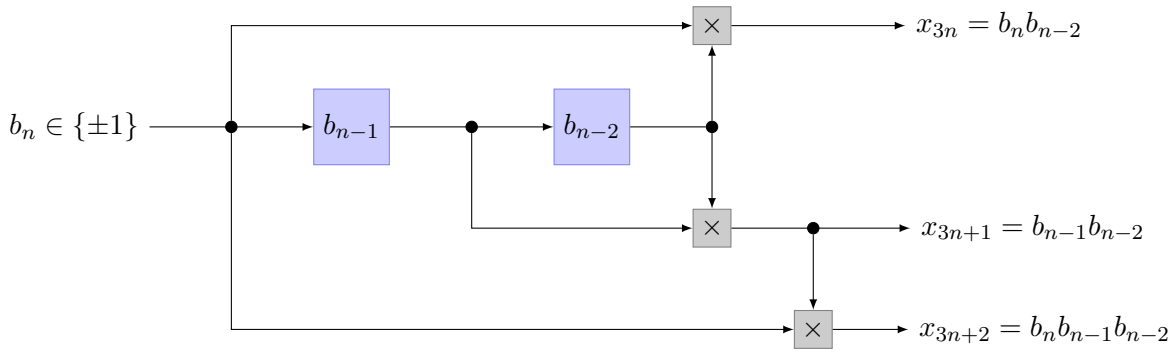
Handout 27 Principles of Digital Communications
Problem Set 12 May 18, 2016

PROBLEM 1. Consider the following convolutional encoder:



(a) Draw the state diagram and the detour flow graph.

(b) Suppose that the serialized encoder output symbols are scaled so that the resulting energy per bit is $\mathcal{E}_b$ and are sent over the discrete-time AWGN channel of noise variance $\sigma^2 = \frac{N_0}{2}$. Derive an upper bound to the bit-error probability assuming that the decoder implements the Viterbi algorithm.

PROBLEM 2. The following equations describe the output sequence of a convolutional encoder that in each epoch takes $k_0 = 2$ input symbols from $\{\pm 1\}$ and outputs $n_0 = 3$ symbols from the same alphabet.

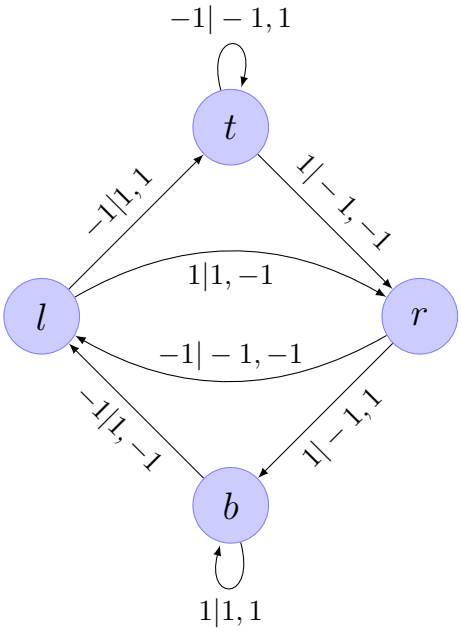
$$\begin{aligned} x_{3n} &= b_{2n} b_{2n-1} b_{2n-2} \\ x_{3n+1} &= b_{2n+1} b_{2n-2} \\ x_{3n+2} &= b_{2n+1} b_{2n} b_{2n-2} \end{aligned}$$

(a) Draw an implementation of the encoder based on delay elements and multipliers.

(b) Draw the state diagram.

(c) Suppose that the serialized encoder output symbols are scaled so that the resulting energy per bit is $\mathcal{E}_b$ and are sent over the discrete-time AWGN channel of noise variance $\sigma^2 = \frac{N_0}{2}$. Derive an upper bound to the bit-error probability assuming that the decoder implements the Viterbi algorithm.

PROBLEM 3. For the convolutional code described by the state diagram shown below:



State Labels
$t = (-1,-1)$
$l = (-1,1)$
$r = (1,-1)$
$b = (1,1)$

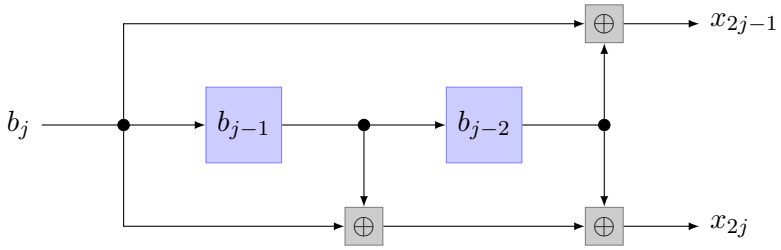
(a) draw the encoder;

(b) as a function of the energy per bit $\mathcal{E}_b$, upper-bound the bit-error probability of the Viterbi algorithm when the scaled encoder output sequence is transmitted over the discrete-time AWGN channel of noise variance $\sigma^2 = \frac{N_0}{2}$.

PROBLEM 4. Consider the convolutional encoder shown below with inputs and outputs over $\{0,1\}$ and addition modulo 2. Its output is sent over the binary erasure channel described by

$$\begin{aligned} P_{Y|X}(0|0) &= P_{Y|X}(1|1) = 1-\epsilon \\ P_{Y|X}(?|0) &= P_{Y|X}(?|1) = \epsilon \\ P_{Y|X}(1|0) &= P_{Y|X}(0|1) = 0, \end{aligned}$$

where $0 < \epsilon < \frac{1}{2}$.



(a) Draw a trellis section that describes the encoder map.

(b) Derive the branch metric and specify whether a maximum likelihood decoder chooses the path with largest or smallest path metric.

(c) Suppose that the initial encoder state is $(0,0)$ and that the channel output is $\{0,?,?,1,0,1\}$. What is the most likely information sequence?

(d) Derive an upper bound to the bit-error probability.