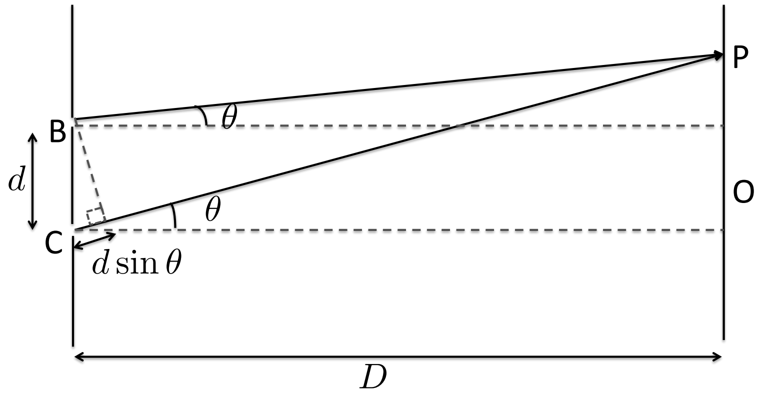
Homework 1: 21 September 2017
Traitement Quantique de l'Information

Exercise 1 *The Young double slit experiment (1803)*

1) The scheme of the experiment is as follows:



If $D >> d$, we use the approximation

$$|\psi(\vec{r}_P)|^2 \approx \frac{A^2}{D^2}\left|e^{\frac{2\pi i}{\lambda}|\vec{r}_B-\vec{r}_P|} + e^{\frac{2\pi i}{\lambda}|\vec{r}_C-\vec{r}_P|}\right|^2.$$

By factoring out the factor whose modulus is 1, we then have

$$|\psi(\vec{r}_P)|^2 \approx \frac{A^2}{D^2}\left|1 + e^{\frac{2\pi i}{\lambda}(|\vec{r}_C-\vec{r}_P|-|\vec{r}_B-\vec{r}_P|)}\right|^2.$$

As shown in the above figure, the difference of lengths between the two beams $|\vec{r}_C - \vec{r}_P| - |\vec{r}_B - \vec{r}_P|$ is $d\sin\theta$. Therefore, we have

$$\begin{aligned}
|\psi(\vec{r}_P)|^2 \approx \frac{A^2}{D^2}\left|1 + e^{\frac{2\pi i d\sin\theta}{\lambda}}\right|^2 &= \frac{A^2}{D^2}\left[\left(1+\cos\left(\frac{2\pi d\sin\theta}{\lambda}\right)\right)^2 + \sin^2\left(\frac{2\pi d\sin\theta}{\lambda}\right)\right] \\
&= \frac{A^2}{D^2}\left[2 + 2\cos\left(\frac{2\pi d\sin\theta}{\lambda}\right)\right] \\
&= \frac{4A^2}{D^2}\cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right),
\end{aligned}$$

where the last line uses $\cos 2\alpha = 2\cos^2\alpha - 1$.

2) The intensity attains its minima at 0 when $\sin\theta = \left(m + \frac{1}{2}\right)\frac{\lambda}{d}$. The intensity attains its minima when the cosine function equals ± 1, whereby $\sin\theta = m\frac{\lambda}{d}$ for some integer m.

3) For $D >> d$, we use the approximation $\tan\theta \approx \theta \approx \sin\theta$ so that the intensity is given by

$$|\psi(\vec{r}_P)|^2 \approx \frac{4A^2}{D^2}\cos^2\left(\frac{\pi d\rho}{D\lambda}\right).$$

As the location of maxima satisfies $\frac{d\rho_m}{D\lambda} = m \in \mathbb{N}$, the distance between two successive minima is

$$\rho_{m+1} - \rho_m = \lambda\frac{D}{d}.$$

With $d = 0.25$mm, $D = 10$m and $\lambda = 652$nm, the $\rho_{m+1} - \rho_m$ is 26.1mm.

Exercise 2 *Modern Young's experiment*

1) For a molecule $p = mv$ and $m = \frac{M_{\text{mole}}}{N_A}$. The De Broglie wavelength is $\lambda = \frac{h}{p} = \frac{hN_A}{M_{\text{mole}}v}$. For an average velocity of 220m/s, the wavelength is 1.511×10^{-10}m.

2) Take the results known for waves. We should observe interference fringes with a distance $\rho_{m+1} - \rho_m = \lambda\frac{D}{d} = 1.89$mm.

3) The wavelength is 5.30×10^{-35}m, out of measurable distance.

Exercise 3 *Photoelectric effect*

According to Einstein's formula, the kinetic energy of the ejected electrons is

$$\frac{1}{2}mv^2 = h\nu - W_0$$

where $W_0 = h\nu_0 = \frac{hc}{\lambda_0}$ is the minimal energy for extraction. The equation can be rewritten as $\frac{hc}{\lambda} = \frac{1}{2}mv^2 + \frac{hc}{\lambda_0}$. Therefore, the necessary wavelength is

$$\lambda = \left(\frac{1}{2}mv^2 + \frac{hc}{\lambda_0}\right)^{-1} hc.$$

Numerics can be calculated using $\frac{1}{2}mv^2 = 1.5$eV. One can find $\lambda = 180$nm.