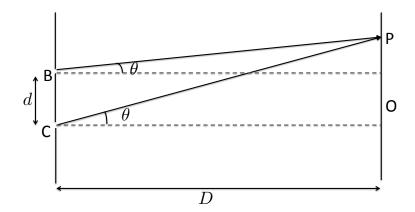
# Homework 1: 22 September 2016 Traitement Quantique de l'Information

## Exercise 1 The Young double slit experiment (1803)

In this exercise we want to calculate the form of the Young interference fringes. A beam of monochromatic light of wave length  $\lambda$  is sent through a double slit, and the light is reflected on a screen at a distance D. The distance between the two slits is d.



We assume that the waves diffracted by each slit have a spherical shape ( $\lambda$  the wavelength and  $\nu$  the frequency):

$$\phi_B(\vec{r}) = A \frac{e^{i\left(\frac{2\pi}{\lambda}|\vec{r}_B - \vec{r}| - 2\pi\nu t\right)}}{|\vec{r}_B - \vec{r}|}, \quad \phi_C(\vec{r}) = A \frac{e^{i\left(\frac{2\pi}{\lambda}|\vec{r}_C - \vec{r}| - 2\pi\nu t\right)}}{|\vec{r}_C - \vec{r}|}.$$

The total wave function at P on the screen is

$$\psi(\vec{r}_P) = \phi_B(\vec{r}_P) + \phi_C(\vec{r}_P).$$

We will use the plane wave approximation for D >> d:

$$\psi(\vec{r}_P) \simeq \frac{A}{D} e^{-2\pi i \nu t} \left( e^{\frac{2\pi i}{\lambda} |\vec{r}_B - \vec{r}_P|} + e^{\frac{2\pi i}{\lambda} |\vec{r}_C - \vec{r}_P|} \right).$$

1) Show that the intensity at P on the screen is equal to

$$|\psi(\vec{r}_P)|^2 \approx \frac{4A^2}{D^2}\cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right).$$

Hint: evaluate first the path difference  $|\vec{r}_C - \vec{r}_P| - |\vec{r}_B - \vec{r}_P|$  for D >> d.

2) Find the condition on  $\sin \theta$  which leads to minima and maxima of the intensity on the screen.

3) Let  $\rho$  be the coordinate on the screen measured from O. We have  $\tan \theta \approx \frac{\rho}{D}$  and since  $\theta$  is small  $\theta \approx \frac{\rho}{D}$ . Compute the distance between two successive minima of the intensity pattern on the screen.

Let d = 0.25mm, D = 10m and  $\lambda = 652$ nm (red light). What is the distance between two successive minima?

## Exercise 2 Modern Young's experiment

Young's double slit experiment has been reformed with Carbon 60 molecules,  $C_{60}$  in 1999. Surprisingly, these molecules behave like waves when they are well isolated from their environment. The more recent experiments have evidenced such a wave-like behavior for bigger molecules with 400 to 1000 atoms.

The diameter of  $C_{60}$  (this molecule has the form of a sphere and contains 60 carbon atoms) is approximately 0.7 nm and a mole containing  $N_A = 6.022 \times 10^{23}$  carbon atoms weights 12 grams.

- 1) Compute the De Broglie wavelength of molecules produced in an oven which have an average velocity of 220m/s. Compare with the size of individual molecules.
- 2) We perform a Young's experiment with d = 100nm and D = 1.25m. What do we observe on the screen assuming a wave like behavior?
- 3) A football weights approximately 450g and the initial velocity of a professional shoot can attain 100km/h. Estimate the De Broglie wavelength.

#### Exercise 3 Photoelectric effect

The maximal wavelength to extract a photoelectron from tungsten is 230nm (ultraviolets). What is the necessary wavelength of light to extract electrons with kinetic energy 1.5eV? What is the speed of these electrons?

#### Useful Constant

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c=2.997\times 10^8 m/s (speed of light) \hbar=1.054\times 10^{-34}~{\rm J\cdot s}~(\frac{h}{2\pi}~{\rm where}~h~{\rm is~Planck's~constant}) m=9.109\times 10^{-34}{\rm kg}~({\rm mass~of~an~electron}) 1{\rm eV}=1.6\times 10^{-19}{\rm J}
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