

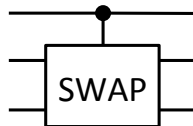
Exercise 1 *Matrix representations of classical gates*

Consider the following representation of a classical bit $0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, and of bit strings $00 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $01 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$, $10 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$, $11 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$, etc.

- (a) Give a matrix representation for the following reversible gates : NOT ; CNOT ; CCNOT.
- (b) Recognize that these are all permutation matrices. What is their inverse matrix ?

Exercise 2 *Fredkin gate*

The SWAP operation takes two input bits and permutes them : $\text{SWAP}(b_1, b_2) = (b_2, b_1)$. The Fredkin gate is a three input controlled SWAP gate and is reversible. The gate swaps the two last bits if the first bit is a 1. Otherwise it leaves the input bits unchanged. One intriguing particularity of the Fredkin gate is that it conserves the number of ones.



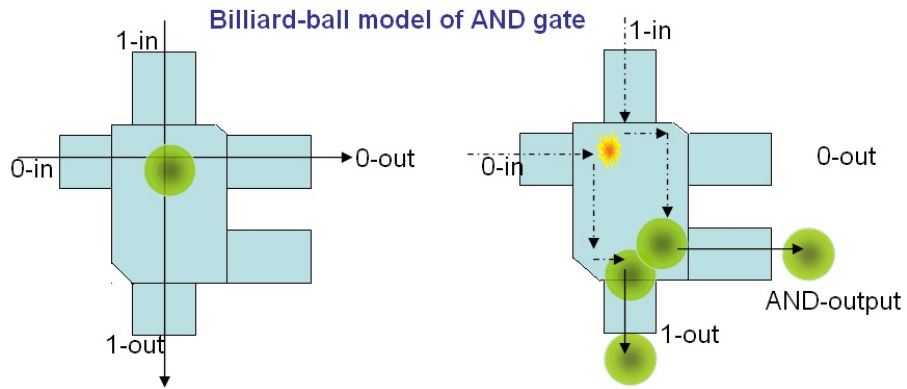
- (a) Show that the irreversible gates AND, OR can be represented in a reversible way from the Fredkin gate.
- (b) Give the matrix representation of the Fredkin gate.
- (c) Represent the Toffoli (CCNOT) gate in terms of {Fredkin, CNOT}.
Hint : You can achieve with at most one Fredkin gate and two CNOT gates.

Exercise 3 *Billiard Ball Model of a classical computation (cultural aside)*

This is a physical model of computation that entirely functions through the laws of elastic collisions. It is a *reversible* and *conservative* model of computation. Here “reversible” means there is no heat disipation. And “conservative” means that the mass or equivalently the number of balls is conserved ; this is analogous to the number of ones conserved in teh Fredkin gate) model of computation.

Here we just illustrate the flavor of this model invented by Fredkin and Toffoli by considering two gates.

- (a) Convince yourself that the following operates as an AND gate. Discuss this with your fellow students.



[Figure from "Billiard-ball computer," Wikipedia, The Free Encyclopedia]

- (b) Convince yourself that the following implements the Fredkin gate.

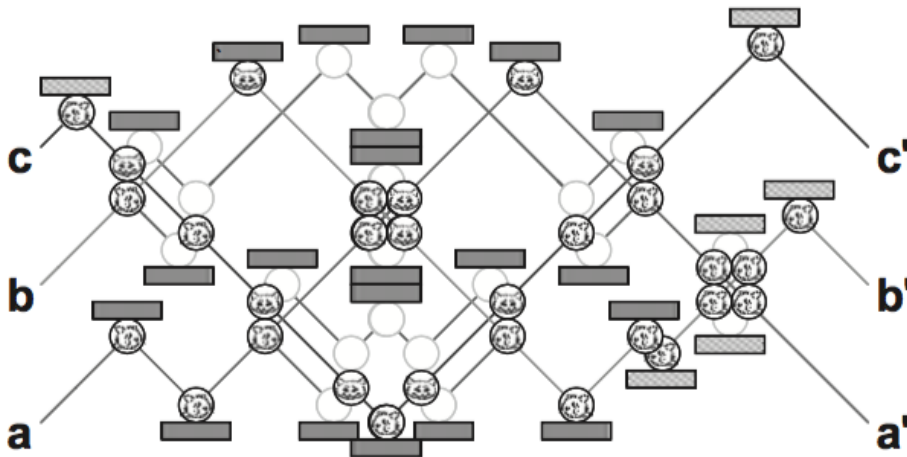


Figure 3.14. A simple billiard ball computer, with three input bits and three output bits, shown entering on the left and leaving on the right, respectively. The presence or absence of a billiard ball indicates a 1 or a 0, respectively. Empty circles illustrate potential paths due to collisions. This particular computer implements the Fredkin classical reversible logic gate, discussed in the text.

[Figure from Nielsen, Michael A., Chuang, Isaac L. (2000). Quantum Computation and Quantum Information. Cambridge, UK : Cambridge University Press.]

