Problem 1.
(a) 2
(b) 2 if \( n \) is even, 3 otherwise.
(c) \( \chi'(W_{n+1}) = n \) for \( n \geq 3 \).

Problem 2. Since the edges incident with any single vertex must be assigned different colors,
\[ \chi' \geq \Delta. \]
This is a property that holds in general for simple graphs.
Assume that \( \chi' = \Delta \). We say that color \( i \) is represented at vertex \( v \) if some edge incident with \( v \) has color \( i \). Then, every color is represented at every vertex. However, any set of edges of the same color gives a matching and hence covers an even number of vertices. With an odd number of vertices it is not possible that any color covers every vertex, so this contradicts \( \chi' = \Delta \). We conclude that \( \chi' \geq \Delta + 1 \).

Problem 3. Consider a coloring of the edges using the colors \( 1, 2, \ldots, q \) and let \( E_i \) denote the set of edges with color \( i \). Clearly, each of the \( E_i \)'s defines a matching. Then
\[ m = |E_1| + |E_2| + \ldots + |E_q| \leq q m^*. \]
Since there exists a coloring with \( \chi' \) distinct colors, the result follows.

Problem 4. Assume w.l.o.g. that \( m \geq n \), and therefore \( \Delta(K_{m,n}) = m \). As in Problem 2, we have that \( \chi' \geq \Delta \). Hence, if we exhibit an edge coloring which uses \( m \) colors and s.t. no two edges incident on the same vertex have the same color, we are done.
Let \( u_0, \ldots, u_{m-1} \) be the vertices on the left-hand side and \( v_0, \ldots, v_{n-1} \) the vertices on the right-hand side. Also, let \( c_0, \ldots, c_{m-1} \) be \( m \) distinct colors. In \( K_{m,n} \), every vertex on the left-hand side is connected to every vertex on the right-hand side. Let \( e_{i,j} \) be the edge connecting vertex \( u_i \) to vertex \( v_j \), for all \( 0 \leq i \leq m-1, 0 \leq j \leq n-1 \). Then color edge \( e_{i,j} \) by color \( c_{(i+j) \mod m} \).

We now need show that this coloring is correct, i.e., no vertex has any incident edges colored by the same color. First consider a vertex \( u_i \). The set of edges incident to \( u_i \) is \( e_{i,0}, \ldots, e_{i,n-1} \) and these edges are assigned colors \( c_{(i+0) \mod m}, \ldots, c_{(i+n-1) \mod m} \). Since \( n \leq m \), for \( 0 \leq x \leq n-1 \), it holds that the \( (i+x) \mod m \) correspond to \( n \) distinct elements of \( 0, \ldots, m-1 \). Therefore our coloring assigned different colors to each of the \( n \) edges. On the other hand, consider a vertex \( v_j \). The set of edges incident to \( v_j \) is \( e_{0,j}, \ldots, e_{m-1,j} \) and is assigned colors \( c_{(0+j) \mod m}, \ldots, c_{(m-1+j) \mod m} \). By the same argument, for \( 0 \leq x \leq m-1 \), we get that the \( (x+j) \mod m \) form a permutation of \( 0, \ldots, m-1 \) (in fact, they correspond to a cyclic shift of the latter set \( j \) positions to the left) and therefore the colors assigned to the \( m \) edges are distinct. We conclude that our edge coloring is valid.

Problem 5. Since \( G \) is 3-regular then it must have an even number of vertices. Suppose \( G \) is Hamiltonian, then any Hamiltonian cycle of \( G \) is even, so we can color its edges properly with 2 colors, say red and blue. Now each vertex is incident with 1 red edge, 1 blue edge and 1 uncolored edge. The uncolored
edges form a matching of $G$, so we can color all of them with the same color, say green. Thus, $\chi'(G) = 3$, which gives a contradiction.