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Handout 11 Principles of Digital Communications
Quiz 1 Mar. 16, 2018

PROBLEM 1. Consider a binary hypothesis test. Our observation Y is a n-dimensional vector $(Y_1, ..., Y_n)$. When $H = 0$, $Y_i = 1 + Z_i$. When $H = 1$, $Y_i = -1 + Z_i$, where $\{Z_i\}$ are i.i.d. $\mathcal{N}(0, \sigma^2)$. For each of the following, indicate if it is a sufficient statistic.

(a) $T_1 = \sum_i Y_i^2$.

(b) $T_2 = \sum_i (Y_i + 1)^2$.

(c) $T_3 = (T_1, T_2)$.

(d) $T_4 = \sum_i Y_i$.

(e) $T_5 = \text{sign}(T_4)$.

PROBLEM 2. Suppose we have m hypotheses, and when $H = i$, the observation Y is a n-dimensional vector given by $Y = c_i + Z$ where Z is $\mathcal{N}(0, \sigma^2 I_n)$. Consider another observation $\tilde{Y}$ which is $2n$-dimensional, and equals $\tilde{c}_i + W$ where W is $\mathcal{N}(0, \tilde{\sigma}^2 I_{2n})$ and $\tilde{c}_i = (c_i, c_i)$ is the vector where c_i is repeated twice. For instance, if $n = 2$ and $c_i = (a, b)$, $\tilde{c}_i = (a, b, a, b)$.

(a) Let d_{ij} be the distance between c_i and c_j and let $\tilde{d}_{ij}$ be the distance between $\tilde{c}_i$ and $\tilde{c}_j$. How are d_{ij} and $\tilde{d}_{ij}$ related?

(b) Suppose you are given the choice to observe Y or $\tilde{Y}$. Explain how you would make this choice (in terms of σ^2 and $\tilde{\sigma}^2$), so as to achieve the smallest error probability in guessing H. [Hint: use the result in (a).]

Problem 3. Suppose Z_1 and Z_2 are independent, Z_1 is uniformly distributed in the interval $[-2, 2]$, and Z_2 is $\mathcal{N}(0, \sigma^2)$. Suppose that under hypothesis i, $i = 0, 1$, the observation Y is $c_i + Z$ with $c_0 = (1, 1)$, $c_1 = (-1, -1)$ and $Z = (Z_1, Z_2)$. With equally likely hypotheses, sketch the decision regions on the (Y_1, Y_2) plane and find the probability of error.