Exercise 1 The Young double slit experiment (1803)

1) The scheme of the experiment is as follows:

\[
|\psi(\vec{r}_P)|^2 \approx \frac{A^2}{D^2} \left| e^{\frac{2\pi i}{\lambda} |\vec{r}_B - \vec{r}_P|} + e^{\frac{2\pi i}{\lambda} |\vec{r}_C - \vec{r}_P|} \right|^2.
\]

By factoring out the factor whose modulus is 1, we then have

\[
|\psi(\vec{r}_P)|^2 \approx \frac{A^2}{D^2} \left| 1 + e^{\frac{2\pi i}{\lambda} (|\vec{r}_C - \vec{r}_P| - |\vec{r}_B - \vec{r}_P|)} \right|^2.
\]

As shown in the above figure, the difference of lengths between the two beams $|\vec{r}_C - \vec{r}_P| - |\vec{r}_B - \vec{r}_P|$ is $d \sin \theta$. Therefore, we have

\[
|\psi(\vec{r}_P)|^2 \approx \frac{A^2}{D^2} \left| 1 + e^{\frac{2\pi i d \sin \theta}{\lambda}} \right|^2 = \frac{A^2}{D^2} \left[ 2 + 2 \cos \left( \frac{2\pi d \sin \theta}{\lambda} \right) \right] = \frac{4A^2}{D^2} \cos^2 \left( \frac{\pi d}{\lambda} \sin \theta \right),
\]

where the last line uses $\cos 2\alpha = 2 \cos^2 \alpha - 1$.

2) The intensity attains its minima at 0 when $\sin \theta = \left( m + \frac{1}{2} \right) \frac{\lambda}{d}$. The intensity attains its minima when the cosine function equals $\pm 1$, whereby $\sin \theta = m \frac{\lambda}{d}$ for some integer $m$. 


3) For $D >> d$, we use the approximation $\tan \theta \approx \theta \approx \sin \theta$ so that the intensity is given by

$$|\psi(\vec{r}_P)|^2 \approx \frac{4A^2}{D^2} \cos^2 \left( \frac{\pi d \rho}{D \lambda} \right).$$

As the location of maxima satisfies $\frac{d \rho}{d \lambda} = m \in \mathbb{N}$, the distance between two successive minima is

$$\rho_{m+1} - \rho_m = \frac{\lambda D}{d}.$$ 

With $d = 0.25\text{mm}$, $D = 10\text{m}$ and $\lambda = 652\text{nm}$, the $\rho_{m+1} - \rho_m$ is 26.1mm.

**Exercise 2 Modern Young’s experiment**

1) For a molecule $p = mv$ and $m = \frac{M_{\text{mole}}}{N_A}$. The De Broglie wavelength is $\lambda = \frac{\hbar}{p} = \frac{\hbar N_A}{M_{\text{mole}}v}$. For an average velocity of 220m/s, the wavelength is $1.511 \times 10^{-10}\text{m}$.

2) Take the results known for waves. We should observe interference fringes with a distance $\rho_{m+1} - \rho_m = \lambda \frac{D}{d} = 1.89\text{mm}$.

3) The wavelength is $5.30 \times 10^{-35}\text{m}$, out of measurable distance.

**Exercise 3 Photoelectric effect**

According to Einstein’s formula, the kinetic energy of the ejected electrons is

$$\frac{1}{2}mv^2 = h\nu - W_0$$

where $W_0 = h\nu_0 = \frac{hc}{\lambda_0}$ is the minimal energy for extraction. The equation can be rewritten as $\frac{hc}{\lambda} = \frac{1}{2}mv^2 + \frac{hc}{\lambda_0}$. Therefore, the necessary wavelength is

$$\lambda = \left( \frac{1}{2}mv^2 + \frac{hc}{\lambda_0} \right)^{-1}hc.$$ 

Numerics can be calculated using $\frac{1}{2}mv^2 = 1.5\text{eV}$. One can find $\lambda = 180\text{nm}$.  

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