m at motor shaft, the gear ratio k_G , the wheel radius r_e and the gear efficiency η_G (see chap. 5).

$$Z = \eta_G \frac{M_m}{k_G r_e} 10^{-3} \quad [\text{kN}] \qquad (3.17)$$

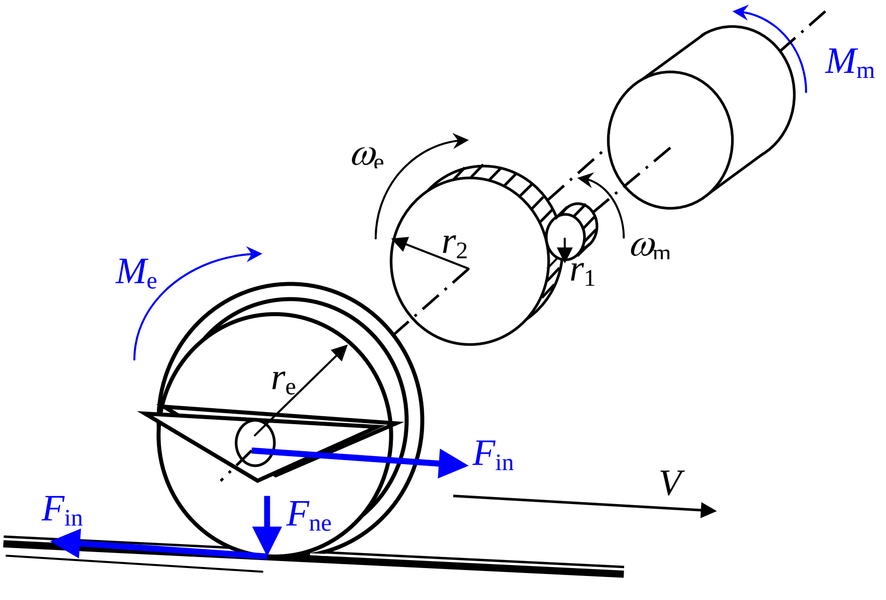

Fig. 3.12 & 3.14 From rotation to translation, on adhesion.

Because of the adhesion conditions of steel wheels on steel rails, the traction effort and braking effort are limited. The limit depends on the adhesion's factor μ_r and the force F_{ne} of the axle perpendicularly to rail surface. In a first approximation, the force can be considered as a quarter of the weight a four-wheel locomotive, but it is not a constant in reason of the dynamic of bogies and body and their suspensions, when a tractive effort at the coupling and the efforts at wheel rims cause a rotation torque. The adhesion's factor depends on the translation speed of the vehicle and the gliding speed of wheel on rails, from empiric laws described on figures 3.15 and 3.17.

$$Z < \mu_r F_{ne} \quad [\text{kN}] \tag{3.24b}$$

If the condition (3.24b) is not respected, the concerned axle begins to glide: This movement will increase if the driver ore an anti-gliding device does not reduce strongly and quickly the motor torque.

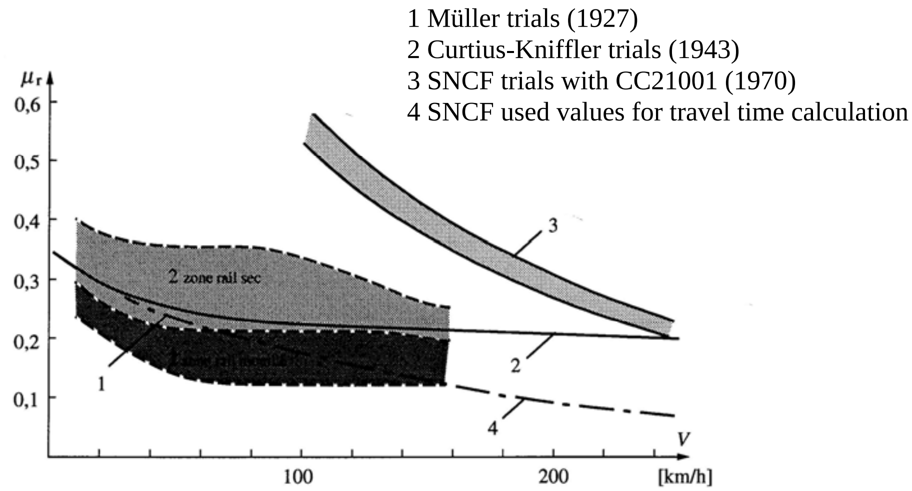

Fig. 3.15 Adhesion factor in function of the translation speed.

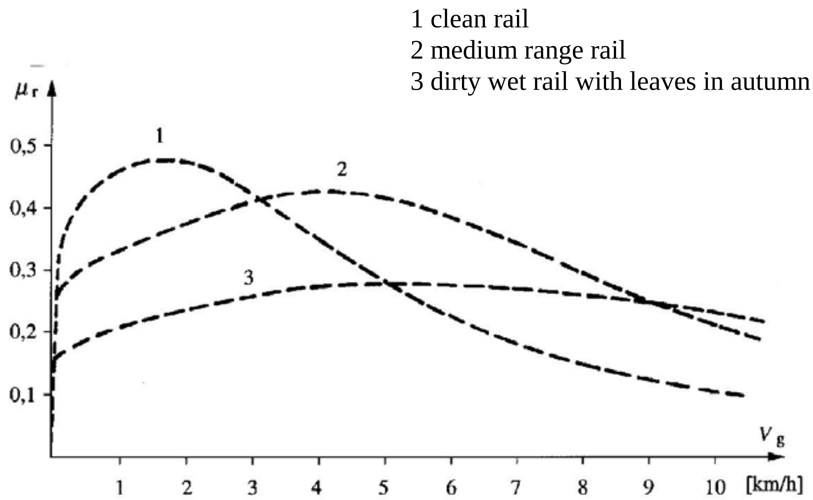

Fig. 3.17 Adhesion factor in function of the gliding speed.

The adhesion factor of a rubber tire on a roadway – metro or trolleybus – is quite double as a steel wheel on steel rail, between 0,55 and 0,62 depending of the surface type.

The necessary efforts Z maximal are at start.

$$Z = m * a + F_d + F_f \tag{3.1b}$$

The SNCF uses an empiric method to estimate effort, with a modified ramp i_{corr}.

$$Z = m\ g\ i_{corr}\ 10^{-3} \quad [kN] \tag{3.12b}$$

As example, the values are given for a freight train for an acceleration of 0,03 [m/s^2] :

$$\begin{aligned} i_{corr} &= 1{,}225\,(i + 2{,}2) && \text{if} \quad i \geq 7\ [‰] \\ i_{corr} &= 4{,}35 + i && \text{if} \quad i < 7\ [‰] \end{aligned} \tag{3.14b}$$

The *coupling effort* F_{att} is the effort at wheels rims, minus the necessary effort used for the traction engine itself.

$$F_{att} = Z - (m_{loc} * a + F_{d\_loc} + F_{f\_loc}) \tag{3.44}$$

The « UIC » coupling is built for a minimal breaking force of 850 kN. From this value, railway operators uses security margin and give a service limit (SNCF : 360 kN, CFF : 650 kN). Reinforced screw coupling exist (limit at 1350 kN) with same geometry as standard coupling. With an automatic coupling, the limit reaches 1500 kN.

The limits of adherence or of coupling have to be carefully observed at star.