Problem 1. What is the length of the maximum matching in the cycle graph on $n$ vertices? Can you give a closed form expression?

Problem 2. Show that the cube (defined in Problem Set 3) has a perfect matching.

Problem 3. Show that a tree cannot have two distinct perfect matchings. (Two matchings are distinct if there exists an edge that is contained in one matching but not the other.)

Problem 4. Two people play a game on a graph $G$ by alternately selecting distinct vertices $v_1, v_2, v_3, \ldots$ such that for $i > 0$, $v_i$ is adjacent to $v_{i-1}$. The last player who is able to select a vertex wins. If player 1 is the first to choose a vertex, show that $G$ has a perfect matching if and only if there is a winning strategy for player 2.