Discrete Structures EPFL, Fall 2014

Graded Problem Set 2

Date: 26.09.2014 Due date: 3.10.2014

Instructions : Please write the solution for each problem on a separate piece of paper. Sort these pieces of paper, with Problem 1 on top. Add this page as cover page, fill in your name, Sciper number, and the list of collaborators, and staple them all together on the left top.

Rules : You are allowed and encouraged to discuss these problems with your colleagues. However, each of you has to write down her solution in her own words. If you collaborated on a homework, write down the name of your collaborators and your sources in the space below. No points will be deducted for collaborations. But if we find similarities in solutions beyond the listed collaborations we will consider it as cheating. Please note that EPFL has a VERY strict policy on cheating and you might be in BIG trouble. It is simply not worth it.

Grading : You will get 5 bonus points if you solve part c) of Problem 6.

Collaborators and sources :

Name : ....

Sciper : ....

Problem 1 ________________________________ ... / 12

Problem 2 ________________________________ ... / 24

Problem 3 ________________________________ ... / 24

Problem 4 ________________________________ ... / 10

Problem 5 ________________________________ ... / 20

Problem 6 ________________________________ + ... / 15

TOTAL ________________________________ ... / 100

Problem 1.

a) Write a proposition equivalent to $p \vee \neg q$ that uses only p, q, $\neg$ and $\wedge$.

b) Write a proposition equivalent to $p \to q$ using only p, q, $\neg$ and $\vee$.

c) Prove that $(q \wedge (p \to \neg q)) \to \neg p$ is a tautology using propositional equivalence and the laws of logic.

Problem 2. Suppose $P(x, y)$ means "$x + 2y = xy$", where x and y are integers. Determine the truth value of the following statements. Justify your answers.

1. $P(1, -1)$.
2. $P(0, 0)$.
3. $\exists y P(3, y)$.
4. $\forall x \exists y P(x, y)$.
5. $\exists x \forall y P(x, y)$.
6. $\forall y \exists x P(x, y)$.
7. $\exists y \forall x P(x, y)$.
8. $\neg \forall x \exists y \neg P(x, y)$.

Problem 3. Suppose the variables x and y represent real numbers, and

$$L(x, y) = \text{“}x < y\text{”} \qquad Q(x, y) = \text{“}x = y\text{”} \qquad E(x) = \text{“}x \text{ is even”} \qquad I(x) = \text{“}x \text{ is a funny number”}.$$

Write the statements using these predicates and any needed quantifiers.

1. Every funny number is even.
2. If $x < y$ then x is not equal to y.
3. There is no largest real number,

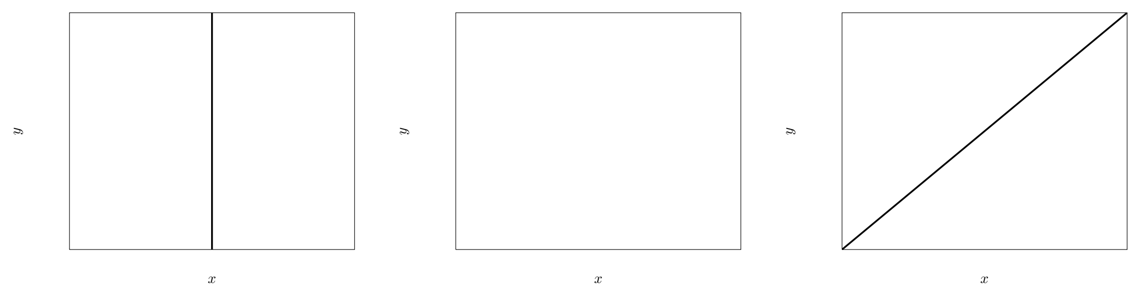
Problem 4. Show that the premises "Jean is a student in my class" and "No student in my class is from England" imply the conclusion "Jean is not from England".

Problem 5. Determine whether the following arguments are valid. Justify your answer.

a) If n is a real number such that $n > 2$, then $n^2 > 4$. Suppose that $n \leq 2$. Then $n^2 \leq 4$.

b) If you are not in the tennis tournament, you will not meet Ed.
If you aren't in the tennis tournament or if you aren't in the play, you won't meet Kelly.
You meet Kelly or you don't meet Ed.
It is false that you are in the tennis tournament and in the play.
From the above statements, we can conclude that you are in the tennis tournament.

Problem 6. Let x and y be real numbers in $[0, 1]$. Let $P(x, y)$ be a predicate. We put a dot on the point (x, y) on the unit square if $P(x, y)$ is true. For each of the figures below, write down a proposition involving $P(x, y)$ and any needed quantifier which "characterizes" the figure. E.g., if the figure was all black, you should write $\forall x \forall y P(x, y)$.



(a) (b) (c) Bonus