Solution to Graded Problem Set 2

Date: 26.09.2014 Due date: 3.10.2014

Problem 1.

a) $\neg(\neg p \wedge q)$. Indeed, $\neg(\neg p \wedge q) \equiv \neg\neg p \vee \neg q \equiv p \vee \neg q$. The first equivalence is obtained by applying De Morgan Law and the second equivalence is obtained by applying double negation law.

b) $\neg p \vee q$. Indeed, $p \to q \equiv \neg p \vee q$ by definition of implication.

c) Apply rules successively:

$$
\begin{array}{lll}
 & (q \wedge (p \to \neg q)) \to \neg p & \\
\equiv & (q \wedge (\neg p \vee \neg q)) \to \neg p & \text{definition of implication} \\
\equiv & ((q \wedge \neg p) \vee (q \wedge \neg q)) \to \neg p & \text{distributive law} \\
\equiv & ((q \wedge \neg p) \vee \mathrm{F}) \to \neg p & \text{negation law} \\
\equiv & (q \wedge \neg p) \to \neg p & \text{identity law} \\
\equiv & \neg(q \wedge \neg p) \vee \neg p & \text{definition of implication} \\
\equiv & (\neg q \vee p) \vee \neg p & \text{De Morgan's law} \\
\equiv & \neg q \vee (p \vee \neg p) & \text{associative law} \\
\equiv & \neg q \vee \mathrm{T} & \text{negation law} \\
\equiv & \mathrm{T} & \text{domination law}
\end{array}
$$

Problem 2.

1. True. Indeed, $P(1, -1)$ means that $1 - 2 = -1$, which is true.
2. True. Indeed, $P(0, 0)$ means that $0 + 0 = 0$, which is true.
3. True. By solving the equation $3 + 2y = 3y$, we obtain $y = 3$, from which we deduce that $P(3, 3)$ is true. Hence, $\exists y P(3, y)$.
4. False. Pick $x = 2$. Then, there exists no y s.t. $x + 2y = xy$. Hence, it is not true that $\forall x \exists y P(x, y)$, since for $x = 2$, there exists no such y.
5. False. Pick $y = 1$. Then, there exists no x s.t. $x + 2y = xy$. Hence, it is not true that $\exists x \forall y P(x, y)$, since for any x the choice $y = 1$ makes $P(x, y)$ false.
6. False. Pick $y = 1$. Then, there exists no x s.t. $x + 2y = xy$. Hence, it is not true that $\forall y \exists x P(x, y)$, since for $y = 1$ there is no x which makes $P(x, y)$ true.
7. False. Pick $x = 2$. Then, there exists no y s.t. $x + 2y = xy$. Hence, it is not true that $\exists y \forall x P(x, y)$, since for any y the choice $x = 2$ makes $P(x, y)$ false.
8. False. Indeed, $\neg\forall x \exists y \neg P(x, y) \equiv \exists x \forall y P(x, y)$ and $\exists x \forall y P(x, y)$ is false (see point 5).

Problem 3.

1. $\forall x(I(x) \to E(x))$.
2. $\forall x \forall y(L(x, y) \to \neg Q(x, y))$.
3. $\forall x \exists y L(x, y)$.

Problem 4. Let $R(x)$ be the predicate "x is a student in my class" and $P(x)$ be the predicate "x is from England". The following is the proof :

1. $\nexists x(R(x) \land P(x)) \equiv \forall x(R(x) \to \neg P(x))$	hypothesis
2. $R(\text{Jean}) \to \neg P(\text{Jean})$	universal instantiation on (1)
3. $R(\text{Jean})$	hypothesis
4. $\neg P(\text{Jean})$	modus ponens on (2) and (3)

Problem 5.

a) Let p be the predicate "$n > 2$" and q be the predicate "$n^2 > 4$". Then, the argument can be written as

$$(p \to q) \land \neg p \to \neg q.$$

This argument is not valid. This type of incorrect reasoning is called the *fallacy of denying the hypothesis.*

b) Let p be the predicate "you are in the tennis tournament", q be the predicate "you meet Ed", r be the predicate "you are in the play", and s be the predicate "you meet Kelly".

Consider the following set of propositions:

(i) $\neg p \to \neg q$

(ii) $(\neg p \lor \neg r) \to \neg s$

(iii) $s \lor \neg q$

(iv) $\neg(p \land r)$

(v) p

Then, the argument claims that the first four propositions imply the last one. This argument is not valid, since we can find a set of truth assignments s.t. the last proposition is false, while the first four are true (i.e., the premises can be all true with the conclusion being false).

Indeed, assume that p is false. Then, if s is false and q is false, the first four propositions are true.

Problem 6.

a) $\exists x \forall y P(x, y)$

b) $\forall x \forall y \neg P(x, y)$

c) $\forall x P(x, x)$

Note the solutions to this problem might not be unique.