Discrete Structures EPFL, Fall 2014

Solution to Problem Set 1

Date: 19.09.2014 Not graded

Problem 1.

a) True.

b) True.

c) True.

d) True.

e) False.

Problem 2.

a)

p	q	r	Value
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	T
T	F	F	T
T	F	T	F
T	T	F	T
T	T	T	T

b) If the three coins are fair, each line in the table is equally likely to occur. Out of the 8 possible cases, 4 make the proposition true and the other 4 make the proposition false. Thus, the probabilities of the proposition being true and false are both 50%.

Problem 3. There are many possible solutions. For example:

a) $(\neg p) \wedge (\neg q) \wedge (\neg r)$;

b) $\neg(p \vee q \vee r) \equiv (\neg p) \wedge (\neg q) \wedge (\neg r)$;

c) $(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r)$;

d) $(p \wedge \neg p) \wedge q \wedge r$.

Problem 4. Let $p =$ "system in multiuser state", $q =$ "system operating normally", $r =$ "kernel functioning", $s =$ "system in interrupt mode".

The propositions can then be rewritten as:

1. $p \iff q$,
2. $\neg r \to \neg q$,

3. $\neg r \wedge s$,

4. $\neg p \vee s$,

The set of propositions is indeed consistent. Suppose that the system is not in multiuser state, is not operating normally and is in the interrupt mode and suppose also that the kernel is not functioning. Then all propositions in the set are true. More formally, to fulfill 3 we need that s is true and r is false. Then, 4 is automatically true. To fulfill 2, we need that q is false, because r is false. To fulfill 1, we need that p and q have the same truth value.

Problem 5.

a) Let us set p and r to *false* and q to *true*. Then $(p \to q) \to r$ is false, since $p \to q$ is true and r is false. On the other hand $p \to (q \to r)$ is true because p is false. We just found a truth assignment for which the two propositions have different truth values, and so they are not equivalent!

b) Apply rules successively:

$$\begin{aligned}
& (p \to q) \wedge (\neg p \to (\neg q \to (p \wedge \neg p))) & \\
\equiv\ & (p \to q) \wedge (\neg p \to (\neg q \to \mathrm{F})) & \text{negation law} \\
\equiv\ & (\neg p \vee q) \wedge (\neg\neg p \vee (\neg\neg q \vee \mathrm{F})) & \text{definition of implication} \\
\equiv\ & (\neg p \vee q) \wedge (p \vee (q \vee \mathrm{F})) & \text{double negation law} \\
\equiv\ & (\neg p \vee q) \wedge (p \vee q) & \text{identity law} \\
\equiv\ & (\neg p \wedge p) \vee q & \text{distributivity law} \\
\equiv\ & \mathrm{F} \vee q & \text{negation law} \\
\equiv\ & q & \text{identity law}
\end{aligned}$$

Problem 6. If two or more people tell the truth, then we have a contradiction. Hence, there is at most one person that tells the truth. If all people are lying, then the last person is actually right: contradiction! If there is exactly one person that tells the truth, then 2013 statements are false. Thus, the 2013-th person tells the truth (and the remaining ones lie).

If we replace "exactly" by "at least", notice that if the $n+1$-th person tells the truth, so does the n-th person and, as a consequence, all the people that come before him/her. Suppose exactly the first m people tell the truth. Then, there are $2014 - m$ people that lie, which makes the first $2014 - m$ people tell the truth. Hence, $m = 1007$.

Problem 7. Let $p =$ "you tell false" and $q =$ "ruins on the left". The question to be answered is the XOR of p and q, i.e. $(p \vee q) \wedge \neg(p \wedge q)$. If the villager says yes (=true), the ruins are on the left, otherwise they are on the right, regardless of the fact that he says the truth or not.