Discrete Structures | EPFL, Fall 2014

Graded Problem Set 6

Date: 24.10.2014 | Due date: 31.10.2014

Instructions: Please write the solution for each problem on a separate piece of paper. Sort these pieces of paper, with Problem 1 on top. Add this page as cover page, fill in your name, Sciper number, and the list of collaborators, and staple them all together on the left top.

Rules: You are allowed and encouraged to discuss these problems with your colleagues. However, each of you has to write down her solution in her own words. If you collaborated on a homework, write down the name of your collaborators and your sources in the space below. No points will be deducted for collaborations. But if we find similarities in solutions beyond the listed collaborations we will consider it as cheating. Please note that EPFL has a VERY strict policy on cheating and you might be in BIG trouble. It is simply not worth it.

Grading: You will receive 10 bonus points additionally if you solve Problem 6.

Collaborators and sources:

First name: ....

Last name: ....

Sciper: ....

Problem 1 ... / 20

Problem 2 ... / 20

Problem 3 ... / 15

Problem 4 ... / 20

Problem 5 ... / 25

Problem 6 ... / 10

TOTAL ... / 100

Problem 1. Let $f(n) := \sum_{i=2}^{n} \frac{1}{i \log i}$, where $\log(\cdot)$ denotes the natural logarithm. Show that $f(n) = \Theta(\log \log n)$.

Problem 2. Let $f(n)$ and $g(n)$ be arbitrary functions from $\mathbb{N}$ to $\mathbb{R}^+$. Prove or disprove (by showing a counter example) the following:

a) $\max\{f(n), g(n)\} = \Theta\left(f(n) + g(n)\right)$.

b) $f(n) + g(n) = \Theta\left(\min\{f(n), g(n)\}\right)$.

Problem 3. Mark the following statements with True or False (and give a reason for your choice)

a) $2^{n+1} = O(2^n)$.

b) $2^{2n} = O(2^n)$.

c) If $f : \mathbb{N} \to \mathbb{R}^+$ and $g : \mathbb{N} \to \mathbb{R}^+$ are two arbitrary functions $f(n) = O\left(g(n)\right)$ and $h : \mathbb{R}^+ \to \mathbb{R}^+$ is an increasing continuous bijective function, then $h\left(f(x)\right) = O\left(h\left(g(x)\right)\right)$.

Problem 4. Prove or disprove the following:

a) The sum of two prime numbers is always a prime number.

b) The sum of two irrational numbers is irrational.

c) If a and b are non-zero rational numbers, then a^b is rational.

d) $n^2 - n + 17$ is always prime for all integers n.

e) If p and q are prime numbers larger than 2, then $pq + 1$ is never prime.

Problem 5.

a) What is the multiplicative inverse of 7 modulo 11?

b) What is the multiplicative inverse of 6 modulo 8?

c) What is the multiplicative inverse of 5 modulo 8?

d) Compute the largest integer power of 6 that divides 73!.

e) Compute the remainder of the division of $9^{123456789}$ by 17.

Problem 6. (Bonus) Show that there exist infinitely many prime numbers.