Exercise Set 3 : 8 March 2018
Calcul Quantique

Exercise 1 *Production of Bell states*

a) Check the following identity using Dirac's notation :

$$|B_{xy}\rangle = (CNOT)(H \otimes I)|x\rangle \otimes |y\rangle.$$

where $x, y \in \{0, 1\}$ and $|B_{xy}\rangle$ are the Bell states.

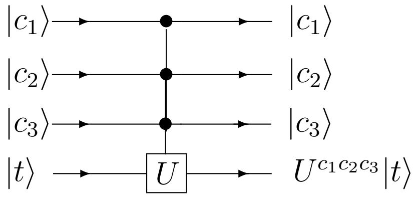
b) Represent the corresponding circuit.

c) Represent the circuit corresponding to the inverse identity :

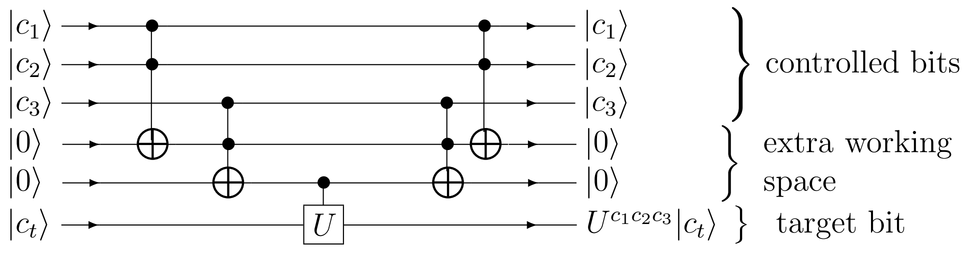
$$|x\rangle \otimes |y\rangle = (H \otimes I)(CNOT)|B_{xy}\rangle$$

Exercise 2 *Construction of a multi-control-U.*

Verify that the multi-control-U :

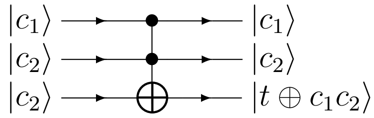


can be realized with the Toffoli gate (control-control-NOT) and a simple control-U.

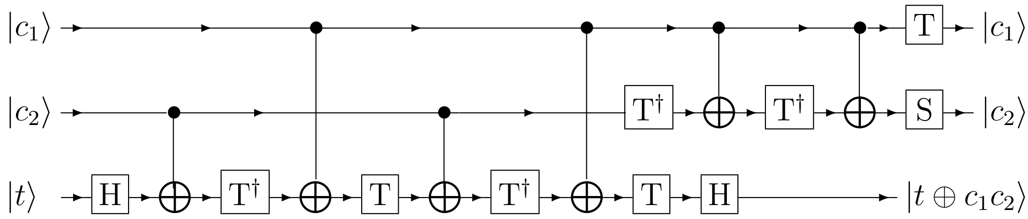


Exercise 3 *Construction of the Toffoli gate from a control-NOT (Indication : long calculation).*

Verify that the control-control-NOT also called Toffoli gate :



is equivalent to the following circuit made of $CNOT$, H, T and S



Exercise 4 *Unitary representation of a reversible computation.*

Classically a Boolean function f with inputs $(x_1, \ldots, x_n) \in \{0,1\}^n$ and output in $\{0,1\}$ can be computed reversibly as

$$\tilde{f}(x_1, \ldots, x_n; y) = (x_1, \ldots, x_n; y \oplus f(x_1, \ldots, x_n))$$

where $y \in \{0,1\}$ is a single storage bit.

This can be implemented in a quantum circuit thanks to the following unitary operation

$$U_f |x_1, \ldots, x_n; y\rangle = |x_1, \ldots, x_n; y \oplus f(x_1, \ldots, x_n)\rangle\rangle$$

a) What is the Hilbert space relevant for this implementation. Prove that U_f is indeed a unitary matrix.

b) Generalize this discussion to the case where the output of f in $\{0,1\}^m$ (there are m output bits).