Exercise 1 The Young double slit experiment (1803)

In this exercise we want to calculate the form of the Young interference fringes. A beam of monochromatic light of wave length $\lambda$ is sent through a double slit, and the light is reflected on a screen at a distance $D$. The distance between the two slits is $d$.

We assume that the waves diffracted by each slit have a spherical shape ($\lambda$ the wavelength and $\nu$ the frequency):

$$\phi_B(\vec{r}) = A e^{i \left( \frac{2\pi}{\lambda} |\vec{r}_B - \vec{r}| - 2\pi \nu t \right) / |\vec{r}_B - \vec{r}|}, \quad \phi_C(\vec{r}) = A e^{i \left( \frac{2\pi}{\lambda} |\vec{r}_C - \vec{r}| - 2\pi \nu t \right) / |\vec{r}_C - \vec{r}|}.$$

The total wave function at $P$ on the screen is

$$\psi(\vec{r}_P) = \phi_B(\vec{r}_P) + \phi_C(\vec{r}_P).$$

We will use the plane wave approximation for $D >> d$:

$$\psi(\vec{r}_P) \simeq \frac{A}{D} e^{-2\pi i \nu t} \left( e^{\frac{2\pi i}{\lambda} |\vec{r}_B - \vec{r}_P|} + e^{\frac{2\pi i}{\lambda} |\vec{r}_C - \vec{r}_P|} \right).$$

1) Show that the intensity at $P$ on the screen is equal to

$$|\psi(\vec{r}_P)|^2 \approx \frac{4A^2}{D^2} \cos^2 \left( \frac{\pi d}{\lambda} \sin \theta \right).$$

Hint: evaluate first the path difference $|\vec{r}_C - \vec{r}_P| - |\vec{r}_B - \vec{r}_P|$ for $D >> d$.

2) Find the condition on $\sin \theta$ which leads to minima and maxima of the intensity on the screen.
3) Let \( \rho \) be the coordinate on the screen measured from \( O \). We have \( \tan \theta \approx \frac{\rho}{D} \) and since \( \theta \) is small \( \theta \approx \frac{\rho}{D} \). Compute the distance between two successive minima of the intensity pattern on the screen.

Let \( d = 0.25 \text{mm}, \ D = 10 \text{m} \) and \( \lambda = 652 \text{nm} \) (red light). What is the distance between two successive minima?

**Exercise 2 Modern Young’s experiment**

Young’s double slit experiment has been reformed with Carbon 60 molecules, \( C_{60} \) in 1999. Surprisingly, these molecules behave like waves when they are well isolated from their environment. The more recent experiments have evidenced such a wave-like behavior for bigger molecules with 400 to 1000 atoms.

The diameter of \( C_{60} \) (this molecule has the form of a sphere and contains 60 carbon atoms) is approximately 0.7 nm and a mole containing \( N_A = 6.022 \times 10^{23} \) carbon atoms weighs 12 grams.

1) Compute the De Broglie wavelength of molecules produced in an oven which have an average velocity of 220 m/s. Compare with the size of individual molecules.

2) We perform a Young’s experiment with \( d = 100 \text{nm} \) and \( D = 1.25 \text{m} \). What do we observe on the screen assuming a wave like behavior?

3) A football weights approximately 450g and the initial velocity of a professional shoot can attain 100 km/h. Estimate the De Broglie wavelength.

**Exercise 3 Photoelectric effect**

The maximal wavelength to extract a photoelectron from tungsten is 230 nm (ultraviolets). What is the necessary wavelength of light to extract electrons with kinetic energy 1.5 eV? What is the speed of these electrons?

**Useful Constant**

\[ c = 2.997 \times 10^8 \text{ m/s} \] (speed of light)
\[ h = 1.054 \times 10^{-34} \text{ J s} \ (\frac{h}{2\pi} \text{ where } h \text{ is Planck’s constant}) \]
\[ m = 9.109 \times 10^{-34} \text{ kg} \] (mass of an electron)
\[ 1\text{eV} = 1.6 \times 10^{-19} \text{ J} \]