MULTIPLE-CHOICE QUESTIONS

1. B.
2. C.
3. B.
4. B.
5. A.
6. i) F  ii) F  iii) F  iv) T
7. i) B  ii) C  iii) B  iv) B  v) A  vi) A
8. D
9. i) B  ii) C  iii) A  iv) B
10. A
11. C
13. B
14. The statement is true. The proof follows.
Since \( f_1 = \Theta(f_2) \), there exists \( x_0 \) and constants \( c_1 > 0 \) and \( c_2 \) s.t. for all \( x \geq x_0 \),
\[
    c_1|f_2(x)| \leq |f_1(x)| \leq c_2|f_2(x)|.
\]
Since \( f_1(x) > 0 \), then \( c_2 > 0 \). Indeed, if \( c_2 \leq 0 \), the inequality above cannot be satisfied.
As the function \( h(x) = x^{-13} \) is decreasing for all \( x > 0 \), we obtain that
\[
    h(c_1|f_2(x)|) \geq h(|f_1(x)|) \geq h(c_2|f_2(x)|),
\]
which implies that
\[
    (c_2)^{-13}(|f_2(x)|)^{-13} \leq (|f_1(x)|)^{-13} \leq (c_1)^{-13}(|f_2(x)|)^{-13}.
\]
Consequently, there exists \( x'_0 \) and constants \( c'_1 > 0 \) and \( c'_2 \) s.t. for all \( x \geq x'_0 \),
\[
    c'_1(|f_2(x)|)^{-13} \leq (|f_1(x)|)^{-13} \leq c'_2(|f_2(x)|)^{-13}.
\]
Indeed, it is enough to take \( x'_0 = x_0 \), \( c'_1 = (c_2)^{-13} > 0 \), and \( c'_2 = (c_1)^{-13} \).

2. The statement is false. Indeed, pick \( f_1(x) = x \) and \( f_2(x) = 2x \). Then, clearly \( f_1(x) = \Theta(f_2) \). By definition \( g_1(x) = 11^x \), and \( g_2(x) = 11^{2x} = 121^x \). Therefore, it is not true that \( g_1 = \Omega(g_2) \).

15. 

**Base step.** If \( n = 0 \), the left hand side and the right hand side of the equality are both 0.

**Induction step.** Assume that \( \sum_{i=0}^{n} i^3 = \frac{n^2(n+1)^2}{4} \). Then,
\[
\sum_{i=0}^{n+1} i^3 = \sum_{i=0}^{n} i^3 + (n+1)^3 = \frac{n^2(n+1)^2}{4} + (n+1)^3 = \frac{n^2(n+1)^2 + 4(n+1)^3}{4} = \frac{(n+1)^2(n^2 + 4(n+1))}{4} = \frac{(n+1)(n+2)^2}{4}.
\]